5

INTERCONNECTED SYSTEMS AND FEEDBACK

5.0 Introduction

Thus far we have used the words "circuit," "network," and "system" more or less interchangeably. But the connotations of these words are rather different. "Circuit" has perhaps the most clearly electrical overtones of the three—charged carriers circulating around closed loops. "Network" emphasizes the reticulated topological characteristics of the structure with little reflection of the dynamic implications of the constituent elements. "System," on the other hand, suggests that it will be productive to consider the composite *hierarchically*—as a systematic interconnection of subsystems and, at least potentially, as an element in a supersystem. The power and limitations of the hierarchical "systems approach" to complex structures is an important subtheme of this book; this is a good point to begin explaining why.

Physical science has been most effective in dealing with those phenomena that can be successfully *analyzed*—that is, resolved into constituents, taken apart, reduced to or understood as nothing but the interactions of their components. Indeed, this success (together with the possibility of summarizing the effects of past experiences by the present state of energy distribution throughout the system) essentially defines what we mean by a "physical system." And the inverse of this analysis process—*synthesizing* a complex structure through an appropriate interconnection of elements to realize some overall purpose—is the essence of technological design. In contrast, science and engineering have been much less successful in either understanding or manipulating social, economic, political, or biological systems, where the whole has characteristically a tendency to be greater than—or at least to appear different from—the sum of the parts (and where the past often seems to be more significantly reflected in the current structure of the system than in the distribution of energy or its equivalent within a time-invariant structure).

Another characteristic of physical systems is that the analytic and synthetic processes are usually most effective if carried out in stages rather than all at once. A television receiver, for example, is best understood at the highest level as a combination of amplifiers, mixers, oscillators, filters, gates, detectors, etc., each of which is composed of integrated circuits, transistors, resistors, capacitors, etc., which are in turn composed of various basic materials of appropriate sizes, shapes, and juxtapositions. To attempt the analysis of such a receiver in one step by solving Maxwell's equations to determine the electrodynamics of all the various shapes and substances making up the receiver would be both extremely foolish—human understanding seems to require the intermediate hierarchical levels—as well as extremely difficult. It is an intriguing fact that the actual numerical effort required to solve a large problem is usually greater if the problem is solved all at once than if it is first "torn" into a small number of subproblems that are solved separately with arbitrary boundary conditions and then interconnected.

At each level in a hierarchical analysis/synthesis process, we seek to combine functional descriptions of the subsystems and structural information as to how the subsystems are interrelated in order to derive a functional description of the larger system, which in turn can be combined with functional descriptions of other systems and structural information about their interrelations to derive a functional description of a still larger supersystem, and so on. Thus in the preceding chapters we have been studying how to combine functional descriptions of LTI electrical circuit elements (that is, constitutive relations such as Ohm's Law) and structural statements about the circuit topology (for example, as derived from Kirchhoff's Laws) to obtain an overall functional description of the circuit (for example, the system function). In this chapter we shall explore some of the properties of supersystems resulting from interconnecting LTI systems described by system functions—with particular emphasis on the simplest nontrivial interconnection, the *feedback system.*

As we move up a system hierarchy, the precise physical principles characterizing element behavior at the lowest levels usually become less and less important. We have based our discussion to this point primarily on electrical circuit elements, although it should be clear that mathematically similar input-output descriptions will apply to many other situations in which the underlying physics is mechanical, chemical , thermal, acoustical, hydrodynamic, etc., rather than electrical. Indeed, as we hinted in Chapter 1, analogies follow almost trivially for any of the energy-flow/work aspects of general thermodynamic systems. In addition, nonphysical models having the same or similar mathematical structure are widely used in the social and biological sciences. A common example is a macroeconomic model in which the input-output variables may be such attributes as tax and monetary policies, interest rates, unemployment and inflation indices, and the gross national product. Reflecting the increased scope of our discussion, we shall in the sequel de-emphasize electrical quantities such as voltages and currents through more regular use of non-connotative symbols such as $x(t)$ and $y(t)$ for inputs and outputs.

5.1 Elementary System Interconnections; Effects of Loading

The simplest possible system interconnections are the *cascade* (or *series*) and the *parallel* arrangements shown as block diagrams in Figure 5.1–1. If the subsystems are LTI systems characterized by their system functions, then the combinations in Figure 5.1–1 are equivalent (at least as far as input-output ZSR behavior is concerned) to a single LTI system with system function given by the product or sum of the component system functions, as shown. Note in particular that, since a product is commutative, the overall input-output system function is independent of the order in which LTI systems are cascaded. There is little that is new here—we have in fact been using block diagrams in this way without elaboration since the beginning of this book.

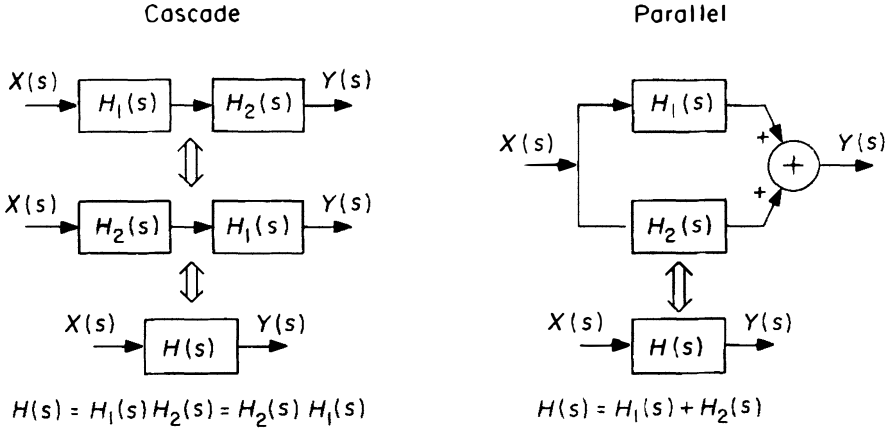

Figure 5.1–1. Cascade and parallel system connections.

It is important to point out, however, that although these formulas are undeniably correct interpretations of the intent of the block diagrams shown, they may not correctly describe similar interconnections of the corresponding circuits. Thus consider the two LTI 2-ports shown in Figure 5.1–2. Here we assume that $H_1(s)$ and $H_2(s)$ describe the open-circuit ($I_b(s) = I_d(s) = 0$) voltage-transfer ratios as indicated. If these 2-ports are directly cascaded as in Figure 5.1–3, we cannot conclude in general that the equivalent system function is $H(s) = H_1(s)H_2(s)$, since $I_b(s)$ is not necessarily equal to zero under the connected condition, and hence $V_b(s)/V_a(s)$ is not necessarily $H_1(s)$ as before. Moreover, the system function of the simple cascade will depend in general on the order in which the 2-ports are arranged.

If we want the system function of the cascaded 2-ports to be the product of the individual system functions and independent of the order—which may be extremely convenient from a design point of view—there are several things we can do. One is to insert a voltage follower as an *isolating amplifier* or *buffer*

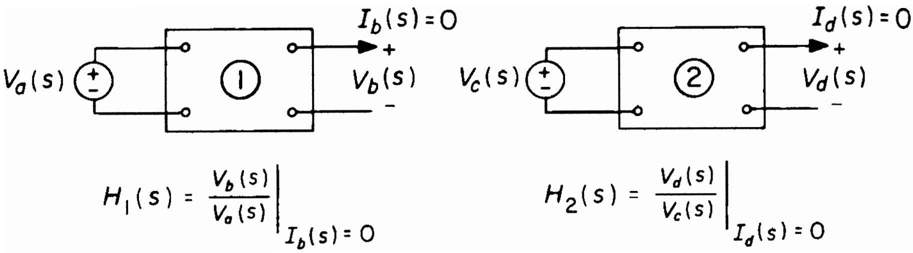

Figure 5.1–2. LTI 2-ports described by open-circuit voltage transfer ratios.

