6.3100: Dynamic System Modeling and Control Design

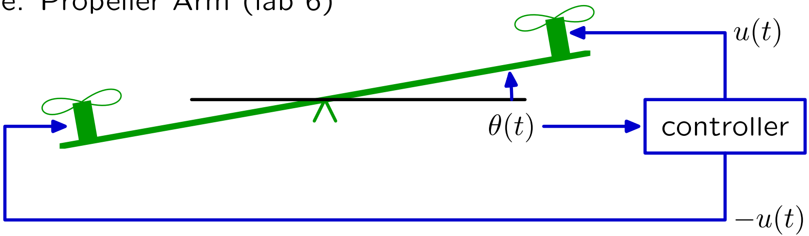
Noise Performance

December 02, 2024

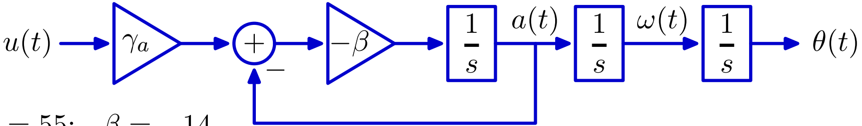
Noise Performance of Feedback Controllers

Today's goal: Analyze effects of **noise** on feedback controllers.

Example: Propeller Arm (lab 6)



Model of Plant (transfer function):



$\gamma_a = 55; \quad \beta = -14$

State-Space Model:

$$\frac{d}{dt}\begin{bmatrix}\theta\\ \omega\\ a\end{bmatrix} = \underbrace{\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & 0 & \beta\end{bmatrix}}_{\mathbf{A}}\begin{bmatrix}\theta\\ \omega\\ a\end{bmatrix} + \underbrace{\begin{bmatrix}0\\ 0\\ -\beta\gamma_a\end{bmatrix}}_{\mathbf{B}} u(t); \qquad y(t) = \underbrace{\begin{bmatrix}1 & 0 & 0\end{bmatrix}}_{\mathbf{C}}\begin{bmatrix}\theta\\ \omega\\ a\end{bmatrix}$$

Performance of Feedback Controllers

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Example: Propeller Arm (lab 6)

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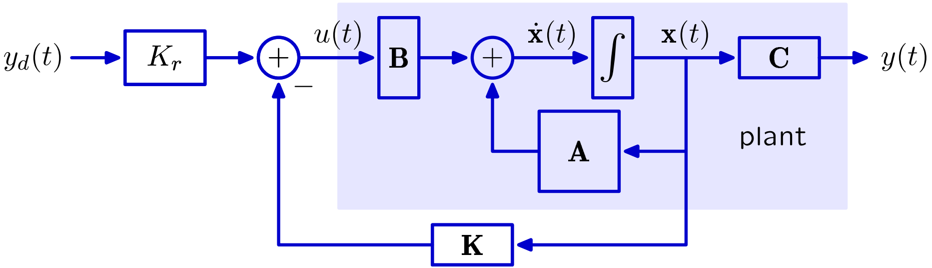
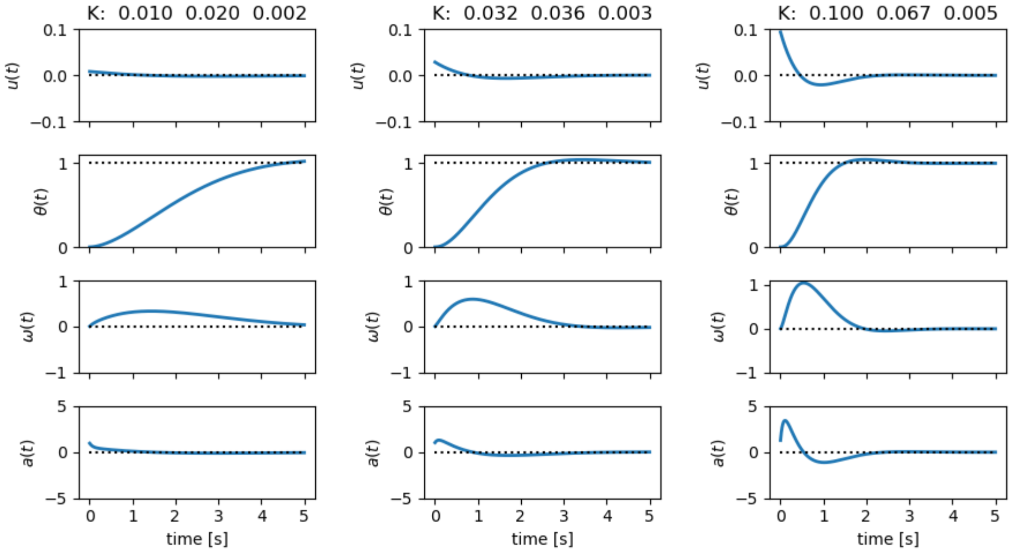
State-Space model with Full-State Feedback Controller



