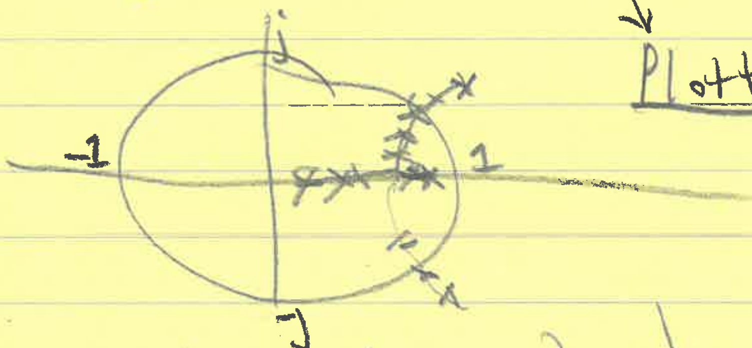
6.3100   11/06/23

In D.T.

$$\vec{x}[n] = A(k)\,\vec{x}[n-1] + B(k)\,y_d[n]$$

controller dependent

$\mathrm{eig}(A(k))$ = nat freqs → Plotted vs ~~K~~ k_p or k_d



$$\min_{K}\left(\max\left(|\lambda_i|\right)\right)$$

small may → faster

In C.T.

$$Y = \underbrace{\frac{K(s)H(s)}{1+K(s)H(s)}}_{G(s) = \frac{n(s)}{d(s)}} Y_d$$

$n(s)$ ← num poly

$d(s)$ ← den poly

$$\mathrm{real}\big(\underbrace{\mathrm{roots}(d(s))}_{\text{nat freqs}}\big) < 0$$

more neg → faster

2

Good Control: make
$$G_{dist}(j\omega) = \left|\frac{1}{1+K(j\omega)H(j\omega)}\right| \simeq 0$$
$$G(j\omega) = \left|\frac{K(j\omega)H(j\omega)}{1+K(j\omega)H(j\omega)}\right| \simeq 1$$

Good Tracking
$$K(j\omega)H(j\omega) >> 1$$ for as many ω's as possible

$K(j\omega)H(j\omega)$ never near -1

OR when $|K(j\omega)H(j\omega)| = 1$ $\angle K(j\omega)H(j\omega) > -180°$

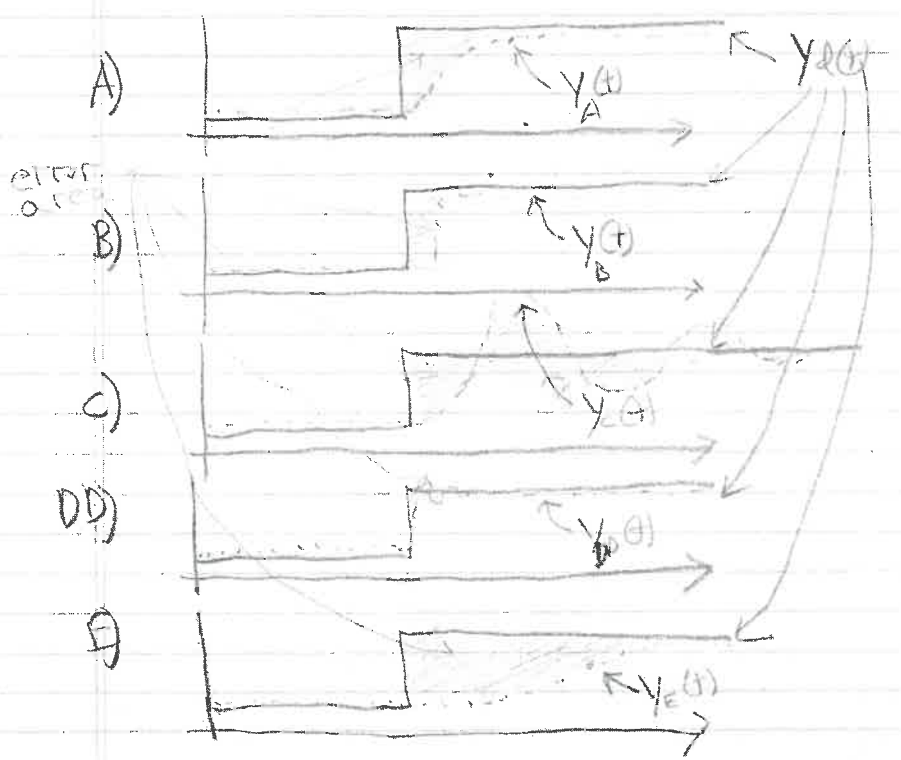
Poles & Phase Margin Not Directly quantity of interest!

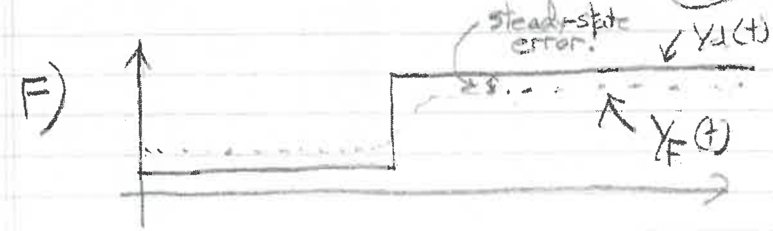
What do we want to optimize when designing a controller?

Phase Margin, Pole location, etc are <u>indirect</u> measures of tracking error.

Direct Tracking Metric?

Example Closed-Loop Step Response





Candidate Metrics Controller Should Minimize

1) $\max_{\tau \in [0,\infty)} \left| y_d(\tau) - y(\tau) \right|$

<u>Bad Choice</u>: Same value for Case A, B, C, D, E, and smaller for Case F

2) $\int_0^\infty \left(y_d(\tau) - y(\tau) \right) d\tau$ Integral of error

<u>Bad Choice</u>: Value is small for case "C", because positive and negative areas cancel

<u>But</u>: Is infinite in case F, because of steady-state error!

(5)

3) $$\int_0^\infty (y_d(\tau) - y(\tau))^2 \, d\tau$$

Good Choice:

Small for A, B, DD
Large for C, E
Infinite for F!

Differentiable: (so better than $\int |y_d(\tau) - y(\tau)| \, d\tau$)

But: Can still be small when there is overshoot (Case DD)

(6)

If we like the integral of the squared error metric

$$\int_0^\infty (y_d(\tau) - y(\tau))^2 \, d\tau = \text{integral of squared error}$$

Can we design a controller, $K(s)$, that minimizes the integral of the squared error?

Not by hand, but using computational tools, yes.

Need a ~~form~~ Standard Form

System Description in some standard form → Optimizer (Numerical procedure) → Gains

parameters for the metric → Optimizer

Standard form

State-Space Descriptor System

Metric Parameterization

Weights on squared error
and
Squared input (make clear later)

Controller

State feedback and input weights

Outline For Next Few Weeks

Single input Single Output (SISO) State-Space Systems with Measured State Feedback

Complete physical model, all states measured

Feedback gains using pole placement.

Win, Place LQR LAB

Feedback Gains by metric minimization (LQR)

Learn and Observe LAB

State Estimation using observers, Switch to D.T.

↓

Output measured, Model estimated, State estimated, Gains computed