Postlab 1: Fast and Spurious

Problems

1. Latent Inconsistency

In Lab 1 (Fast and Spurious), we worked on motor speed control where the system is described by a first-order model:

$$\omega[n] = \omega[n-1] + \Delta T\Big(\beta\omega[n-1] + \gamma c[n-1]\Big)$$

Based on measurements, we calculated the system parameters β and γ. However, in Checkoff 5, we collected data and observed an inconsistency between the model and the measurement. Demonstrate the inconsistency by plotting speed as a function of time under the following two conditions:

1a. K_p is set to its largest stable value, `REPEATS`=3, `ticksperupdate`=8, `FREQ`=0.2, AMP=1, and `ticksperestimate`=8, `disturbA`=0.

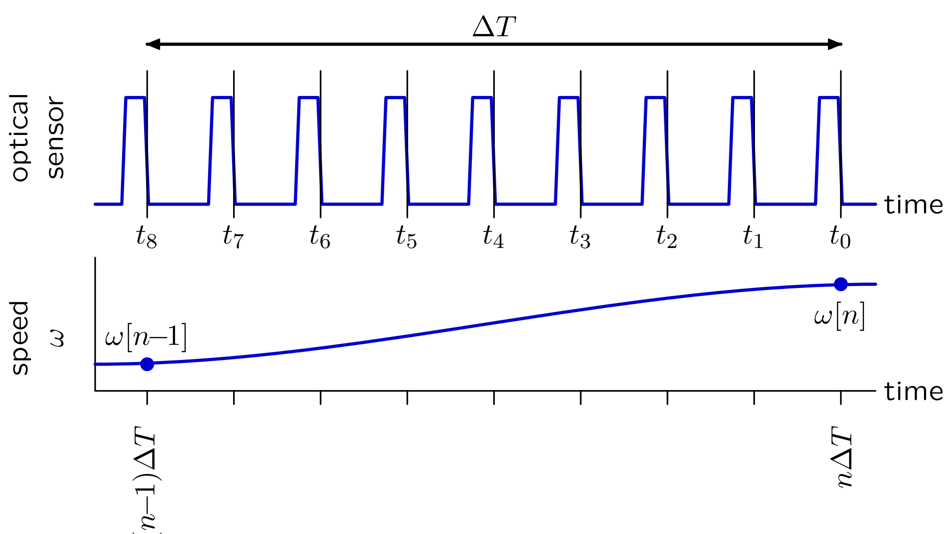
1b. K_p is set to its largest stable value, `REPEATS`=3, `ticksperupdate`=8, `FREQ`=0.2, `AMP`=1, and `ticksperestimate`=1, `disturbA`=0.

1c. Describe the inconsistency that is demonstrated by the previous plots.
Hint: You should have discussed this point with the TAs during Checkoff 5.

2. Estimating Speed

The goal of this Post-Lab is to analyze and understand the origin of the inconsistency that was seen in the previous part. The major factor that contributes to this inconsistency stems from the way we measure rotor speed. In the model, ω represents the current value of the angular velocity of the rotor. However, our experiment hardware does not provide a direct measurement of ω. Instead, we must estimate the angular velocity from the time between pulses of light detected by the optical sensor.

The following figure illustrates our method. When `ticksperupdate=8`, the control loop runs once after each set of 8 pulses. If we label the current time as t_0 and previous times as t_1, t_2, $\cdots$, then the sample time $\Delta T = t_0 - t_8$.



Let $\widetilde{\omega}$ represent our estimate of angular speed based on the experimentally determined times t_i. If `ticksperestimate=8`, the time for 8 pulses to occur is equal to the time for one turn of the rotor (since there are 8 blades on the rotor). Therefore the angular speed is approximately 1 revolution divided by the time for 8 pulses of light:

$$\widetilde{\omega}[n] \approx \frac{1}{t_0 - t_8} \text{ revolutions per second}$$

Notice however that this estimate matches the true speed ω best at a point midway between the sample times $t_0 = n\Delta T$ and $t_8 = (n{-}1)\Delta T$ – i.e., at t_4. If we make a piecewise linear approximation of speed, then

$$\widetilde{\omega}[n] \approx \frac{\omega[n] + \omega[n-1]}{2}$$

In our original formulation of the model, the control signal $c[n]$ was proportional to the difference between $\omega_d[n]$ and $\omega[n]$:

$$c[n] = K_p(\omega_d[n] - \omega[n])$$

However, the controller does not have direct access to $\omega[n]$, and a better model for the experiment hardware is

$$c[n] = K_p(\omega_d[n] - \widetilde{\omega}[n]) = K_p\left(\omega_d[n] - \frac{\omega[n] + \omega[n-1]}{2}\right)$$

Simulate a new model for the experiment hardware using this controller instead of our original controller. Use the measured parameters β and γ you obtained from Lab 1, and suppose $\omega_d[n] = 1$ and $\omega[0] = 0$.

2a. Provide a plot of $\omega[n]$ as a function of n using the programming language of your choice (e.g., MATLAB, Python, ...).

2b. Briefly describe the important features of your plot, and how they relate to the inconsistency that was described in part 1 of this post-lab.

3. Estimating Speed With Less Delay

In the previous section, we estimated the rotor speed by computing the time for the rotor to spin one full term, i.e., $t_0 - t_8$ in the previous figure. This method introduces significant delay in the feedback loop, and makes the system behave more like a second-order system than as a first-order system. In this section, we investigate an estimate based on a single pulse period to see if restricting the estimate to more recent times can better preserve the first-order nature of the hardware that we expected.

If `ticksperestimate=1`, our hardware computes a different estimate of rotor speed:

$$\widehat{\omega}[n] = \frac{1/8}{t_0 - t_1}$$

The 1/8 in the numerator results because the rotor rotates just one-eighth of a turn in the time from t_1 to t_0. As with $\widetilde{\omega}[n]$ we can relate the estimated speed to the model speed $\omega[n]$ by linear interpolation:

$$\widehat{\omega}[n] = a\omega[n] + b\omega[n-1]$$

However, the constants a and b are no longer $\frac{1}{2}$.

3a. What are the values of a and b needed for $\widehat{\omega}$? Briefly explain your reasoning.

Determine the control system equation that results from these new values of a and b.

3b. Enter the new system equation. Briefly show how you derived this equation by providing a few intermediate steps.

Write a program to compute $\omega[n]$ as a function of n when $\omega_d[n] = 1$ and $\omega[0] = \omega[1] = 0$.

3c. Provide a plot of $\omega[n]$ as a function of n.

3d. Briefly describe the important features of your plot, and how they relate to the inconsistency that was described in part 1 of this post-lab.

Comparing the two simulated results, which case shows faster convergence? More broadly, if we want our control system to convert the fastest, is there an optimal way for measuring the velocity $\widetilde{\omega}$? If we are free to choose a and b, what are the optimal values?

Hint: you need to solve this problem numerically (i.e., sweep through different combinations of a and b and identify the optimal pair.)

Please show your analysis (possibly with graphs) below.

3e. What values of a and b provide the fastest convergence? Briefly explain.