PostLab 3: A Disturbance in the Force

In this week's lab (A Disturbance in the Force), we studied continuous-time control and sinusoidal steady-state response. We characterized system resonance properties by measuring the magnitude and phase of the angular displacements of the propeller system. In this post-lab, we will investigate an inverse problem. Given the system's resonance measurements, can we implement numerical methods to estimate system parameters?

Problems

1. **Sweep Data**

   For the last checkoff in this week's lab, you were asked to collect frequency response data for the propeller arm system. These data are organized in five columns. The first column is frequency. We will ignore the 2nd and the 3rd columns. The fourth and fifth columns show the sine and cosine components of the output angle $\theta_a(t)$ which were computed as follows:

   $$\int \theta_a(t) \sin(\omega t) dt$$

   $$\int \theta_a(t) \cos(\omega t) dt$$

   Please make two plots. The first plot should show five traces (one for each column of data) as functions of row number. The second plot should show the fourth and fifth columns as functions of frequency.
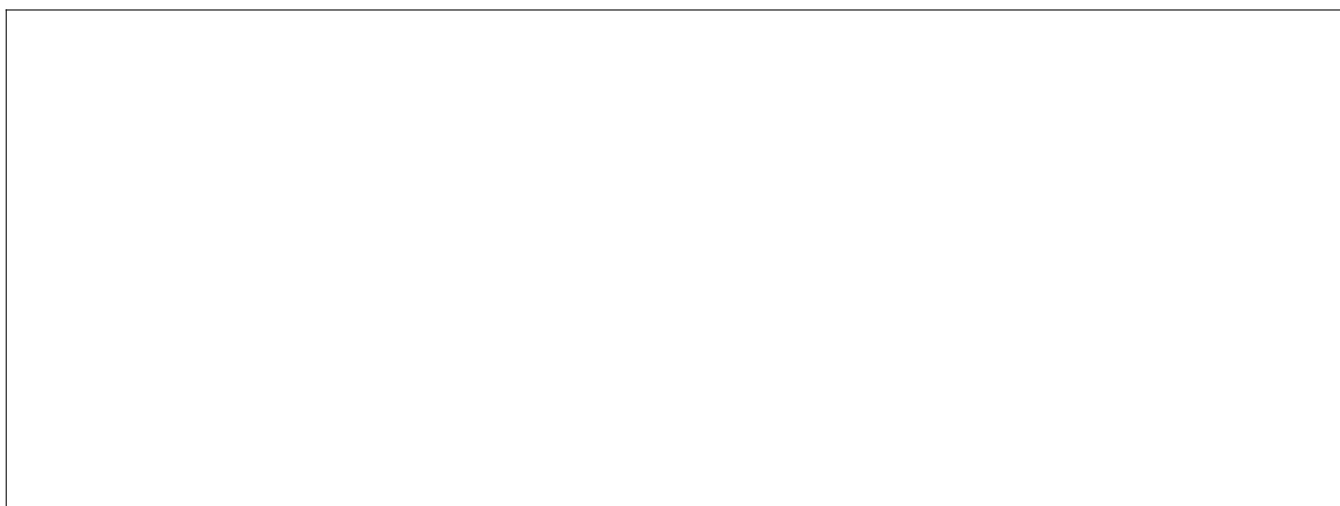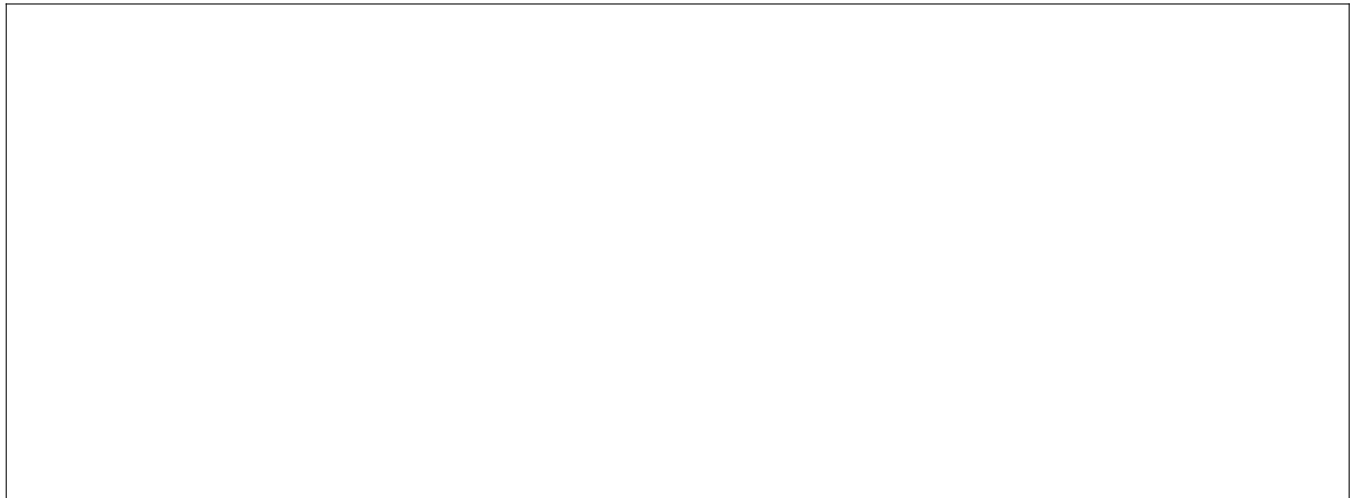
2. Frequency Response

Next, solve for the amplitude and phase of the closed-loop transfer function. The amplitude and phase are given by the following MATLAB expressions:

$$|G(j\omega)| = \sqrt{x(:,4)^2 + x(:,5)^2}$$

$$\angle(G(j\omega)) = \tan^{-1} \frac{x(:,5)}{x(:,4)}$$

Please note that the angle is given by the arctangent of the ratio of the cosine component to the sine component because the input to the propeller system was a sine function (rather than a cosine function).

