In D.T.

\[ x[n] = a(k) x[n-1] + b(k) y_d[n] \]

controller dependent

\[
\text{eig}(A(k)) = \text{nat freqs}
\]

Plotter vs \( K_p \) or \( K_d \)

\[
\min_{k=1}^{\text{max}(x_i)}
\]

small may \( \rightarrow \) faster

In C.T.

\[
I = \frac{K(s) H(s)}{1 + K(s) H(s)} Y_d
\]

\[
G(s) = \frac{\text{num poly}}{\text{den poly}}
\]

\[
\text{real}(\text{roots}(d(s))) < 0
\]

\[
\text{not freqs}
\]

more \( \implies \text{faster} \)
Good Control: \( \text{dist}(\omega) = \left| \frac{1}{1 + K(\omega)H(\omega)} \right| \approx 0 \)

make \( G(\omega) = \left| \frac{2K(\omega)H(\omega)}{1 + K(\omega)H(\omega)} \right| \approx 1 \)

Good Tracking: \( K(\omega)H(\omega) \gg 1 \) for as many \( \omega \) as possible

\( K(\omega)H(\omega) \) never near \(-1\)

OR when \( |K(\omega)H(\omega)| = 1 \) \( \angle K(\omega)H(\omega) > -\infty \)

Poles & Phase Margin, Not Directly Quantity of Interest!

What do we want to optimize when designing a controller?
Phase Margin, Pole location, etc are indirect measures of tracking error.

**Direct Tracking Metric?**

**Example Closed-Loop Step Response**

1. \[ \text{max} \left( \text{Re}(s), \infty \right) \left| Y_d(s) - Y(s) \right| \]
   - Bad Choice: Same value for Case A, B, G, D, E, and smaller for Case F

2. \[ \int_0^\infty (Y_d(t) - Y(t)) \, dt \]
   - Integral of error
   - Bad Choice: Value is small for case C; because positive and negative areas cancel
   - But: Is infinite in case F because of steady-state error!
3) \[ \int_{0}^{\infty} (y_d(t) - y(t))^2 \, dt \]

**Good Choice:**
- Small for A, B, DD
- Large for SE
- Infinite for F!

Differentiable: (so better than
\[ \int |y_d(t) - y(t)|^2 \, dt \]

**But:** can still be small when there is overshoot (Case DD)

If we like the squared error metric:
\[ \int_{0}^{\infty} (y_d(t) - y(t))^2 \, dt = \text{Integral of the squared error} \]

**Can we design a controller?** (\(K_0\))
that minimizes the integral of the squared error?

Not by hand, but using computational tools, yes.
Need a Standard Form

System Description in Some Standard Form → Optimize (Numerical Procedure) → Control

- Parameters for the Metric

Standard Form

State-Space Descriptor System

Metric Parameterization

Weights on squared error and squared input

Controller

State feedback and input weights

Outline For Next Few Weeks

- Single input Single Output (SISO)
  - State-Space Systems and with Measured State Feedback
    - Feedback gains using pole placement
    - Feedback gains by Metric Minimization (LQR)
  - Win, Place LQR LAB
  - Feedback Gains by Metric Minimization (LQR)

- State Estimation Using observers, Switch to D.T.
  - Output measured
  - State estimated
  - Gains computed

Learn and Observe LAB