Performance of Feedback Controllers

Increasing gains $\mathbf{K}$ can speed convergence. $\surd$



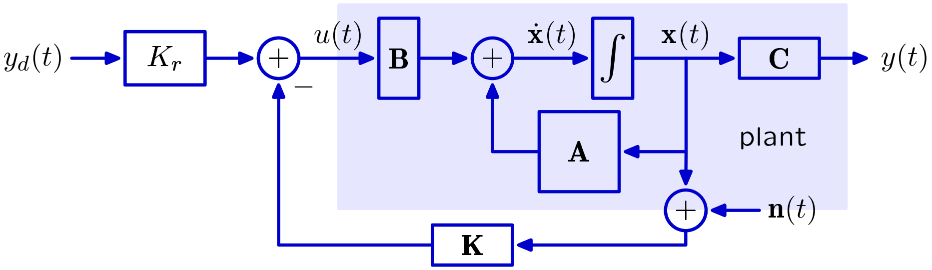
Unfortunately, high gains can also have detrimental effects. High gains can decrease stability (as we saw with root locus and gain/phase margins). ✗

Noise Performance

High gains can also increase sensitivity to noise.

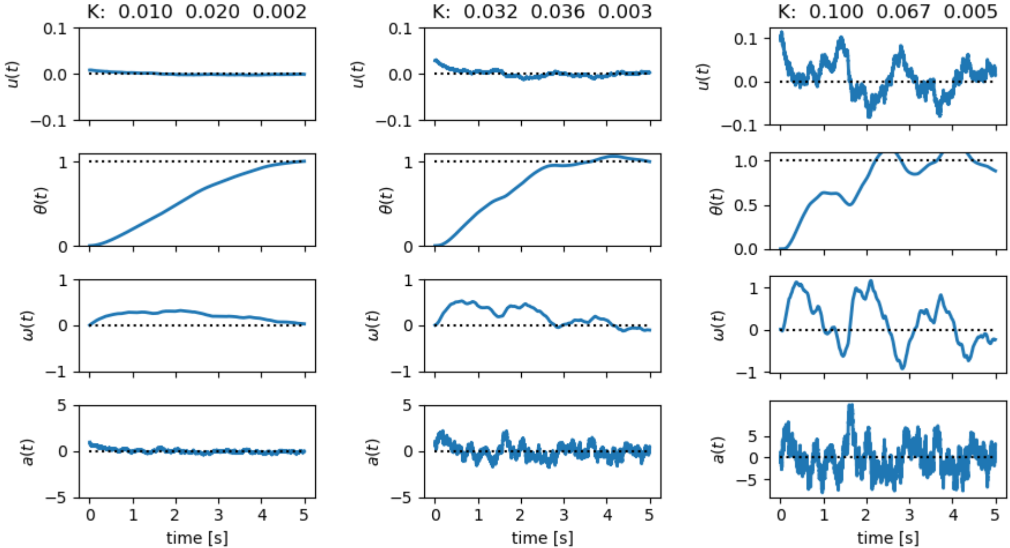
Today we will analyze effects of **noise** on feedback systems.

There are many possible sources of noise. Start with **sensor noise**, where the state $\mathbf{x}(t)$ reported to the controller differs from that seen in the plant.



Noise Performance

Increasing gains $\mathbf{K}$ speeds convergence but also increases noise sensitivity. Columns show results for same sensor noise but increasing values of $\mathbf{K}$.

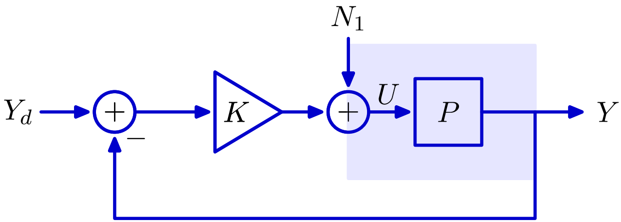


Same sensor noise generates more output noise at high gains than at low.

Noise Analysis

Consider a simpler example to understand effects of noise.

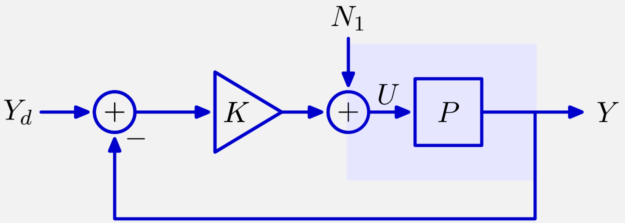
constant system (P) + proportional control K + noise N_1 at input to plant.



Check Yourself

For the feedback system shown below, let

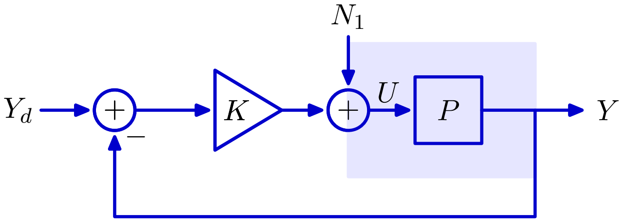
$$H_Y = \frac{Y}{Y_d} \text{ when } N_1 = 0 \text{ and } H_N = \frac{Y}{N_1} \text{ when } Y_d = 0$$



Which (if any) of the following are true?