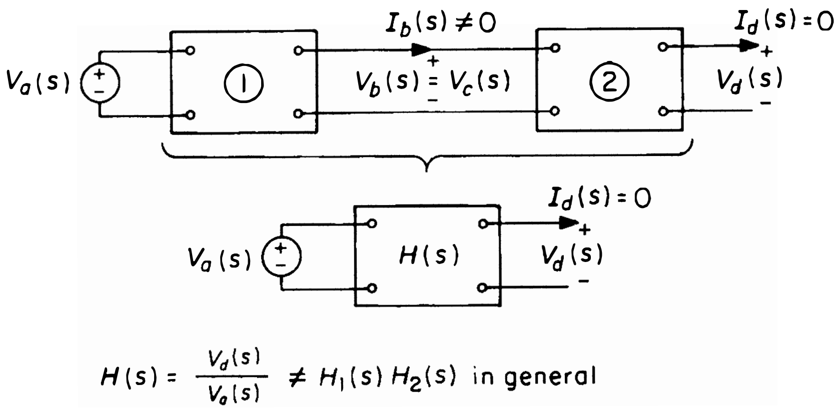

Figure 5.1–3. Cascade interconnection of 2-ports of Figure 5.1–2.

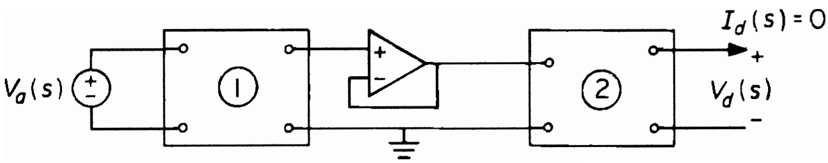

Figure 5.1–4. Use of isolating amplifier in a cascade connection of 2-ports.

between the two 2-ports, as shown in Figure 5.1–4. If this voltage follower is designed into or considered part of the second 2-port, then we are ensuring that the output current of the first 2-port is zero even when it is connected to the second 2-port. If the voltage follower is designed into or considered part of the first 2-port, then we are ensuring that the output voltage of the first 2-port is independent of the current drawn. Either way the result is $H(s) = H_1(s)H_2(s)$. And if we always insert such an isolating amplifier between 2-ports, or if both 2-ports have such voltage followers as input or output stages, then the result of cascading will be independent of the order. An example of the use of isolation in this way is provided by the cascading of Sallen-Key circuits to synthesize

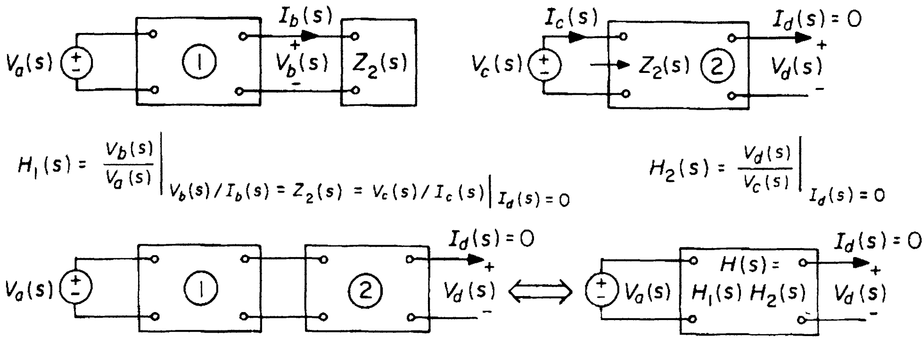

Figure 5.1–5. Redefinition of $H(s)$ to include the effect of loading.

pole-only systems, as discussed in Problem 4.9.*

Another way to make the system function of the cascade equal to the product of the system functions is to measure or calculate the system function $H_1(s)$ of the first 2-port under the condition that it be loaded by an impedance equal to the driving-point impedance of the second 2-port, rather than loaded by an open circuit. Then the system function of the cascade, as shown in Figure 5.1–5, will be $H(s) = H_1(s)H_2(s)$ as desired (although interchanging the order of cascading will not yield the same result unless $H_2(s)$ is similarly defined and appropriate load impedances are provided). Often, if a series of 2-ports are designed to be connected together in various combinations—examples are audio components such as amplifiers, mixers, attenuators, and microphones, and microwave waveguide or coaxial-cable components—it is convenient to design each 2-port so that it achieves its desired characteristics when driven by a Thévenin source and loaded by a resistance of a certain standard value such as 50 Ω or 300 Ω. If this is done, and if the output stage is always loaded by its specified impedance, then the effect of cascading is independent of the order of the components for LTI subsystems.

Cascade and parallel connections of subsystems to make larger systems are extremely important in science and engineering. Nevertheless the class of composite systems is much larger than simply those that can be built up out of successive applications of just these two operations. The simplest example of a more complex system is the *feedback loop* explored in the next section.

*In practice the order in which systems are cascaded is often extremely important—even if isolating amplifiers are employed—for reasons having to do with the extent to which our idealizations are valid. Thus if we cascade a large amplifier and a large attenuator, putting the amplifier first may overload the input stages of the attenuator, whereas putting the attenuator first may lead to such a low signal level at the input to the amplifier that incidental "pick-up," power-supply ripple, and thermal noise may become important. To escape this Scylla-Charybdis situation in, for example, long-distance telephone circuits (see Section 5.2), amplifiers are distributed every few kilometers, so that the signal level is never allowed to drop too low or rise too high. Of course, even in principle the effect of cascading is independent of order only if both systems are linear and time-invariant; the design of modulating and detecting systems is critically sensitive to this fact.

5.2 Simple Feedback Loops

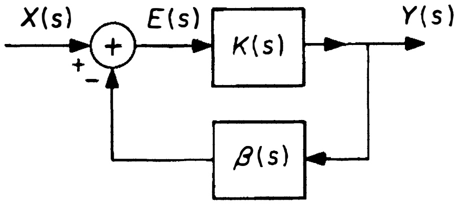

Figure 5.2–1. Simple feedback loop.

An LTI system composed of two LTI subsystems interconnected as shown in Figure 5.2–1 is called a simple *feedback loop.* To find the overall input-output system function $H(s) = Y(s)/X(s)$, write two equations

$$E(s) = X(s) - \beta(s)Y(s), \qquad Y(s) = K(s)E(s)$$

and eliminate the intermediate variable $E(s)$ to obtain

$$\boxed{H(s) = \frac{K(s)}{1+\beta(s)K(s)}.} \tag{5.2–1}$$

This is a sufficiently important formula to warrant engraving on your memory. Note that the plus sign in the denominator is a result of choosing the minus sign on the lower (feedback) input to the adder in the block diagram of Figure 5.2–1; if the feedback sign in the diagram had been plus, the denominator sign would have been minus.

