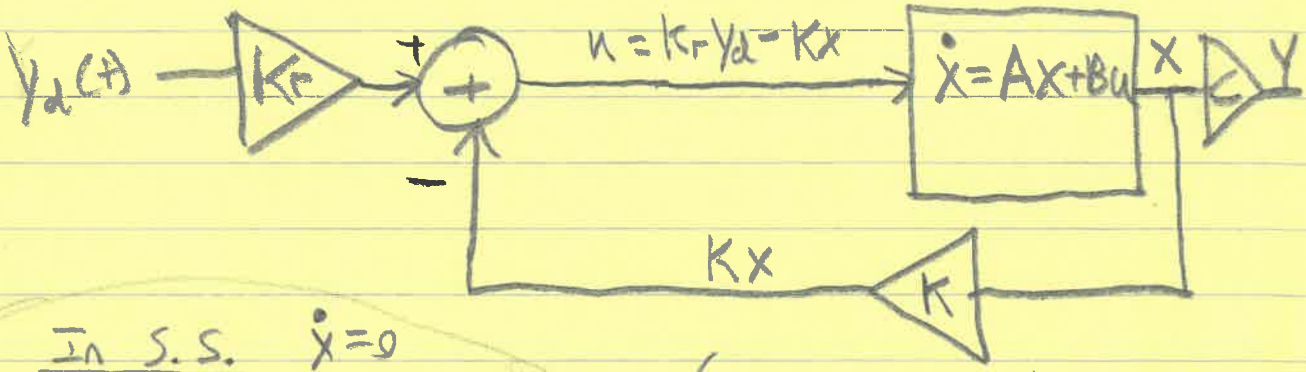
11/13/23 6.310

Steady-State Match



In S.S. $\dot{x} = 0$

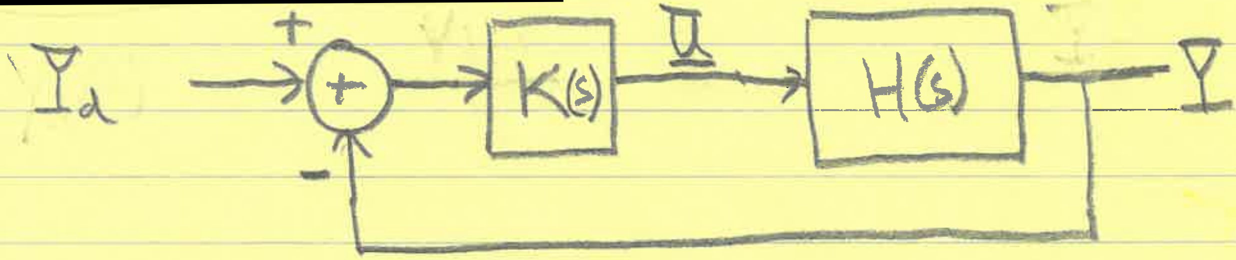
$$-(A-BK)x = BK_r y_d$$

$$x = -(A-BK)^{-1}BK_r y_d$$

$$y = Cx = -C(A-BK)^{-1}BK_r y_d$$

$$y = y_d \text{ if } K_r = \left(-C(A-BK)^{-1}B\right)^{-1}$$

$$\left\{\begin{aligned} \dot{x} &= Ax + B(K_r y_d - Kx) \\ &= (A-BK)x + BK_r y_d \\ y &= Cx \end{aligned}\right. \qquad K_r = -1/C(A-BK)^{-1}B$$



Compare to Transfer Function Approach

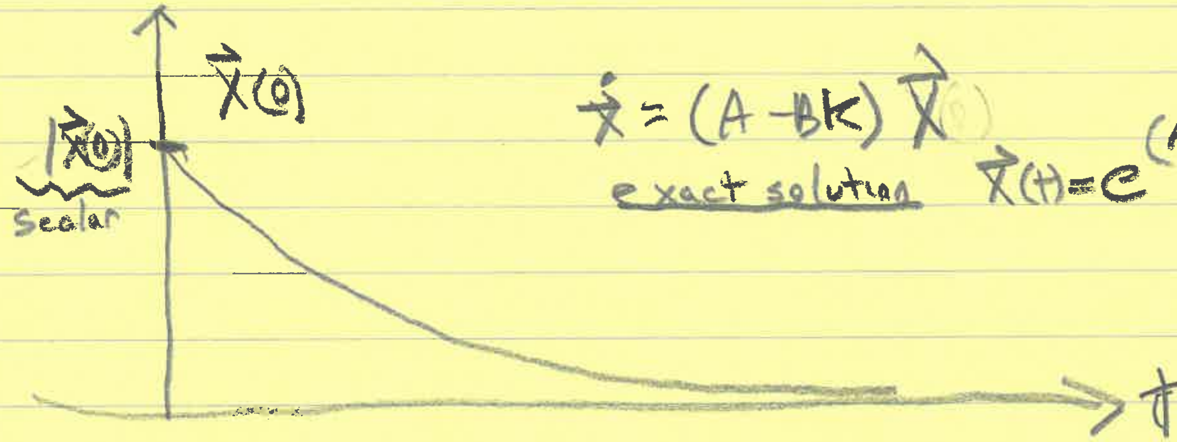
$$Y = \underbrace{\frac{K(s)H(s)}{1+K(s)H(s)}}_{G(s) = \frac{n(s)}{d(s)}} Y_d$$