1. $\lim_{K\to\infty} H_Y = 1$
2. $\lim_{K\to\infty} H_N = 0$
3. $\lim_{K\to\infty} \frac{H_N}{H_Y} = 0$
4. $\frac{H_N}{H_Y} = \frac{1}{K}$
5. none of the above

Check Yourself



Use Black's equation to find the two transfer functions:

$$H_Y = \frac{Y}{Y_d} = \frac{KP}{1+KP}$$

$$H_N = \frac{Y}{N} = \frac{P}{1+KP}$$

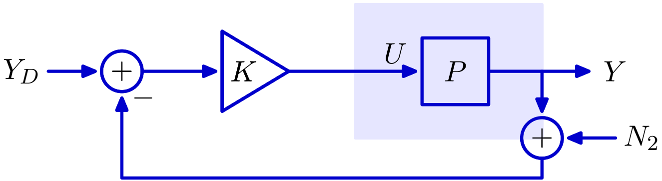
Then

1. $\lim_{K\to\infty} H_Y = 1$ ✓
2. $\lim_{K\to\infty} H_N = 0$ ✓
3. $\lim_{K\to\infty} \frac{H_N}{H_Y} = 0$ ✓
4. $\frac{H_N}{H_Y} = \frac{1}{K}$ ✓
5. none of the above ✗

The transfer function for the noise is K times smaller than that for Y_d.
Increasing the gain K **decreases** the noise seen at the output Y.

Noise Analysis

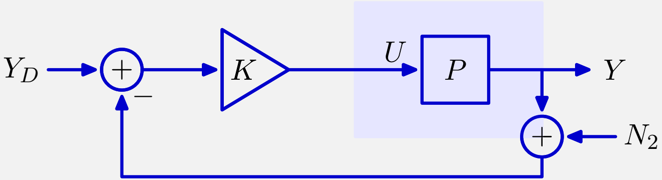
Consider a similar system with noise N_2 at its output.



Check Yourself

For the feedback system shown below, let

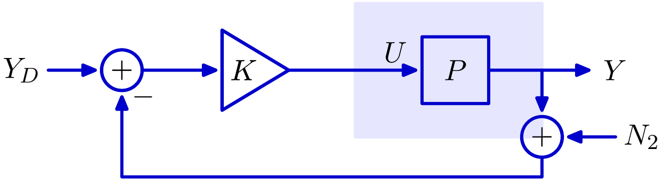
$$H_Y = \frac{Y}{Y_d} \text{ when } N_2 = 0 \text{ and } H_N = \frac{Y}{N_2} \text{ when } Y_d = 0$$



Which (if any) of the following are true?

1. $\lim_{K\to\infty} H_Y = 1$
2. $\lim_{K\to\infty} H_N = 0$
3. $\lim_{K\to\infty} \frac{H_N}{H_Y} = 0$
4. $\frac{H_N}{H_Y} = \frac{1}{K}$
5. none of the above

Check Yourself



Use Black's equation to find the two transfer functions:

$$H_Y = \frac{Y}{Y_d} = \frac{KP}{1+KP}$$

$$H_N = \frac{Y}{N} = \frac{-KP}{1+KP}$$

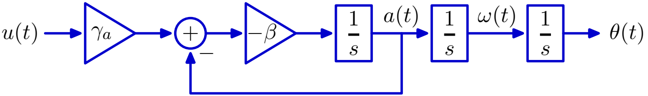
Then

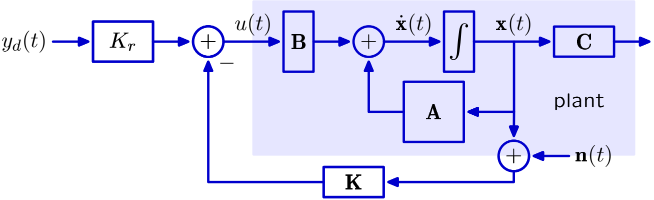
1. $\lim_{K\to\infty} H_Y = 1$ ✓
2. $\lim_{K\to\infty} H_N = 0$ ✗
3. $\lim_{K\to\infty} \frac{H_N}{H_Y} = 0$ ✗
4. $\frac{H_N}{H_Y} = \frac{1}{K}$ ✗
5. none of the above ✗

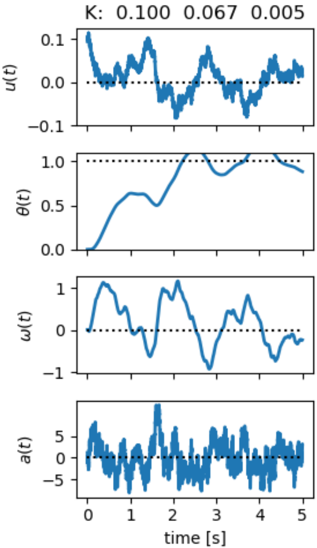
The transfer function for the noise has the same magnitude as that for Y_d. Increasing the gain K **does not decrease** the noise seen at the output Y.