There is little in the deceptively simple formula (5.2–1) to suggest the design magic that is hidden there. One additional key step is necessary. Suppose we choose the subsystems $K(s)$ and $\beta(s)$ so that the *loop gain* $K(s)\beta(s)$ has a magnitude much greater than 1. Then*

$$H(s) \approx \frac{K(s)}{\beta(s)K(s)} = \frac{1}{\beta(s)} \quad \text{if } |\beta(s)K(s)| \gg 1. \tag{5.2–2}$$

*Except in certain idealized situations, it is generally impossible to make $|\beta(s)K(s)| \gg 1$ for all complex s. Nevertheless the approximation $H(s) \approx 1/\beta(s)$ will still be valid and useful if the range of s for which $|\beta(s)K(s)| \gg 1$ includes all those s for which the $\mathcal{L}$-transform of the input $X(s)$ has a significant magnitude.

Two aspects of this approximate formula have significance for design:

a) If the loop gain is large, the overall system function is not dependent significantly on the properties of the feedforward path, $K(s)$;

b) If the loop gain is large, the overall system function is approximately equal to the reciprocal of the feedback-path system function, $\beta(s)$.

Historically, the usefulness of the second of these features was not widely recognized until the late 1930's when it became the basis for what is now called classical control theory, which we shall explore in the next chapter. However, the importance of the first feature—that feedback reduces the effect of fluctuations and distortions in the feedforward path—has been at least intuitively understood for a long time. Norbert Wiener, for example, identified the first conscious appreciation of the value of a closed-loop feedback system as appearing in a treatise on the fly-ball steam-engine speed governor published by James Clerk Maxwell in 1868.* But much the same idea, applied to the regulation of water-clocks, was described by Archimedes in the third century B.C.; no doubt the first "conscious appreciation" of the value of feedback is far older. Indeed the "inventor" of feedback, whoever he or she was, probably did not consider the "invention" as anything subtle—just a straightforward application of common sense.

The modern theory of feedback systems essentially begins with the work of H. S. Black and his associates (notably H. Nyquist and H. W. Bode) at the Bell Telephone Laboratories in the late 1920's. Black was working on the design of amplifiers for long-distance telephone lines.† The first transcontinental system (1914) used #8 (3 mm diameter) copper wire weighing about half a ton per mile. Even so, accumulated losses due to the resistance of 3000 miles of wire amounted to about 60 dB; three to six vacuum tube amplifiers were used to boost the signal amplitude. It was appreciated that if more amplifiers could be used, then the attentuation resulting from smaller wire would be acceptable, leading to potentially significant cost reductions. But the amplifiers of that day had limited bandwidth and introduced substantial nonlinear distortion; the compound effect of cascading more than a very small number of these amplifiers was intolerable. Black's challenge was to invent a better amplifier; his invention (1927)‡ of the *negative feedback amplifier* was so successful that by 1941 the first coaxial-cable system could use 600 cascaded amplifiers, each with a gain of 50 dB (that is, the cascaded cable losses amounted to a fantastic 30,000 dB!) and a bandwidth so much greater than the 1914 amplifiers that 480 telephone channels were available instead of just one.

In his study, Black distinguished two kinds of feedback—*degenerative* (or *negative*) feedback, in which the feedback signal actually reduces the input to the $K(s)$ block (in our notation, this means $\beta > 0$), and *regenerative* (or *positive*) feedback, in which the feedback signal increases the input ($\beta < 0$ or the sign on

*J. C. Maxwell, *Proc. Roy. Soc. (London)* (March 5, 1868).

†H. W. Bode, *Proc. Symp. Active Networks and Feedback Systems* (Polytechnic Institute of Brooklyn, NY: Polytechnic Press, 1960).

‡The first open publication was in *Electrical Engineering, 53* (Jan. 1934): 114–120.

the adder changed to +).* The advantageous effects of regenerative feedback—increasing the gain and (for $|\beta K| > 1$) producing useful oscillations—had been recognized long before Black, but degenerative feedback was generally thought to be deleterious, since obviously it reduced the overall gain.† Black, however, pointed out that if one sought a highly reliable overall system with behavior insensitive to distortions or changes in the always imperfect active elements in the K branch, then it was definitely desirable to design initially an amplifier with more gain than ultimately needed and to reduce the gain by negative (passive) feedback to the desired amount. His argument was essentially that already given: If $|\beta(s)K(s)| \gg 1$, then the overall system function is $H(s) \approx 1/\beta(s)$, independent of $K(s)$ and hence independent of many of the corruptions and limitations of $K(s)$. The best way to appreciate how this works in detail is to consider a number of examples.

5.3 Examples of the Effects of Negative Feedback

Example 5.3–1

Suppose it is desired to build an amplifier having a gain of 10 and capable of supplying some tens of watts to a load such as a loudspeaker. Such an amplifier could be built using a single-stage amplifier employing a power transistor and no feedback, or it could be built using a multistage amplifier employing the same power transistor in the output stage and using feedback to reduce the overall gain to 10. The two possibilities are described by the block diagrams in Figure 5.3–1, where to be specific we have assumed that the part of the multistage amplifier preceding the power stage has a gain of 100.

Now suppose that the gain of the power amplifier is reduced to half its initial value (as a result perhaps of aging of the active elements, or changes in loading, or temperature, or power supply voltages). The overall gain of the non-feedback amplifier is then also reduced by 50%, whereas that of the feedback amplifier has only been changed by 1%.

*The distinction between positive and negative feedback is clear enough if β and K are real constant multipliers (gains or attenuations) but is less clear if $\beta(s)$ and $K(s)$ are complex functions of s (as we shall usually assume).

†Black's patent application was delayed for more than nine years in part because the concept was so contrary to established beliefs that the Patent Office initially did not believe it would work. They treated the application "in the same manner as one for a perpetual motion machine." See H. S. Black, "Inventing the negative feedback amplifier," *IEEE Spectrum* (Dec. 1977): 54–60.

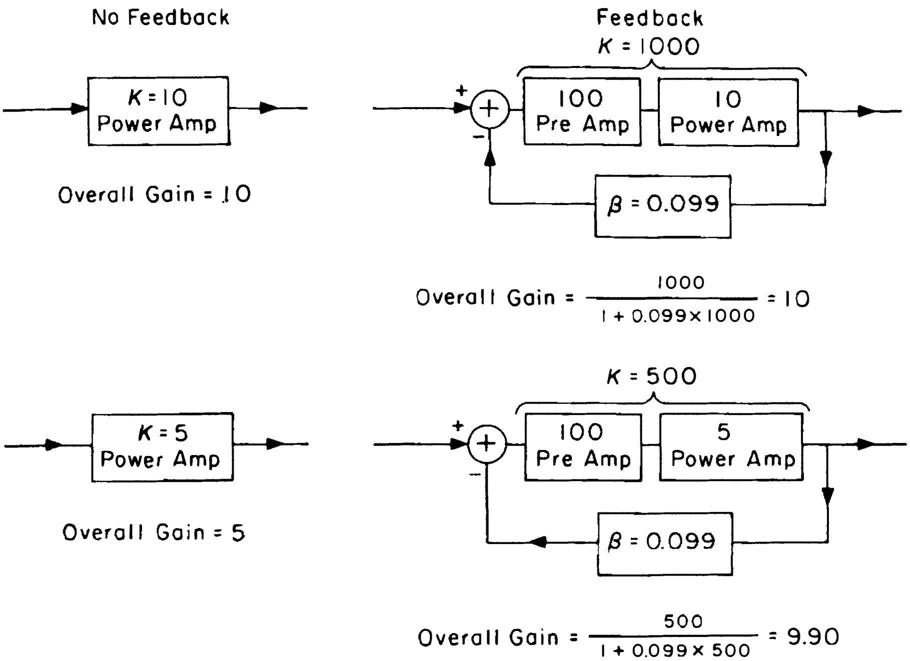

Figure 5.3–1. Gain change effects in a power amplifier with and without feedback.

