9/25/23 6.3100 1L

Summary

1st Order:

D.E. $y[n] = \lambda y[n-1] + \gamma u[n-1]$

Plug & Chug Soln $y[n] = \lambda^n \underbrace{y[0]}_{\text{Known}} + \sum_{m=0}^{n-1} \lambda^{(n-1)-m} \gamma \underbrace{u[m]}_{\text{Known}}$

→ Nat Freq: λ

Zero Input Response

2nd Order No-input

$$y[n] = a_{-1} y[n-1] + a_{-2} y[n-2]$$

Guess $y[n] = c\lambda^n$

$\lambda_1, \lambda_2 \in \text{Roots}(\lambda^2 - a_{-1}\lambda - a_{-2} = 0)$ $y[n] = c_1\lambda_1^n + c_2\lambda_2^n$

→ Nat Freqs: λ_1, λ_2

State Space

$$\#\text{states}\updownarrow \overset{\xleftrightarrow{1}}{\begin{bmatrix} \vec{x}[n] \end{bmatrix}} = \updownarrow \overset{\xleftrightarrow{\#\text{ states}}}{\begin{bmatrix} A \end{bmatrix}} \begin{bmatrix} \vec{x}[n-1] \end{bmatrix} + \updownarrow \overset{\xleftrightarrow{1}}{\begin{bmatrix} B \end{bmatrix}} u[n-1]$$

$\#\text{states} = N_s$

Like 1st Order Soln $\vec{x}[n] = A^n \vec{x}[0] + \sum_{m=0}^{n-1} A^{((n-1)-m)} B u[m]$

Like 2nd Order Guess $\begin{bmatrix} \vec{x}[n] \end{bmatrix} = c\lambda^n \begin{bmatrix} \vec{s} \end{bmatrix}$

2L

Guess?

Plug in $\vec{x}[n] = c\lambda^n \vec{s}$

ZIR $c\lambda^n \begin{bmatrix}\uparrow \\ s \\ \downarrow\end{bmatrix} = A\, c\lambda^{n-1} \begin{bmatrix}\uparrow \\ s \\ \downarrow\end{bmatrix}$

$$\Rightarrow \lambda \begin{bmatrix}\uparrow \\ s \\ \downarrow\end{bmatrix} = A \begin{bmatrix}\uparrow \\ s \\ \downarrow\end{bmatrix}$$

eigenvalue

eigenvector

True for almost all A's

Assume $\underset{\#\text{states}}{}\overset{\#\text{states}}{\begin{bmatrix} & A & \end{bmatrix}}$ has # states unique evals $\lambda_i \neq \lambda_j \; i \neq j$

evectors

Then $\vec{x}[n] = c_1 \lambda_1^n \begin{bmatrix}\uparrow \\ s_1 \\ \downarrow\end{bmatrix} + \dots + c_{N_s} \lambda_{N_s}^n \begin{bmatrix}\uparrow \\ s_{N_s} \\ \downarrow\end{bmatrix}$

evals = Nat. Freqs

N_s = # states

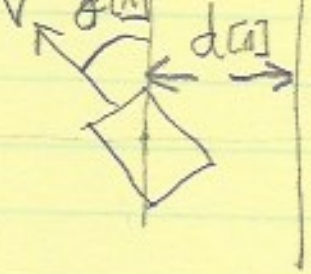
$[n]$ = D.T index

If $|\lambda_i| < 1 \;\; \forall \; i \in \{1, \dots N_s\}$

Then $\lim_{n \to \infty} \vec{x}[n] = 0$ (ZIR case)

①R

Wall Follower



θ small, $\sin\theta \cong \theta$

$$d[n] \cong d[n-1] + \Delta T\, V\theta[n-1]$$

$$\theta[n] = \theta[n-1] + \Delta T \underbrace{\omega[n-1]}_{\text{input}}$$

$$\begin{bmatrix} d[n] \\ \theta[n] \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & \Delta T V \\ 0 & 1 \end{bmatrix}}_{A} \begin{bmatrix} d[n-1] \\ \theta[n-1] \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \Delta T \end{bmatrix}}_{B} \omega[n]$$