Sensor Noise in Propeller Arm System

Why do the noises in $u(t)$, $\theta(t)$, $\omega(t)$, and $a(t)$ look different?







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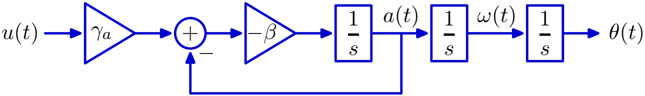
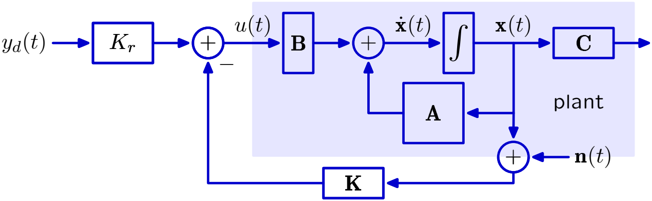
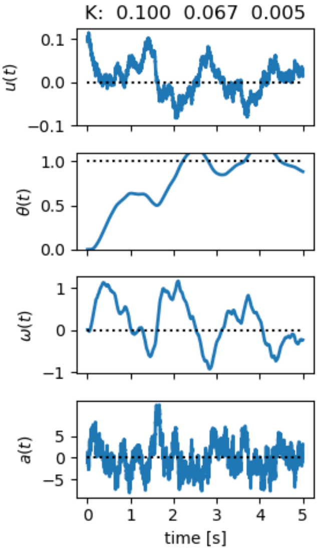


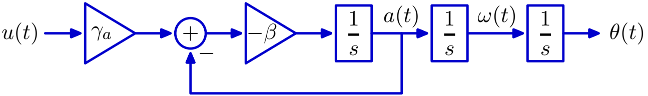


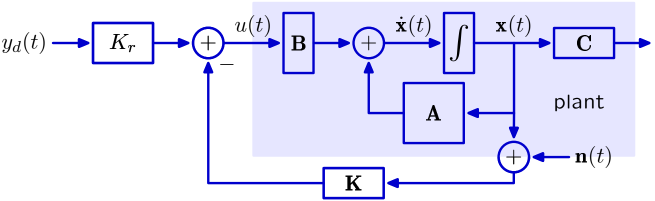


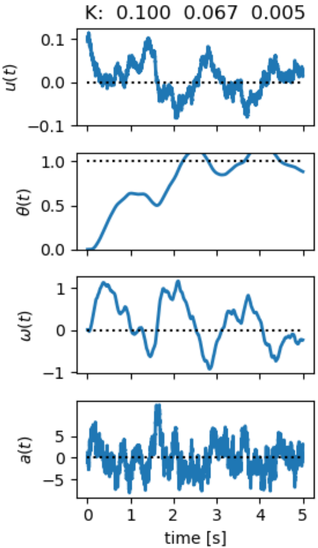
N_a looks biggest. N_ω has fewer high frequencies than N_a. N_θ has fewer high frequencies than N_ω.

Sensor Noise in Propeller Arm System

Why does the noise in $u(t)$, $\theta(t)$, $\omega(t)$, and $a(t)$ look different?



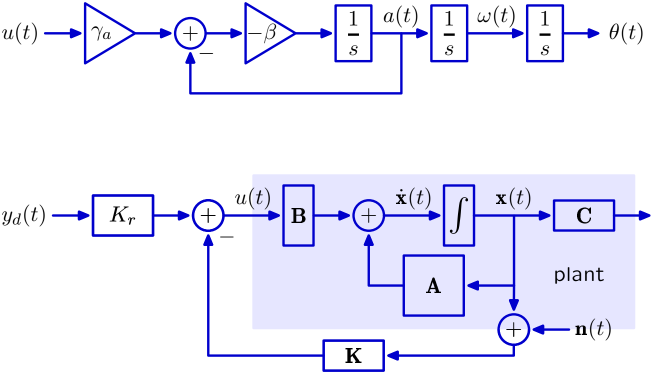


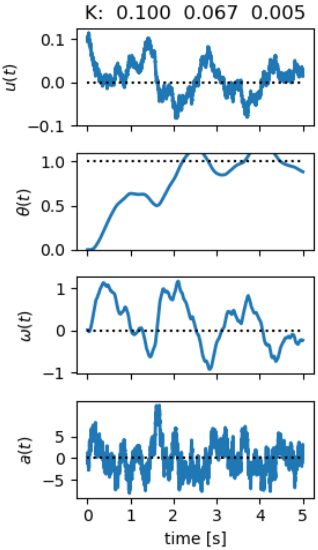


$\mathbf{n}(t)$ is added to $\mathbf{x}(t)$. The result is multiplied by $\mathbf{K}$ and subtracted from $K_r y_d(t)$ to get $u(t)$. $u(t)$ flows through 2 gains and an integrator to get to $a(t)$, through another integrator to get to $\omega(t)$, and another to get to $\theta(t)$.