roots are poles = nat freqs

In S.S.

$$Y_{ss} = \left.\frac{K(j\omega)H(j\omega)}{1+K(j\omega)H(j\omega)}\right|_{j\omega=0} Y_d = \underbrace{\frac{K(0)H(0)}{1+K(0)H(0)}}_{\approx 1 \text{ if } K(0)H(0) \gg 1} Y_d$$

Consider Zero-Input Case



$\dot{\vec{x}} = (A-BK)\vec{x}$

exact solution $\vec{x}(t) = e^{(A-BK)t}\vec{x}(0)$

$$e^{(A-BK)t}\vec{x}(0) = \sum_{i=1}^{\#\text{states}} e^{\lambda_i t}\,[\ \cdots\ \vec{x}(0)]$$

combos of $e^{\lambda_i t}$ $\lambda_i \in \text{evals}(A-BK)$

Note

If $x(0)$ is i^{th} evec. of A

then $e^{At}x(0) = e^{\lambda_i t}x(0)$

Pick K to make the least negative real(eig(A-BK)) as negative as possible

For typical B & K, can make real(λ)'s $\to -\infty$
but $K \to \infty$

place(A, B, [λ_1, ... λ_L])

↑ # states

Problem if K's are too large
u will be too large &
saturate controller

e.g. PWM drivers range $-1 \rightarrow 1$

-1: 100% -5 ; 1: 100% $+5$

No knob to turn to reduce u!

LQR Alternative (Simplified)

input weights

Minimize over all K's $\int_0^\infty \sum q_i x_i^2(t) + r u^2(t) \, dt$

state weights, states, $u = -Kx$ (Zero input case)

Minimization easy to solve (Large linear system)

Exchange picking good K's
for
picking good weights

Note scaling q_i's and r together
does not change result

Steps

1) Determine A & B for state space system

2) 'Pick' q's with $r = 1$ (normalize)

3) $K = \mathrm{lqr}\left(A, B, \begin{pmatrix} q_1 & & 0 \\ & \ddots & \\ 0 & & q_n \end{pmatrix}, r\right)$

4) $K_r = -\left(C(A - BK)^{-1}B\right)^{-1}$

5) Examine step response for

$$\dot{\vec{x}} = (A - BK)\vec{x} + BK_r y_d \qquad y = Cx$$

Examine u

$$u = K_r y_d - K\vec{x}$$

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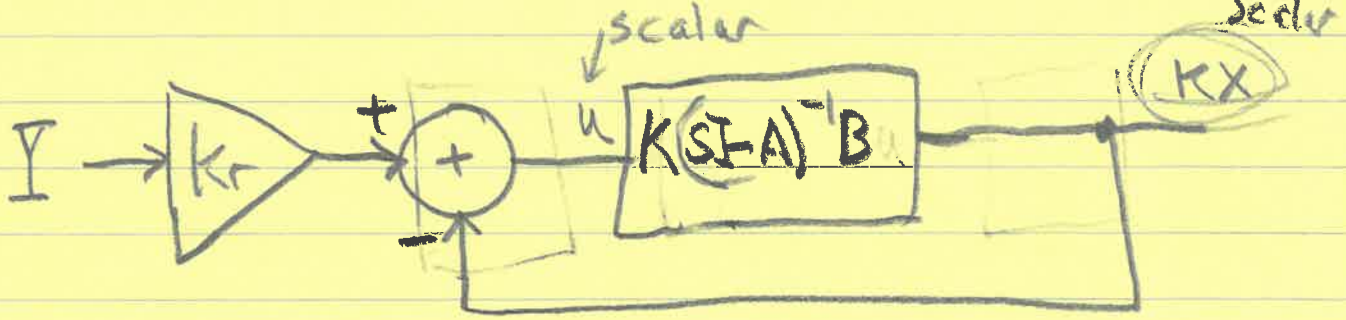

$$KX = \frac{K(sI-A)^{-1}B}{1 + K(sI-A)^{-1}B}\, Y$$

Phase margin