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Bode* formalized the effect illustrated by Example 5.3–1 by defining a quantity called the *sensitivity*, S, of the amplifier as

$$S = \frac{\text{fractional change in gain of overall system}}{\text{fractional change in gain of active element}} = \frac{\dfrac{\Delta H}{H}}{\dfrac{\Delta K}{K}}. \tag{5.3–1}$$

For an amplifier without feedback, $S = 1$. On the other hand, for small changes the sensitivity of a feedback amplifier is

$$S \approx \frac{K}{H}\frac{\partial H}{\partial K} = \frac{\partial \ln H}{\partial \ln K} = \frac{1}{1+\beta K} \tag{5.3–2}$$

*H. W. Bode, *Network Analysis and Feedback Amplifier Design* (New York, NY: Van Nostrand, 1945) p. 52. Our definition is actually the reciprocal of Bode's.

which shows that the effect of negative feedback is to make the system less sensitive (than an unfedback system) by $1/(1+\beta K)$, or for large loop gain by approximately one over the loop gain. Correspondingly, the effect of regeneration is to increase the sensitivity, often leading for large regeneration to instability and oscillation.

Less formally, as we previously argued, for large loop gain ($|\beta K| \gg 1$) the overall gain is

$$H \approx \frac{1}{\beta} \tag{5.3–3}$$

independent of K and dependent only on β, which is usually determined by passive, linear, cheap, and reliable elements. In this same spirit, we can consider the adder in the diagram as a *comparator*—comparing the input x with βy (the inverse of the desired operation, operating on the output); any *error signal* is so heavily amplified that it must be very small. This, of course, is the approach we have been taking all along toward the analysis of ideal op-amp circuits. Our goal in this chapter is to abstract the general principles of feedback that previously we have illustrated only for specific cases.

Example 5.3–2

Real circuits rarely fit the simple feedback-loop model exactly; because of loading effects, the identification of β and K is usually neither easy nor unique. Fortunately, the difficulties become less when the loop gain βK is large—which, of course, is precisely the range of values for which feedback has a significant effect.

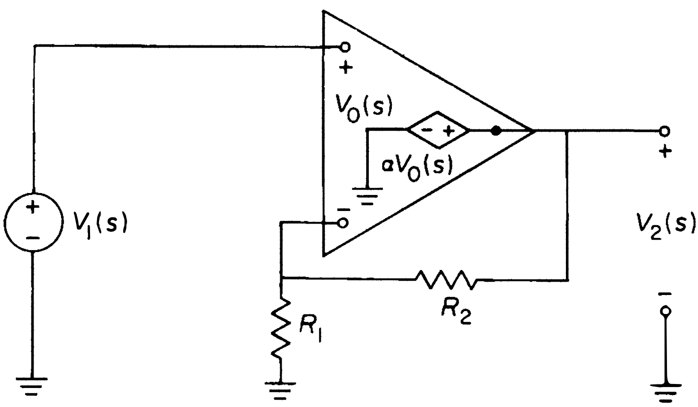

Figure 5.3–2. Non-inverting op-amp amplifier circuit.

a) The non-inverting op-amp amplifier shown in Figure 5.3–2 is a rare exception to this rule. If we model the op-amp as shown with infinite input impedance, zero output impedance, and finite gain α, it is apparent that

$$V_0(s) = V_1(s) - \frac{R_1}{R_1 + R_2} V_2(s), \quad V_2(s) = \alpha V_0(s).$$

These equations correspond exactly with the equations for the simple feedback loop if we identify $K = \alpha$ and $\beta = R_1/(R_1 + R_2)$, so that

$$\frac{V_2(s)}{V_1(s)} = \frac{K}{1+\beta K} = \frac{\alpha}{1 + \dfrac{\alpha R_1}{R_1 + R_2}} \approx \frac{R_1 + R_2}{R_1} = \frac{1}{\beta}$$

independent of α if the loop gain $\beta K = \dfrac{\alpha R_1}{R_1 + R_2} \gg 1$.

b) On the other hand the more common inverting op-amp amplifier circuit shown in Figure 5.3–3 illustrates the more usual situation. Using the same op-amp model as before, we may write

$$V_0(s) = \frac{-R_2}{R_1 + R_2} V_1(s) - \frac{R_1}{R_1 + R_2} V_2(s), \quad V_2(s) = \alpha V_0(s)$$

which are not precisely in the form of the simple feedback-loop equations. If we choose $K = \alpha$ and $\beta = R_1/(R_1 + R_2)$ as before, we are in effect describing the circuit by the block diagram in Figure 5.3–4.

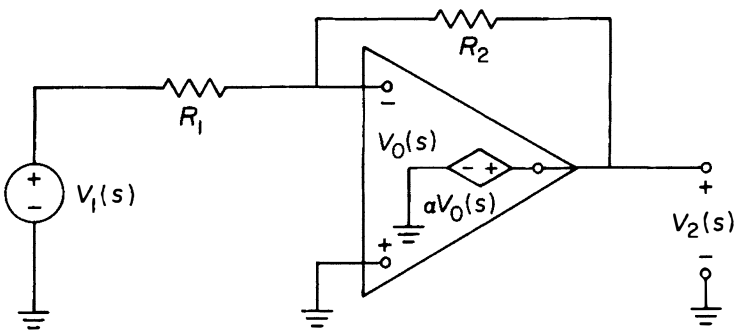

Figure 5.3–3. Inverting op-amp amplifier circuit.

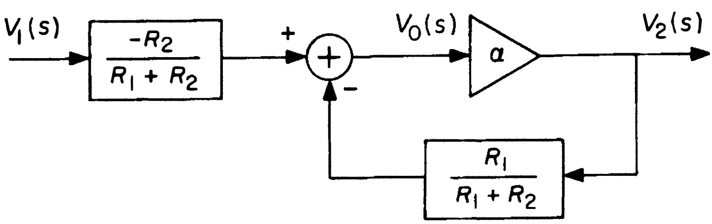

Figure 5.3–4. Block diagram for an inverting op-amp amplifier.

The overall gain is

$$\frac{V_2(s)}{V_1(s)} = -\frac{R_2}{R_1 + R_2} \frac{\alpha}{1 + \alpha \dfrac{R_1}{R_1 + R_2}} \approx -\frac{R_2}{R_1}$$

independent of α if the loop gain $\alpha R_1/(R_1 + R_2) \gg 1$. We note that this large-α gain is not $1/\beta$. Alternatively, we might choose $\beta = R_1/R_2$ and $K = \alpha R_2/(R_1 + R_2)$, which is equivalent to describing the circuit in terms of the block diagram in Figure 5.3–5. Except for a sign, this block diagram is identical to a simple feedback-loop block diagram and does reduce for large K to $-V_2(s)/V_1(s) = 1/\beta$. Obviously, a large number of other choices of β and K corresponding to still other block diagrams are possible; there is no uniquely "correct" or "best" choice. We note, however, that—independent of the choice—the loop gain βK is well-defined and can be computed by "opening" the loop at any convenient spot, applying a unit source, and calculating the signal that returns to the other side of the opening. (Care must be taken that loading effects, if any, are properly accounted for.)

