Problem 1) A) 1)

$$X_d = Z X_d + \frac{1}{2} C_0 \quad (Z = \zeta)$$

$$X_d (1 - Z) = \frac{1}{2} C_0$$

$$C_0 = 2 X_d (1 - Z)$$

$$Z = 0.5 \Rightarrow C_0 = X_d,$$

$$Z = 0.9 \Rightarrow C_0 = 0.2 X_d$$

2) from above, we know in a patient with $Z = 0.9$ $X_d = 5 C_0$.

Lets call $Z = 0.5$ Patient's dose

$C_0 = X_d'$

If we give $C_0 = X_d'$ to $Z = 0.9$ Patient, This Patient has Blood Stream Concentration of $5X_d'$. $5X$ the Expected concentration (Not Good).

B) 1) no oscillation means $\lambda > 0$.

Substitute $C[n-1]$ into model.

$$X[n] = Z\,X[n-1] + \frac{1}{2} K_p \left(X_d - X[n-1] \right)$$

$$X(n) = \frac{1}{2} K_p X_d + \underbrace{\left(Z - \frac{1}{2} K_p \right)} X[n-1]$$

first order model, λ is above

$$\lambda = Z - \frac{1}{2} K_p > 0$$

$K_p < 2 \cdot Z$

Z can range Between 0.5 & 0.9,

Lets be safe and assume $Z = 0.5$

$K_p = 1$

2) In Steady State, $X[n] = X[n-1] = X_{ss}$

$$X_{ss} = \frac{1}{2} K_p X_d + \left(Z - \frac{1}{2} K_p\right) X_{ss}$$

$$X_{ss}\left(1 - Z + \frac{1}{2} K_p\right) = \frac{K_p}{2} X_d$$

$$\frac{X_{ss}}{X_d} = \frac{K_p/2}{1 - Z + \frac{K_p}{2}}$$

$$K_p = 1 \Rightarrow \frac{X_{ss}}{X_d} = \frac{1/2}{\frac{3}{2} - Z} = \frac{1}{3 - 2Z}$$

$$Z = 0.5 \Rightarrow \frac{X_{ss}}{X_d} = \frac{1}{2}$$

$$Z = 0.9 \Rightarrow \frac{X_{ss}}{X_d} = 0.833$$

With P control, our X_{ss} ranges between 50% and 83% of our desired concentration. Much less variable than open loop's 20% – 100% range. This is better.

c) 1)

$$C[n-1] = K_p\left(X_d - X[n-1]\right) + K_i S[n-1]$$

$$X[n] = \frac{1}{2}K_p X_d + \left(Z - \frac{1}{2}K_p\right) X[n-1] + \frac{1}{2}K_i S[n-1]$$

$$S[n] = S[n-1] + X_d - X[n-1]$$

$$\vec{B} = \begin{bmatrix} \frac{K_i}{2} \\ 1 \end{bmatrix}$$

$$\bar{A} = \begin{bmatrix} Z - K_p/2 & \frac{K_i}{2} \\ -1 & 1 \end{bmatrix}$$

2) How do $\bar{A}$, $\vec{B}$, and X_d affect our Steady State $\vec{X}_{ss}$?

If all of our λ's have magnitude < 1, then $\vec{X}[n] = \vec{X}[n-1] = \vec{X}_{ss}$

$$\vec{X}_{ss} = \bar{A}\,\vec{X}_{ss} + \vec{B} \cdot X_d$$

$$\vec{X}_{ss} - \bar{A}\,\vec{X}_{ss} = \vec{B} \cdot X_d$$

$$(I - \bar{A})\,\vec{X}_{ss} = \vec{B} \cdot X_d$$

$$\vec{X}_{ss} = (I - \bar{A})^{-1}\,\vec{B} \cdot X_d$$

If we know X_d and $(I - \bar{A})^{-1}\vec{B}$, we can find $\vec{X}_{ss}$.

OK, what does $(I-\bar{A})^{-1}\vec{B}$ tell us about the relationship between X_d and $\vec{X}_{ss}$?

