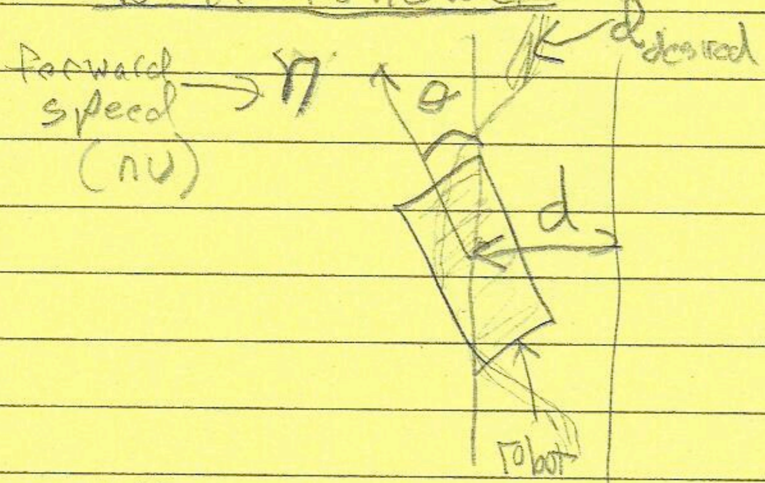
6.3100 10/1/25 (1)

Wall Follower



rotational velocity

Control: ω

In D.T

linearized $\sin(\theta)$

$$d[n] = d[n-1] + \Delta T\, V\, \theta[n-1]$$

$$\theta[n] = \theta[n-1] + \Delta T\, \underbrace{\gamma}_{\omega}\, \underbrace{c[n-1]}_{\text{control}}$$

$$\underbrace{c[n] = K_p\left(d_d[n] - d[n]\right)}_{\text{proportional control}}$$

$$\Downarrow$$

$$\begin{bmatrix} d[n] \\ \theta[n] \end{bmatrix} = \overbrace{\begin{bmatrix} 1 & \Delta T V \\ \Delta T \gamma K_p & 1 \end{bmatrix}}^{A_{dt}} \begin{bmatrix} d[n-1] \\ \theta[n-1] \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \Delta T \gamma K_p \end{bmatrix}}_{B_{dt}} d_d[n]$$

In C.T.

$$\frac{d[n] - d[n-1]}{\Delta T}$$

$$\frac{d}{dt} d(t) = V\, \theta(t)$$

$$\frac{d}{dt} \theta(t) = \gamma\, c(t)$$

$$c(t) = K_p\left(d_d(t) - d(t)\right)$$

$$\frac{d}{dt}\begin{bmatrix} d(t) \\ \theta(t) \end{bmatrix} = \overbrace{\begin{bmatrix} 0 & V \\ \gamma K_p & 0 \end{bmatrix}}^{A_{ct}} \begin{matrix} d(t) \\ \theta(t) \end{matrix}$$

$$\underbrace{\begin{bmatrix} 0 \\ \gamma K_p \end{bmatrix}}_{B_{ct}} d_d(t)$$

2

D.T.

$x[n] = Ax[n-1] + Bu[n-1]$

$$x[n] = \underbrace{A^{n} x[0]}_{ZIR} + \underbrace{\sum_{m=0}^{n-1} A^{n-m-1} Bu[m]}_{ZSR}$$

if $|\lambda_i(A)| < 1 \; \forall i$ then ZIR → 0 as $n \to \infty$

C.T.

$\frac{d}{dt} x(t) = Ax(t) + Bu(t)$

$$x(t) = \underbrace{e^{At} x(0)}_{ZIR} + \underbrace{\int_0^t e^{A(t-\tau)} Bu(\tau) d\tau}_{ZSR}$$

What's e^{At}?

If $A = \underbrace{V}_{evec} \begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix} V^{-1}$ (evals)

$e^{At} = V \begin{bmatrix} e^{\lambda_1 t} & & \\ & \ddots & \\ & & e^{\lambda_n t} \end{bmatrix} V^{-1}$