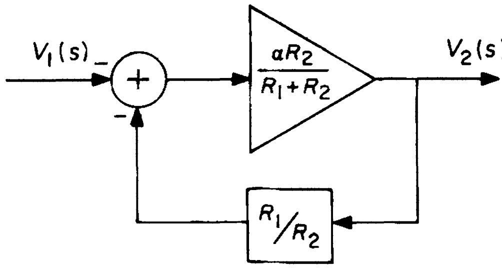

Figure 5.3–5. Another block diagram for an inverting op-amp amplifier.

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Perhaps the most important conclusion to be drawn from the ambiguity in identifying β and K illustrated in Example 5.3–2 is that feedback is clearly and undeniably a useful concept in the design or synthesis of a new system to achieve some desired performance. Whether it is useful in the analysis and understanding of some existing system depends on the particular case. If the system being analyzed was in fact consciously designed as a feedback system, then it will almost certainly be effective to analyze it in these terms. The feedback paths will usually then turn out to be structurally distinct (or nearly so), and simplifying assumptions will suggest themselves that will both reduce the analytical effort and enhance our understanding. But if the system being studied is merely "complicated" (for example, a biological system in which typically "everything influences everything else"), then the utility of feedback as a guide to analysis is more doubtful. For it is always possible to formulate any system analysis problem—even the voltage divider shown in Figure 5.3–6—in feedback terms, but there is certainly no guarantee that such a view will prove helpful. It is worth keeping in mind that, whatever the block diagram, the useful features of feedback to be described in this chapter usually seem to depend on the fact that the K block is an active element, capable of power gain.

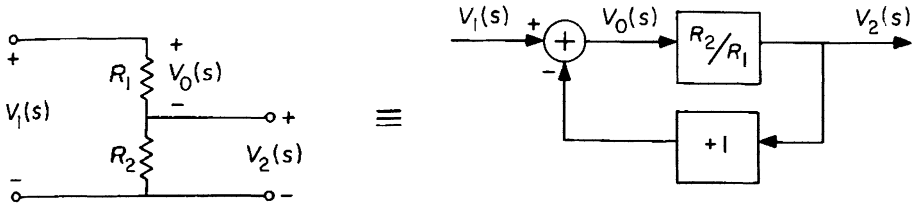

Figure 5.3–6. A voltage divider as a "feedback" system.

Example 5.3–3

To explore the way in which feedback reduces the distorting effects of non-linearities, consider an amplifier whose output $y(t)$ is a non-linear memory-less function of its input $z(t)$:

$$y(t) = f[z(t)].$$

Specifically, suppose $f[\cdot]$ has the shape shown by the solid line in Figure 5.3–7; this graph exhibits *saturation* of the output for large values of the input as well as *dead zone* or *crossover distortion* in that the output is zero until the magnitude of the input exceeds a threshold value. Such characteristics are common in op-amp output stages.

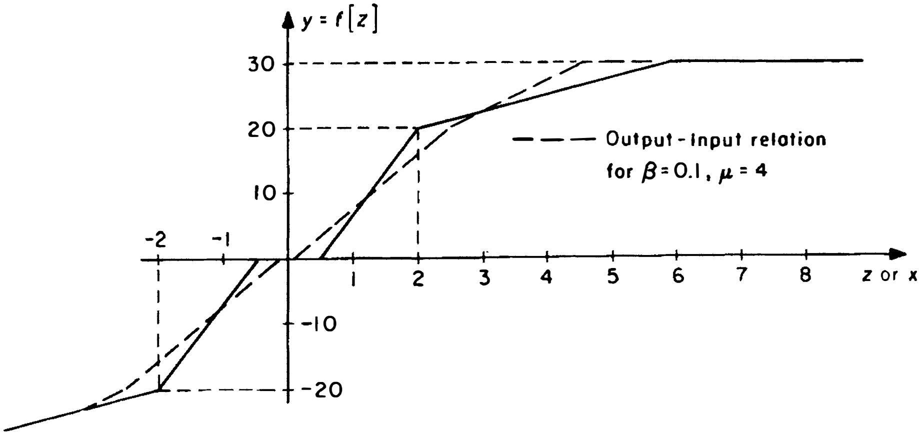

Figure 5.3–7. Non-linear input-output characteristic.

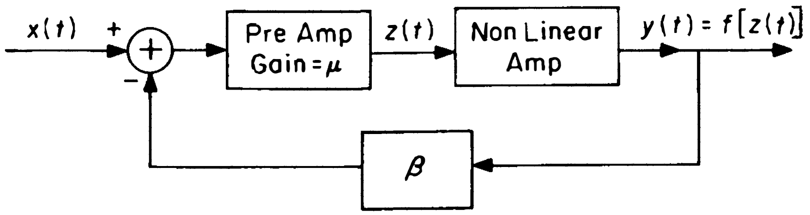

Figure 5.3–8. Feedback amplifier containing a non-linear output stage.

Suppose this amplifier is cascaded with a pre-amplifier whose input contains a fedback fraction of the non-linear amplifier output, as shown in Figure 5.3–8. We readily compute that

$$z(t) = \mu(x(t) - \beta y(t)) = f^{-1}[y(t)]$$

or

$$x(t) = \beta y(t) + \frac{1}{\mu} f^{-1}[y(t)]$$

where $f^{-1}[\ ]$ is the inverse amplifier function shown in Figure 5.3–9. For $y < 30$ and large enough μ, we get approximately

$$y(t) \approx \frac{1}{\beta} x(t)$$

which is a linear relationship. To explore the nature of the approximation, let $\beta = 0.1$ and $\mu = 4$; the output-input relationship is shown dashed in Figure 5.3–7 and is clearly—within the absolute limits imposed by output saturation—a substantial improvement in linearity over the unfedback amplifier.

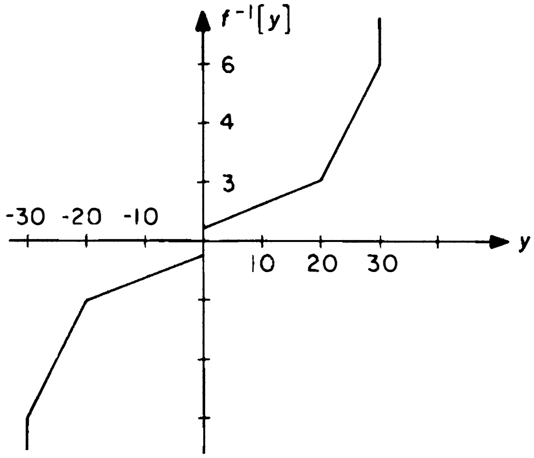

Figure 5.3–9. Inverse amplifier characteristic.

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Example 5.3–4

Consider a power amplifier that is nearly linear but produces a little distortion at full power output. By this we mean that we can describe such an amplifier approximately by the block diagram in Figure 5.3–10, in which $n(t)$ is the difference between the actual distorted output and the output of a linear amplifier with gain K.

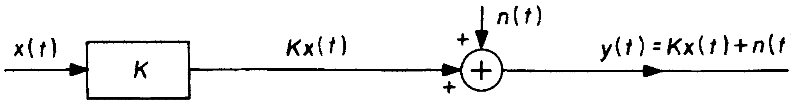


Figure 5.3–10. Block diagram modelling a nearly linear amplifier.

If $n(t)$ is a separate signal, independent of $x(t)$, then this block diagram describes a linear system, and we shall analyze it as if this were the case. But, of course, since the distortion in a non-linear system actually depends upon the input $x(t)$, it must be emphasized that the analysis scheme to be discussed is an acceptable approximation only if the output level is held fixed and if the distortion is small (so that, for example, if we feed back $n(t)$, the "distortion of the distortion" can be ignored).