Sensor Noise in Propeller Arm System

Why does the noise in $u(t)$, $\theta(t)$, $\omega(t)$, and $a(t)$ look different?

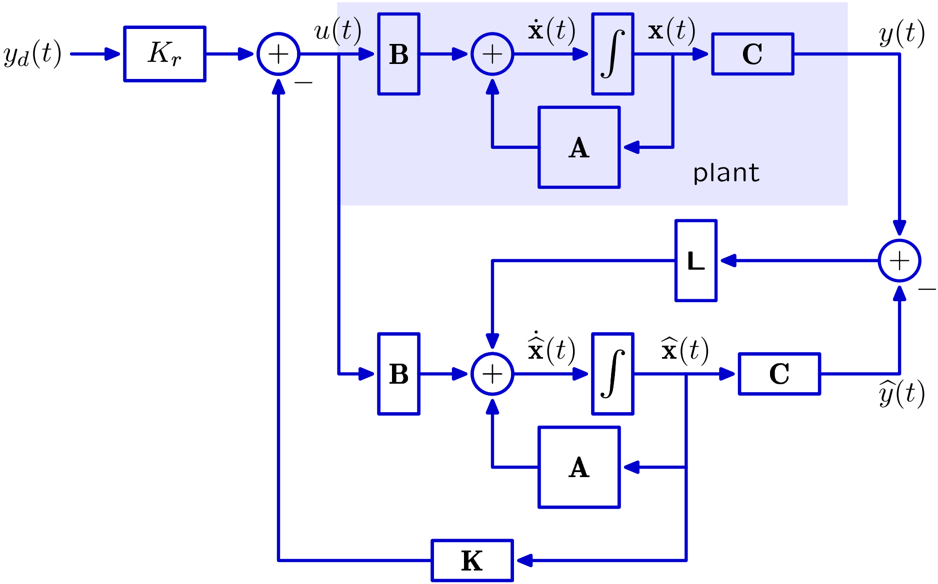




Sensor noise flows through feedback pathways to degrade the state of the plant in ways that can be understood in terms of the structure of the plant.

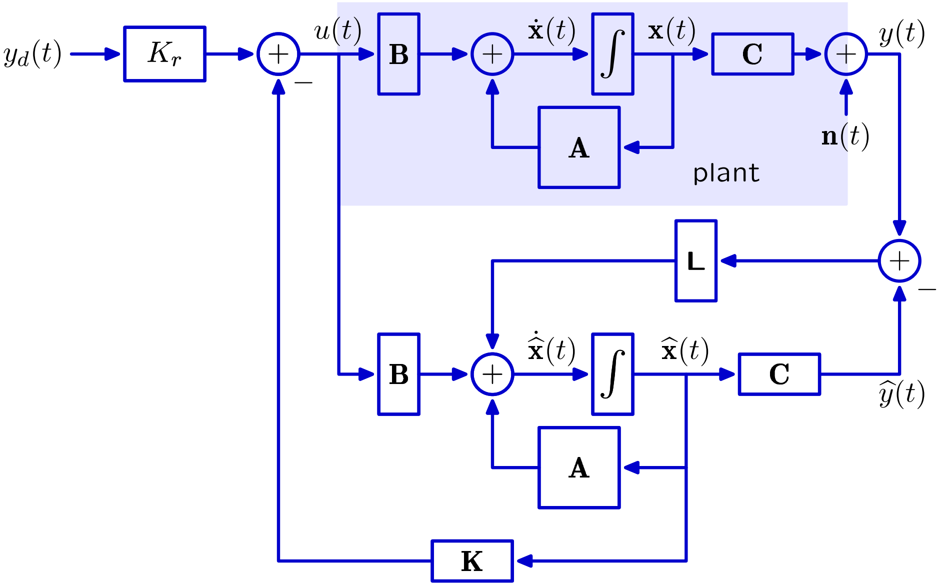
Analysis of Noise in an Observer

How should we model noise in controllers with an observer?