If $\omega[n] = K_p\left(d_d[n] - d[n]\right)$

$$\begin{bmatrix} d[n] \\ \theta[n] \end{bmatrix} = \begin{bmatrix} 1 & \Delta T V \\ 0 & 1 \end{bmatrix} \begin{bmatrix} d[n-1] \\ \theta[n-1] \end{bmatrix} + \begin{bmatrix} 0 \\ \Delta T \end{bmatrix} K_p\left(d_d[n] - d[n]\right)$$

$$\begin{bmatrix} d[n] \\ \theta[n] \end{bmatrix} = \underset{\hat{A}}{\begin{bmatrix} 1 & \Delta T V \\ -\Delta T K_p & 1 \end{bmatrix}} \begin{bmatrix} d[n-1] \\ \theta[n-1] \end{bmatrix} + \begin{bmatrix} 0 \\ \Delta T K_p \end{bmatrix} d_d[n-1]$$

Calculate Eigs of $\hat{A}$

$$\lambda_i \vec{s}_i = \hat{A}\vec{s}_i \Rightarrow \underbrace{(\lambda_i I - A)}_{\text{singular!}}[\vec{s}_i] = 0$$

if $(\lambda I - A)$ sing $\Rightarrow \det(\lambda I - A) = 0$

$$\det\left(\begin{bmatrix} \lambda-1 & -\Delta T V \\ \Delta T K_p & \lambda-1 \end{bmatrix}\right) = 0$$

$$(\lambda-1)^2 + (\Delta T)^2 V K_p = 0$$

$$\underbrace{\lambda^2 - 2\lambda + \left(1 + (\Delta T)^2 V K_p\right) = 0}$$

Sameasequation for nat. freqs

<u>nat freqs</u> $\equiv$ <u>eigenvalues</u>

<u>Suppose</u>

$$\omega[n] = K_p(d_d[n] - d[n]) - K_\theta \theta[n]$$

$$\begin{bmatrix} d[n] \\ \theta[n] \end{bmatrix} = \begin{bmatrix} 1 & \Delta T V \\ & 1 \end{bmatrix}\begin{bmatrix} d[n-1] \\ \theta[n-1] \end{bmatrix} + \begin{bmatrix} 0 \\ \Delta T \end{bmatrix} \omega[n-1]$$

$$= \begin{bmatrix} 1 & \Delta T V \\ -\Delta T K_p & 1-\Delta T K_\theta \end{bmatrix}\begin{bmatrix} d[n-1] \\ \theta[n-1] \end{bmatrix} + \begin{bmatrix} 0 \\ K_p \Delta T \end{bmatrix} d_d[n-1]$$

$$\boxed{3R}$$

$$\text{evals}\left(\begin{bmatrix}1 & \Delta TV \\ -\Delta TK_p & 1-\Delta TK_\sigma\end{bmatrix}\right)$$

$$\det\left(\begin{bmatrix}\lambda-1 & -\Delta TV \\ \Delta TK_p & \lambda-(1-\Delta TK_\sigma)\end{bmatrix}\right)=0$$

$$(\lambda-1)\left(\lambda-(1-\Delta TK_\sigma)\right)+(\Delta T)^2VK_p=0$$

$$\lambda^2-(2-\Delta TK_\sigma)\lambda+\left((1-\Delta TK_\sigma)+(\Delta T)^2VK_p\right)=0$$

$$\lambda_1,\lambda_2\quad\left(1-\frac{\Delta TK_\sigma}{2}\right)\pm\sqrt{\frac{(2-\Delta TK_\sigma)^2}{4}-\left(1-\Delta TK_\sigma+(\Delta T)^2VK_p\right)}$$

$$\left(1-\frac{\Delta TK_\sigma}{2}\right)\pm\sqrt{\frac{4-4\Delta TK_\sigma+\Delta T^2K_\sigma^2}{4}-\left(1-\Delta TK_\sigma+(\Delta T)^2VK_p\right)}$$

$$\left(1-\frac{\Delta TK_\sigma}{2}\right)\pm\Delta T\sqrt{\frac{\Delta\cdot K_\sigma^2}{4}-VK_p}$$

$V=1\quad K_p=0\rightarrow$

$\Delta T=0.1\quad K_\sigma=2$

$\underline{K_\sigma=0}.$

$$1\pm j\Delta T\sqrt{VK_p}$$