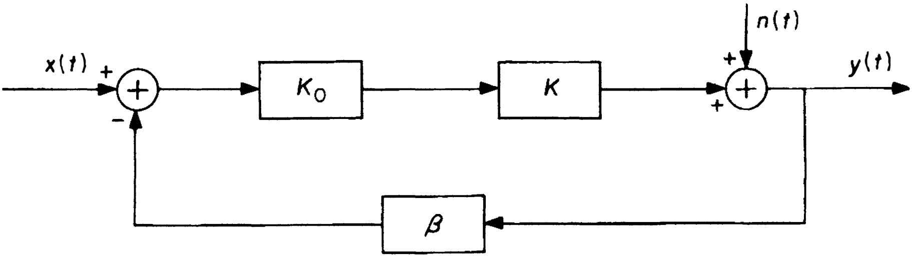


Figure 5.3–11. Feedback added to amplifier model of Figure 5.3–10.

Suppose now we feed back a fraction β of $y(t)$ and add additional gain to K to compensate, as shown in Figure 5.3–11. By superposition (the approximate system is linear), we readily find

$$y(t) = \frac{K_0 K}{1 + \beta K_0 K} x(t) + \frac{1}{1 + \beta K_0 K} n(t).$$

If we adjust K_0 and β so that $K_0/(1 + \beta K_0 K) = 1$, the system has the same gain as before; but for the same amplitude of the desired component of the output, the distortion has been reduced by the factor $1/(1 + \beta K_0 K)$, which can be considerable.
▶▶▶

Of course, the analysis in Example 5.3–4 applies without the approximation to any situation in which $n(t)$ really is an independent added disturbance or noise. It should be observed, however, that feedback can markedly reduce the effect of noise added to the output, but has no effect on noise added to the input and only an intermediate effect on noise added at an intermediate point (such as between K_0 and K in Figure 5.3–11). One typical application of this effect of feedback is in multistage amplifiers in which the d-c supply voltages are obtained by rectifying a-c. In such an amplifier, the voltages for the final stages do not need to be particularly well filtered if substantial feedback is employed.

Example 5.3–5

One of the most important uses of feedback is to reduce the effect on the K circuit of changes in some impedance. The Watt speed governor for steam engines analyzed by Maxwell can be considered a design of this sort, intended to reduce the effects on speed of changes in the mechanical load. A similar electrical example is the design of a voltage regulator (see Problem 5.1). Another example is the design of an amplifier to drive a loudspeaker in a high-fidelity sound reproduction system. Such an amplifier must cope with the fact that a loudspeaker has an input impedance that is a rather wild function of frequency. To achieve an overall flat frequency response, it is usually considered desirable to keep the voltage across the speaker terminals constant as a function of frequency (for constant input voltage to the amplifier)—for a permanent-magnet type speaker, voltage by Faraday's Law controls voice-coil velocity. Consequently, a high-fidelity amplifier should behave as an ideal voltage source.

The effective output impedance of an amplifier can be reduced by feeding back a signal proportional to the amplifier load voltage and comparing it with the amplifier input signal. Consider, for example, the non-inverting op-amp circuit of Example 5.3–2—reproduced in Figure 5.3–12—in which a fraction β of the output voltage is fed back. Assume $R \gg R_0$ for simplicity. The effective output impedance looking back from the terminals of R_L is the value of R_1 in the Thévenin equivalent to the feedback amplifier shown in Figure 5.3–13. In circuits containing controlled sources, the most effective way to compute the Thévenin resistance is usually

$$R_1 = \frac{V_2(s) \text{ when } R_L = \infty}{I_2(s) \text{ when } R_L = 0} = \frac{\text{open-circuit voltage}}{\text{short-circuit current}}.$$

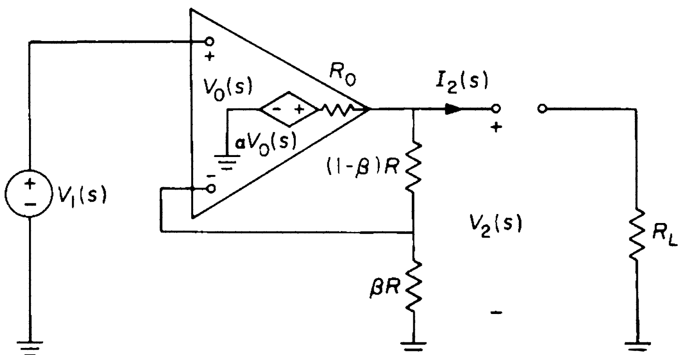


Figure 5.3–12. Non-inverting op-amp amplifier.

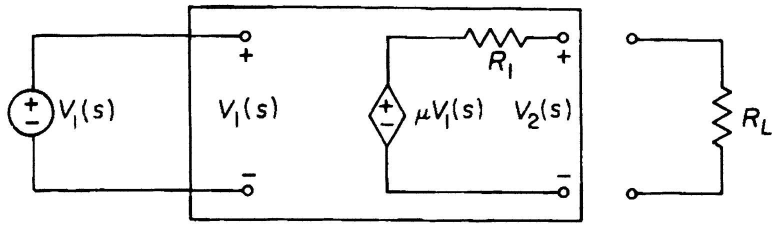


Figure 5.3–13. Thévenin equivalent to non-inverting op-amp amplifier.

The short-circuit current in the circuit of Figure 5.3–12 is easy to find since there is no feedback when the output terminals are shorted. Thus

$$I_2(s)\ \text{(short-circuit)} = \frac{\alpha V_1(s)}{R_0}.$$

Since $R \gg R_0$, on open circuit $V_2(s) \approx \alpha V_0(s)$ and $V_0(s) = V_1(s) - \beta V_2(s)$, so that $V_2(s)/\alpha \approx V_1(s) - \beta V_2(s)$. Solving, we find that

$$V_2(s)\ \text{(open-circuit)} \approx \frac{\alpha V_1(s)}{1+\beta\alpha}$$

and consequently

$$\mu = \frac{\alpha}{1+\beta\alpha} \approx \frac{1}{\beta}.$$

Hence, finally,

$$R_1 = \frac{\alpha V_1(s)}{1+\beta\alpha}\frac{R_0}{\alpha V_1(s)} = \frac{R_0}{1+\beta\alpha}$$

which, if $\beta\alpha \gg 1$ (as it usually is), represents a significant reduction from the unfedback amplifier. For example, a 741 op-amp in a voltage-follower circuit ($\beta = 1$) has $R_0 \approx 75$ Ω and $\alpha \approx 2 \times 10^5$. Then

$$R_1 = \frac{75}{1+2\times 10^5} \approx 3.75 \times 10^{-4}\Omega.$$

▶▶▶

It is perhaps worth observing that the overall gain and the output impedance are identical in theory for the feedback amplifier in Example 5.3–5 and for the unfedback amplifier with the output simply shunted by an appropriate resistor. However, most active elements used as the last stage in a power amplifier must work into an appropriate load impedance to achieve the desired level of power output without saturation or other distortion effects. Thus, in practice, shunting usually cannot be employed to reduce the effective output impedance of the amplifier, but feedback can.