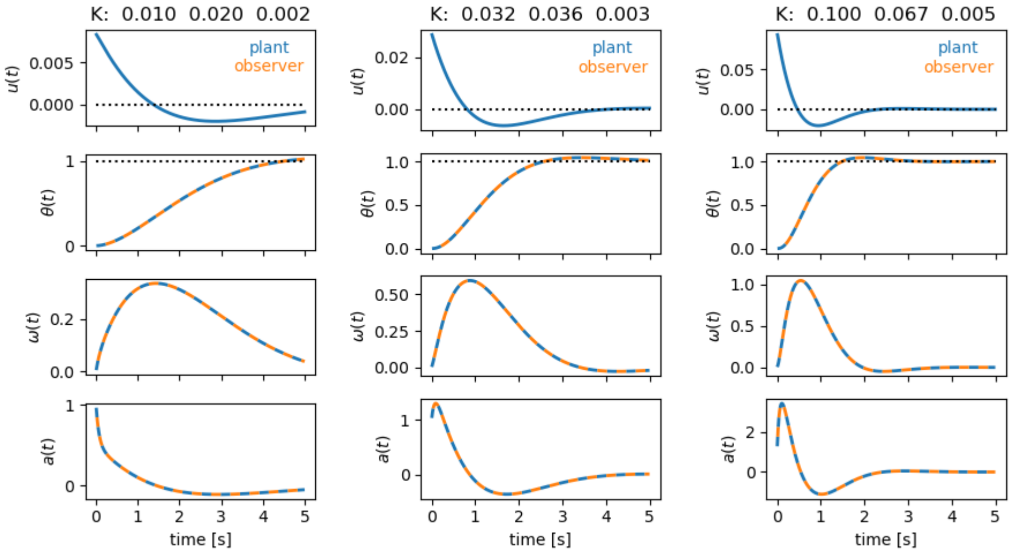
Analysis of Noise in an Observer

Assume the noise affects the output of the plant.



Results Without Noise

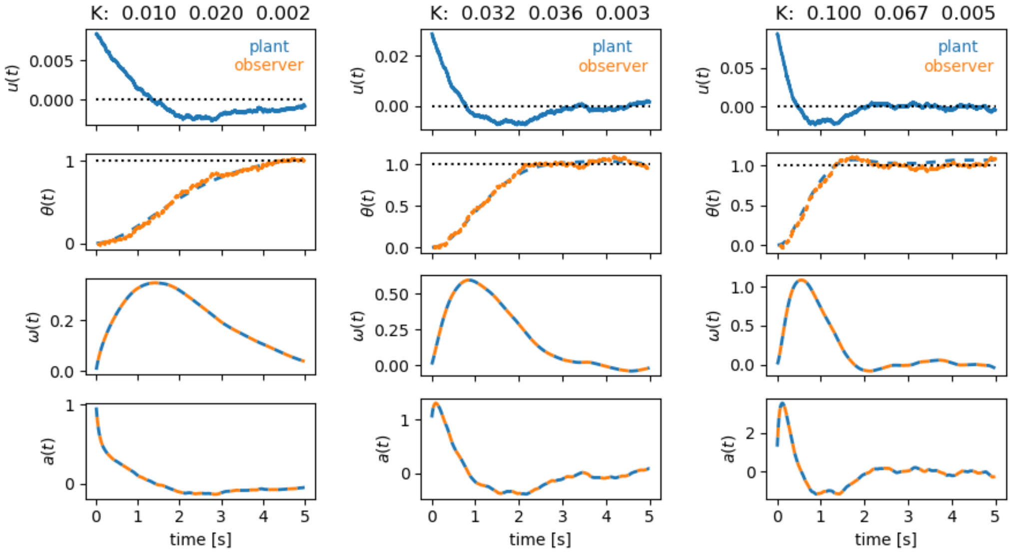
No noise: low gain (left), higher gain (center), and highest gain (right).



When there is no noise, responses of the plant and observer match and higher gains $\mathbf{K}$ speed convergence.

Results With Noise: Observer/Plant Responses Overlaid

With noise: low gain (left), higher gain (center), and highest gain (right).

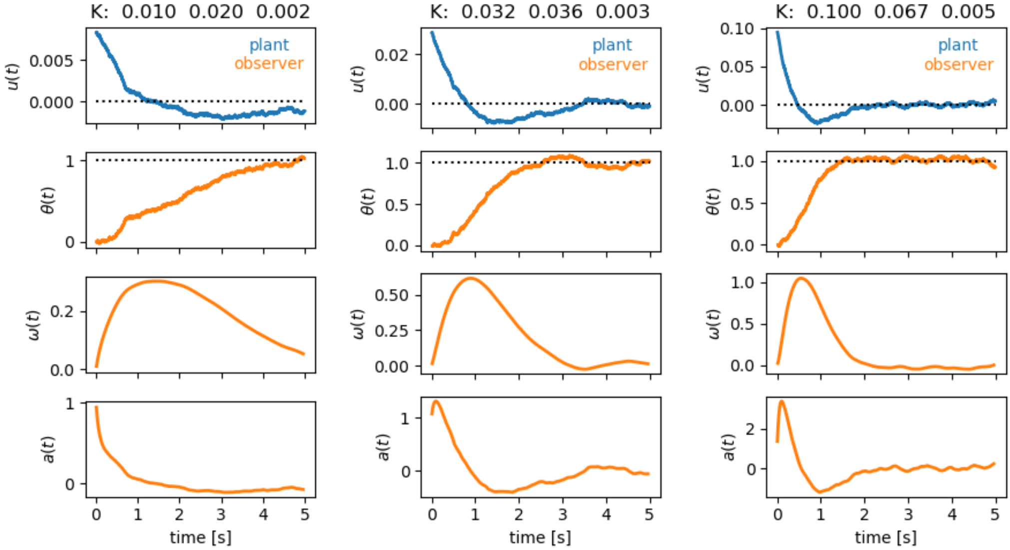


Noise affects the plant and observer differently.