Show plots of the resulting magnitude and phase in the box below.

3. System Parameters

What were the controller parameters for your measured data?

$K_p = $

$K_d = $

$K_i = $

What is the resulting controller transfer function $K(s)$?

$K(s) = $

Draw the block diagram of your control system.

Based on this block diagram, please write the plant transfer function H(s).

$H(s) =$

Here the model parameters γ and β are the unknowns we aim to find. Based on your block diagram, please write the closed loop system transfer function that relates the input $\Theta_d(s)$ and output $\Theta_a(s)$ in the form of $\Theta_a(s) = G(s)\Theta_d(s)$ You can express your answer in terms of $H(s)$ and $K(s)$.

$G(s) =$

Now let's try to formulate a method for solving γ and β. Using the automated sweeper, we measure the magnitude and angle of $G(s)$. Write an expression for $G(j\omega)$ in terms of its magnitude $|G(j\omega)|$ and angle $\angle(G(j\omega))$.

$G(j\omega) =$

Solve for $H(j\omega)$ in terms of $G(j\omega)$.

$H(j\omega) =$

4. **Fitting**

   Implement these in a programming language of your choice and show the magnitude and phase of $H(j\omega)$ below.

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   Note that your calculation of $H(j\omega)$ may be noisy, so we will be careful when performing numerical fitting. To fit the parameters γ and β, first write the magnitude and angle of $H(j\omega)$.

   $|H(j\omega)| = $

   $\angle(H(j\omega)) = $

   We can solve for β by using the relationship:

   $\beta \tan(\angle(H(j\omega))) = \omega$

   Since we have collected many data points, we can use linear regression. Let $A = \tan(\angle(H(j\omega)))$, $b = \omega$, and $x = \beta$. We can write

   $x = A^{-1}b$

   In MATLAB, the linear regression is executed using the line:

   $x = A\backslash b$

   Please implement the linear regression and enter the resulting value of β below.

   $\beta = $

   Following this method, please solve for γ. Note that it is difficult to directly solve for γ because ω^3 varies over a large range. You should take the logarithm of both sides, do linear regression on the logged expression, and then transform it back. Please show your steps below and report the fitted γ value.
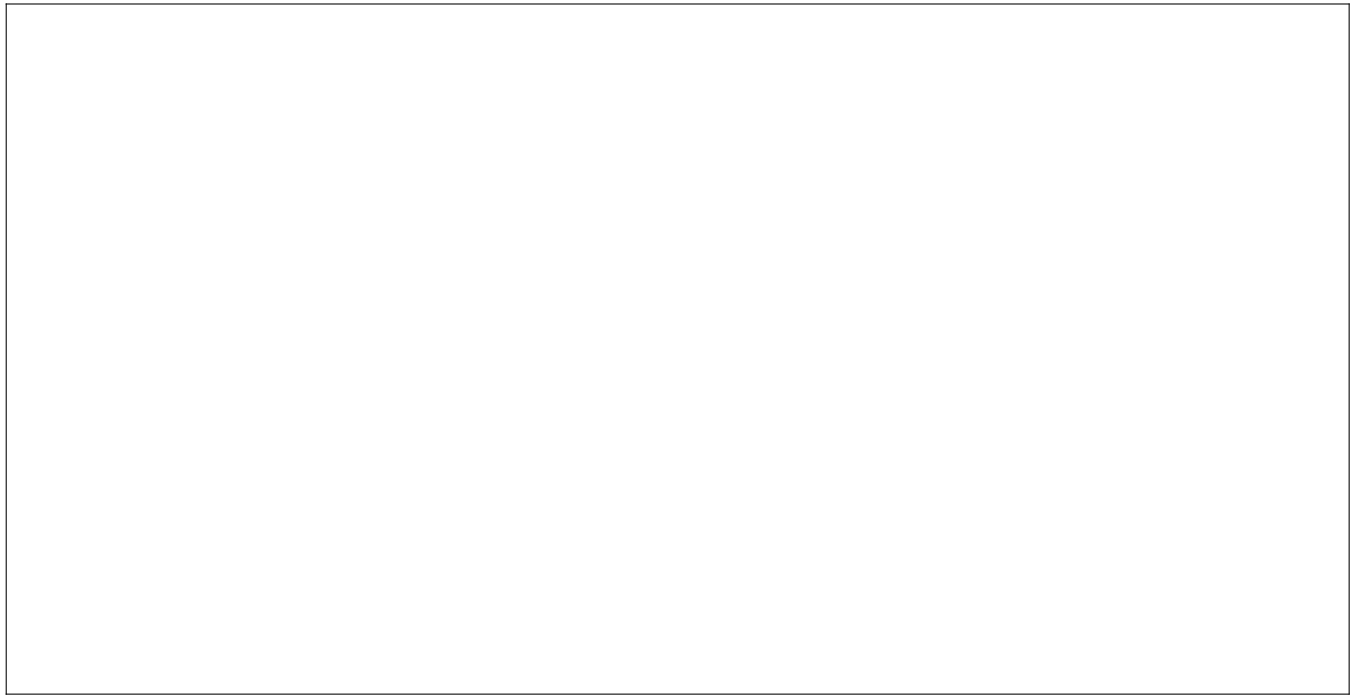
Finally, use the fitted β and γ values to run a simulation. You may find the MATLAB function bode() useful. In your plot, please show the simulated frequency sweep response and the measured response.