Example 5.3–6

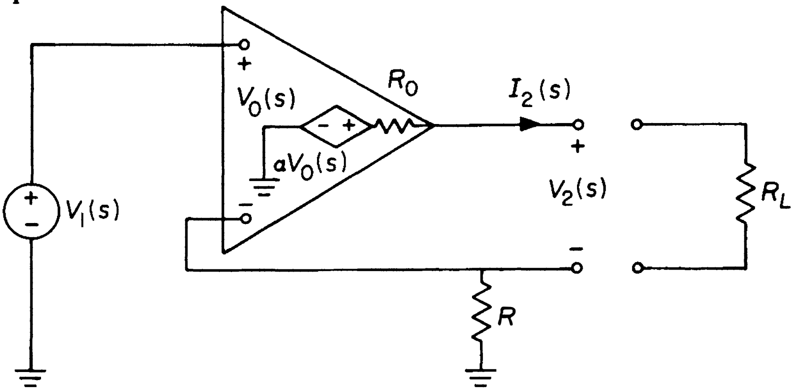


Figure 5.3–14. Amplifier with feedback proportional to output current.

A scheme similar to that of the preceding example can be used to increase the output resistance, that is, to make the amplifier behave more nearly like an ideal current source by feeding back a voltage proportional to the output current. Consider the circuit shown in Figure 5.3–14. On open circuit, there is no feedback. Thus

$$V_2(s) \text{ (open-circuit) } = \alpha V_1(s).$$

On short circuit, $I_2(s) = \alpha V_0(s)/(R_0 + R)$ and $V_0(s) = V_1(s) - RI_2(s)$. Eliminating $V_0(s)$ and solving, we obtain

$$I_2(s) \text{ (short-circuit) } = \frac{\alpha V_1(s)}{R_0 + (1+\alpha)R}.$$

Hence R_1 in the Thévenin equivalent circuit is

$$R_1 = \frac{V_2(s) \text{ (open-circuit)}}{I_2(s) \text{ (short-circuit)}} = \alpha V_1(s) \frac{R_0 + (1+\alpha)R}{\alpha V_1(s)} = R_0 + (1+\alpha)R$$

which can be quite large. This circuit is occasionally used to provide an approximation to an ideal current source, although it has the disadvantage of no common ground between input and output.

▶ ▶ ▶

Example 5.3–7

Feedback can also have a marked effect on the input impedance to various circuits. It is sometimes useful to consider the standard op-amp integrator circuit in Figure 5.3–15 from this point of view.

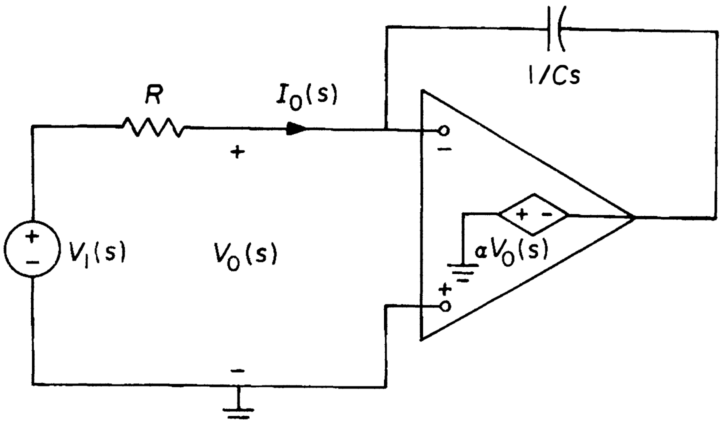


Figure 5.3–15. Integrator circuit.

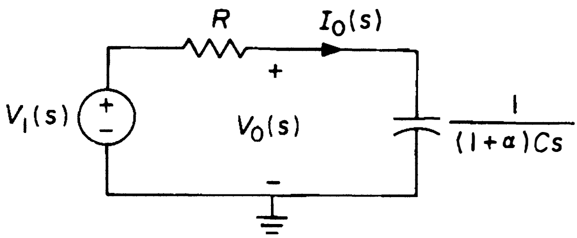


Figure 5.3–16. Equivalent input circuit for the integrator of Figure 5.3–15.

We readily compute that the voltage across the capacitor is $V_0(s)(1+\alpha)$, so that $I_0(s) = CsV_0(s)(1+\alpha)$. Thus from the standpoint of the driving source the op-amp-capacitor combination is equivalent to a capacitor of value $(1+\alpha)C$, as shown in the equivalent input circuit of Figure 5.3–16. The value of $(1+\alpha)C$ can be very large. This, of course, is what makes the circuit function as an approximation to an ideal integrator. But the same idea has other applications. The capacitor C might be simply a stray capacity coupling the output and input of an amplifier stage; the multiplying effect of the gain of the amplifier then creates a large effective input shunt capacitance that can significantly reduce the high-frequency gain of the preceding stage. This is called the *Miller effect* and historically was important in limiting the radio-frequency gain of early vacuum-tube amplifiers, until the invention of the pentode substantially reduced the effective value of C. (For a discussion of the analogous problem in transistors see Problem 5.5.) Another application depends on the fact that the gain of an electronic amplifier can often be changed electrically by adjusting the operating point. If a feedback capacitor is connected around such an amplifier as in Figure 5.3–15, then the combination yields a capacitance whose value can be changed electrically. Such an element is widely useful in electrically-tuned filters, FM modulators, etc.

▸▸▸

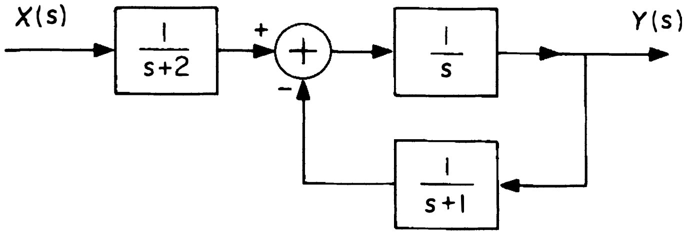
5.4 Summary

The analysis of systems composed of interconnected elements leads naturally to the next hierarchical level—the interconnection of systems to yield supersystems. The arrangement of two subsystems in a simple feedback loop turns out to be particularly interesting as a design tool. Indeed, we agree with J. K. Roberge that "a detailed understanding of feedback is the single most important prerequisite to successful electronic circuit and system design."* In this chapter we have studied the way in which feedback reduces the effects of variations and distortions in the feedforward path because the overall system function for large loop gain is not highly dependent on the characteristics of this path. In the next chapter we shall illustrate how we can use the fact that for large loop gain the overall system function is approximately equal to the reciprocal of the system function describing the feedback path.

*J. K. Roberge, *Operational Amplifiers: Theory and Practice* (New York, NY: John Wiley, 1975). This is an excellent text for further study of the topic of this chapter.

EXERCISES FOR CHAPTER 5

Exercise 5.1



Show that the system function of the block diagram above is

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s+1}{s^3+3s^2+3s+2}.$$

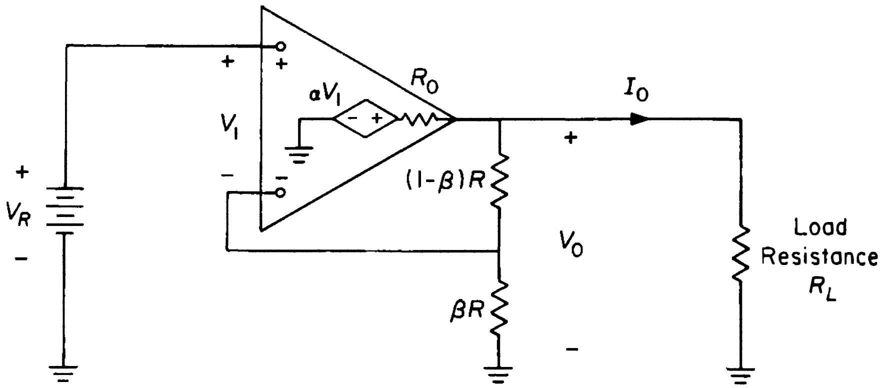
Exercise 5.2

In the non-inverting op-amp amplifier circuit of Example 5.3–2, suppose that R_1 and R_2 are chosen such that the ideal ($\alpha = \infty$) gain is 10. Show that α must be greater than 9.99×10^3 if the actual gain is to be within 0.1% of the ideal value.