But the curves are difficult to separate. Look at them one-at-a-time.

Results With Noise: Observer Responses

With noise: low gain (left), higher gain (center), and highest gain (right).

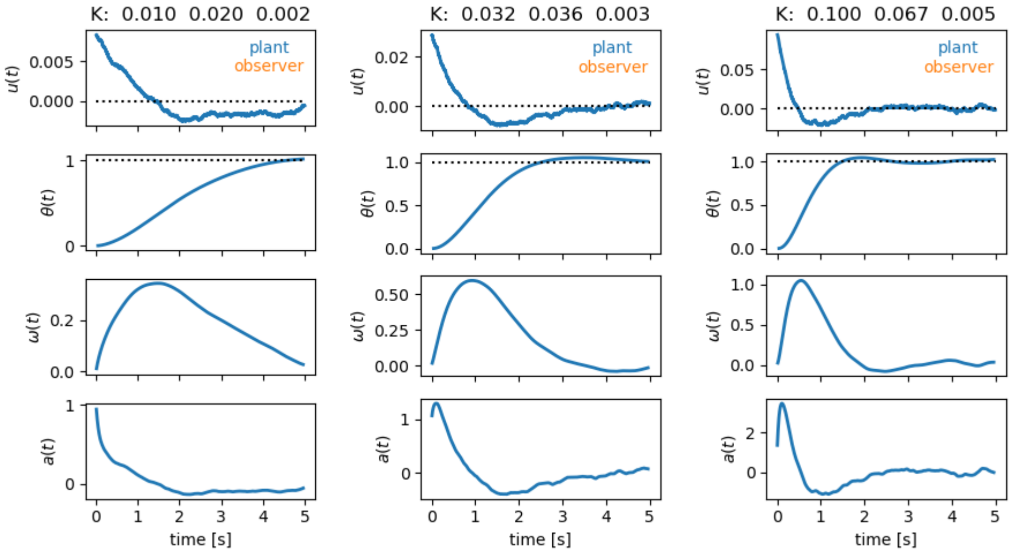


Results for the observer.

Significant noise in control signal $u(t)$ and states $\mathbf{x}(t)$.

Results With Noise: Plant Responses

With noise: low gain (left), higher gain (center), and highest gain (right).

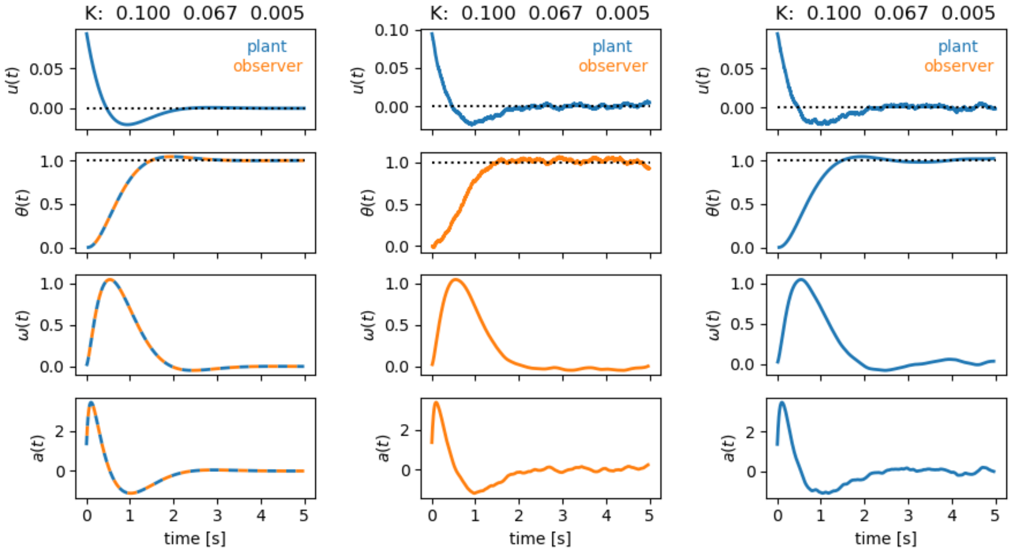


Results for the plant.

Significantly less noise in plant than in observer.

High Gain: No Noise, Observer w/ Noise, and Plant w/ Noise

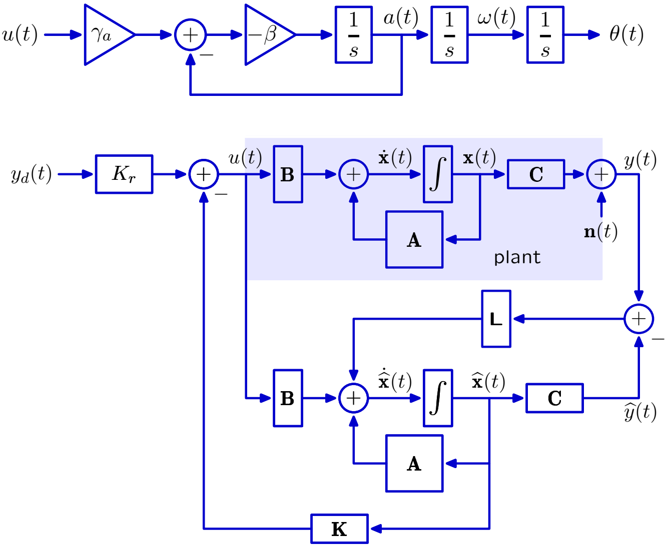
Noise affects plant and observer differently. Why?