Matlab says $(I-\bar{A})^{-1}\vec{B}$ is: $\begin{bmatrix} 1 \\ \frac{2(1-z)}{k_i} \end{bmatrix}$

$$\vec{X}_{ss} = \begin{bmatrix} x[\infty] \\ s[\infty] \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{2(1-z)}{k_i} \end{bmatrix} X_d$$

$$x[\infty] = X_d$$

3) I would use Matlab to plot how the natrual frequencies move as I change K_p and K_i. I would need to make sure all natrual frequencies lie within the unit circle to ensure stability.

D) 1)

$$\bar{E} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$\tilde{A}_{new} = \begin{bmatrix} 2 - K_p/2 & K_i/2 \\ 0 & 1 \end{bmatrix}$$

$$\bar{B} = \begin{bmatrix} K_p/2 \\ 1 \end{bmatrix}$$

D)2) From Matlab:

$$\tilde{A} = \bar{E}^{-1}\bar{A}_{new} = \begin{bmatrix} z - \frac{K_p}{2} & \frac{K_i}{2} \\ \frac{K_p}{2} - z & 1 - \frac{K_i}{2} \end{bmatrix}$$

$$\tilde{B} = \bar{E}^{-1}\bar{B} = \begin{bmatrix} K_p/2 \\ 1 - \frac{K_p}{2} \end{bmatrix}$$

$$(I - \tilde{A})^{-1}\tilde{B} = \begin{bmatrix} 1 \\ \frac{2(1-z)}{K_i} \end{bmatrix}$$

Same as Part C 2

Problem 2)

Our B_{in} Should have a non zero component only in the states directly affected by error[n]. (Sum[n] and α[n])

$$\alpha[n] = (1 + \delta T B)\,\alpha[n-1] - \delta T B \gamma_a \left(K_p e[n-1] + K_d \, del[n-1] + K_i \, sum[n-1] \right)$$

$$Sum[n] = \Delta T e[n] + Sum[n-1]$$

In α[n], The coefficients with e[n] are The coefficients in $\vec{B}_{in}$ (Because $e[n] = \theta_d - \theta[n]$)

The same logic follows for Sum[n].

$$B_{in} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\beta \Delta T \gamma_a K_p \\ \Delta T \end{bmatrix}$$

To check our work, we can make sure that in steady state $\theta[n] = \theta[n-1] = \theta_d$

we use $(\bar{I} - \bar{A})^{-1} \vec{B}_{in}$ to find steady state vector.

$$\begin{bmatrix} \theta[\infty] \\ \theta[\infty-1] \\ w[\infty] \\ \alpha[\infty] \\ Sum[\infty] \end{bmatrix} = \underbrace{(\bar{I} - \bar{A})^{-1} \vec{B}_{in}}\, \theta_d$$

$$\begin{bmatrix} \theta[\infty] \\ \theta[\infty-1] \\ w[\infty] \\ \alpha[\infty] \\ Sum[\infty] \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \theta_d$$

We see $\theta[n] = \theta[n-1] = \theta_d$ in steady state $[n = \infty]$.

To find $\breve{B}_{dist}$, think about how we can "inject" a disturbance force into our system.

On first thought, we might think to inject the disturbance force into α as α represents angular acceleration. In reality, α represents acceleration due to the force coming from the propellers. With an undisturbed system, these two quantities are the same. For this problem, this is not true.

ω is the angular velocity of our arm. All forces contribute to it. It makes the most sense to "inject" our disturbance force into the $\omega[n]$ state.

$$\breve{B}_{dist} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

To be safe, we can make sure a disturbant force will lead to a zero error in θ_a and ω_a.

$$\begin{bmatrix} \theta\,[\infty] \\ \theta\,[\infty-1] \\ \omega\,[\infty] \\ \alpha\,[\infty] \\ Sum\,[\infty] \end{bmatrix} = \left(\bar{I} - \bar{A}\right)^{-1} \vec{B}_{dist}\, u_{dist}$$

$$\begin{bmatrix} \theta\,[\infty] \\ \theta\,[\infty-1] \\ \omega\,[\infty] \\ \alpha\,[\infty] \\ Sum\,[\infty] \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1/\Delta T \\ \dfrac{-1}{\gamma_a K_i \Delta T} \end{bmatrix} u_{dist}$$

We see that Both θ and ω go to Zero from a disturbant force. α does not go to Zero because The Propeller may need to generate a force to Counteract the Steady State distarbance force.