PROBLEMS FOR CHAPTER 5

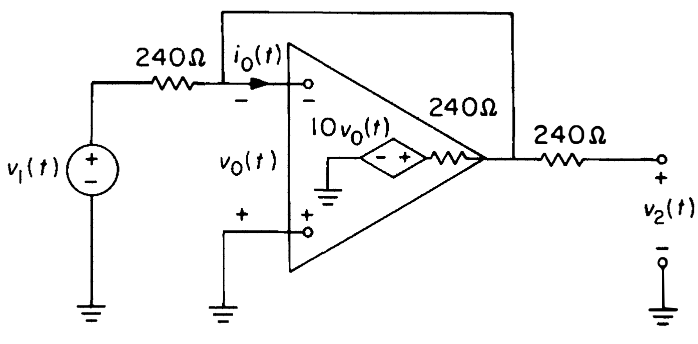
Problem 5.1

If the input to the non-inverting op-amp amplifier of Example 5.3–5 is a constant (e.g., a battery or the voltage across a Zener diode), then the circuit shown below becomes a series *voltage regulator* designed to maintain a fixed output voltage $V_0 \approx V_R/\beta$ independent of load resistance (provided that the output current I_0 is less than some maximum determined by the amplifier). Integrated-circuit voltage regulators such as the LM317 often work on this principle and combine all the relevant elements on a single chip. Sketch the regulation characteristic V_0 vs. I_0 as R_L is varied, and show that the desired properties are achieved as $\alpha \to \infty$. Assume for simplicity that $R \gg R_0$.



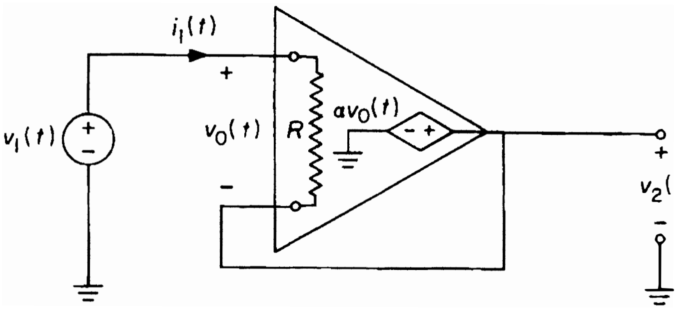
Problem 5.2

Find the Thévenin equivalent circuit at the output of the following feedback amplifier. Assume $i_0(t) = 0$.



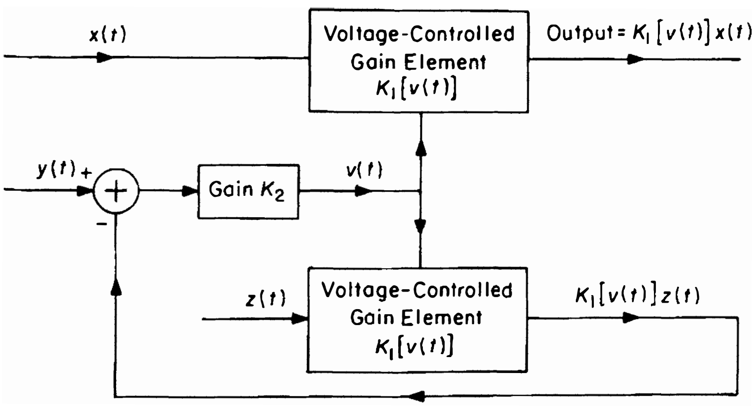
Problem 5.3

Find the input resistance $v_1(t)/i_1(t)$ of the following voltage-follower circuit. Model the op-amp as suggested in the figure.



Problem 5.4

The design of satisfactory analog multipliers is a perennial problem. One useful scheme, shown below, employs two identical voltage-controlled amplifiers or attenuators whose gain is (ideally) an instantaneous monotonic (but not necessarily linear) function of the control voltage $v(t)$.

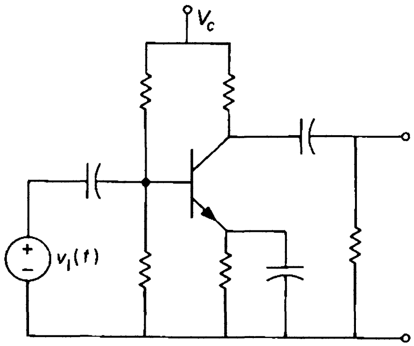


a) Argue that for a sufficiently large value of K_2 the output is approximately equal to $x(t)y(t)/z(t)$ independent of the shape of $K_1[\ ]$.

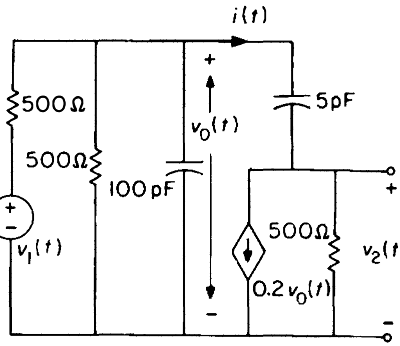
b) Let $z(t) = 1$, $K_1[v(t)] = 1 + v(t)$, and assume that $x(t)$ and $y(t)$ are restricted to the range $0 \leq x(t) \leq 10$, $0 \leq y(t) \leq 10$. Find the minimum value of K_2 such that the output error is never greater than 1% of the maximum value of the product $x(t)y(t)$.

Problem 5.5

This problem explores the effect of collector-to-base capacitance on the frequency response of the common-emitter amplifier stage shown in the figure. Assuming that the npn transistor can be represented near its operating point by a hybrid-π model, the overall equivalent circuit at middle and higher frequencies might appear approximately as shown on the right.



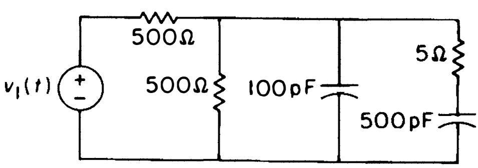
Circuit Diagram
Capacitors shown are low-frequency blocking and bypass capacitors



Equivalent Circuit
Blocking and bypass capacitors have been replaced by short circuits

The two capacitors describe various charge-storage effects in the base region. In particular the 5 pF capacitor represents primarily the capacitance of the back-biased collector-base junction; although its value is small, its effect is amplified because of its location in the feedback path. As mentioned in Example 5.3–7, this is called the *Miller effect.*

a) Calculate the input impedance looking to the right at the point where the current is labelled $i(t)$ and thus show that the impedance presented to the source $v_1(t)$ can be represented by the following equivalent circuit.



b) Determine approximately the half-power frequency of this amplifier stage and compare with the half-power frequency if the collector-base junction capacitance had been zero. The effective multiplication of the value of this capacitance by the gain of the stage can thus be a serious problem.