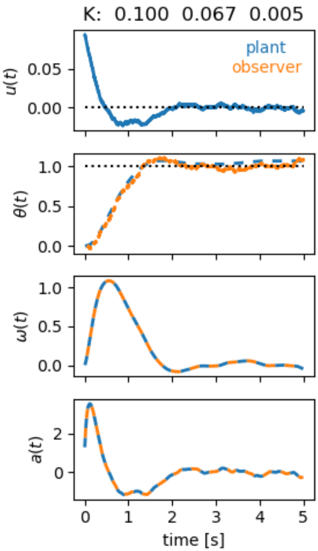


Differences seem greater for $\theta(t)$ than for $\omega(t)$ or $a(t)$. Why?

Analysis of Noise in an Observer

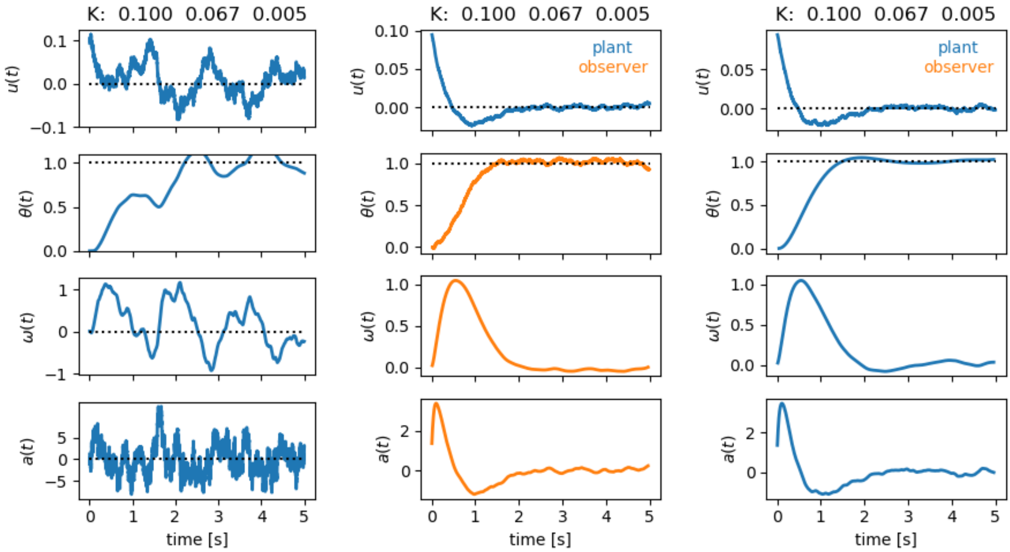
Assess effects of noise by tracing signal flow through the block diagram.





Compare

Effect of noise without observer (left) and with observer (middle and right).



Why is there so much less noise with the observer?

Sensor Noise in Propeller Arm System

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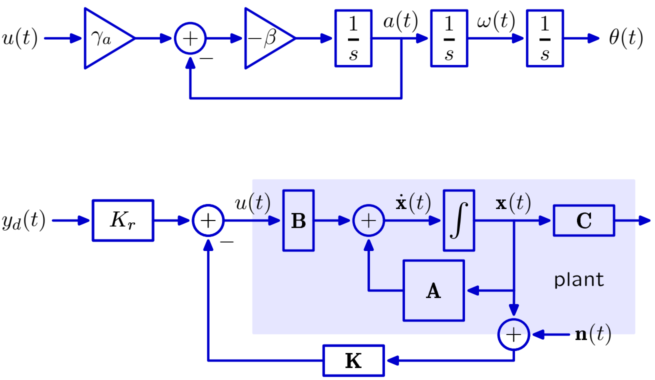

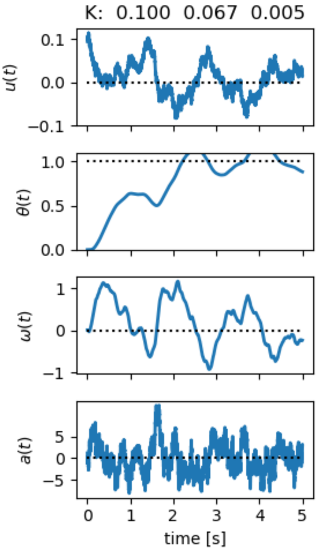

Summary

Noise can be an important consideration in design of feedback controllers.

There are many sources of noise: e.g., noise at the plant's input and output.

Increasing gains can speed convergence but can increase noise sensitivity.

Using an observer can dramatically improve noise performance.