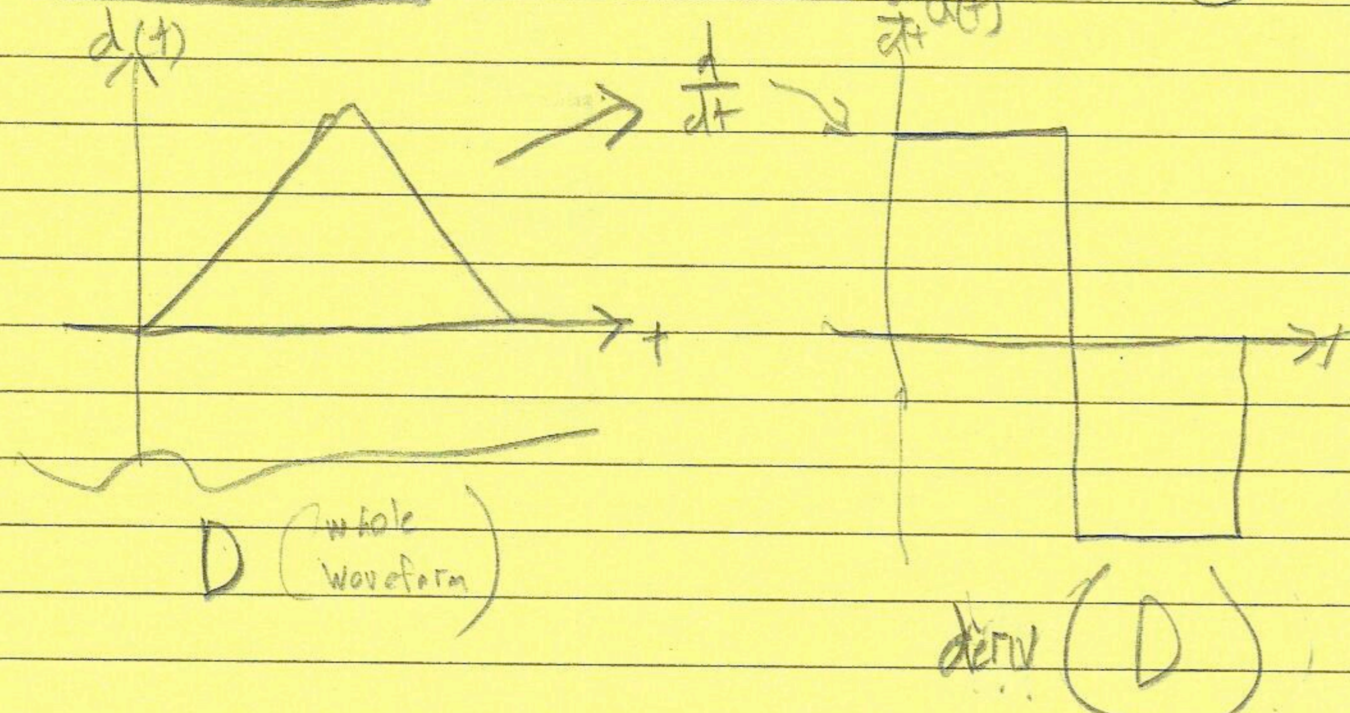
If $Re(\lambda_i(A)) < 0 \; \forall i$

then ZIR → 0 as $t \to \infty$

ZIR is weighted combo

Wait: What about blocks, and intuition and insight and...

Consider



$\frac{d}{dt}$ is an operator, mapping D to its derivative

Use 's' to denote this operator

So $\frac{d}{dt} d(t) = \gamma\,\theta(t) \Rightarrow s D = \gamma\,\Theta$

$\frac{d}{dt} \theta(t) = \gamma\, c(t) \Rightarrow s \Theta = \gamma\, C$

Some identities

$$s^2 Y = s(sY) \Rightarrow \frac{d}{dt}\left(\frac{d}{dt} y(t)\right) = \frac{d^2}{dt^2} y(t)$$

$$(\beta s + \alpha) Y \Rightarrow \beta \frac{d}{dt} y(t) + \alpha y(t)$$

$$(s+\alpha_1)(s+\alpha_2) Y = (s^2 + (\alpha_1+\alpha_2)s + \alpha_1\alpha_2) Y \Rightarrow \frac{d^2}{dt^2} y(t) + (\alpha_1+\alpha_2)\frac{d}{dt} y + \alpha_1\alpha_2\, y(t)$$

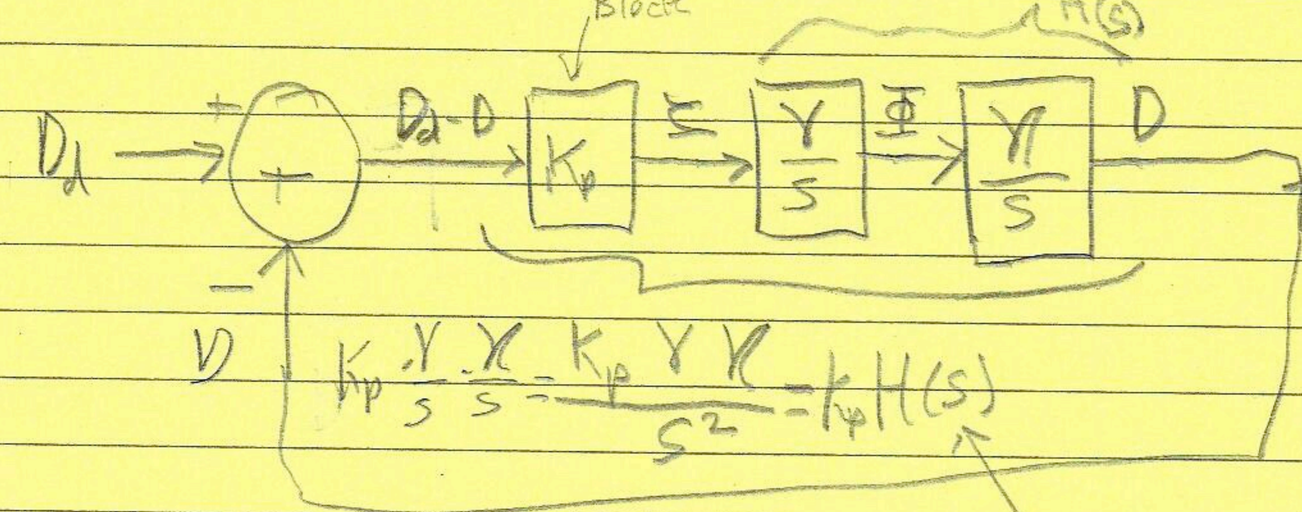
Back to Line Follower

Huh

$$s D = \eta \, \Phi \qquad D = \frac{\eta}{s} \Phi$$

$$s \Phi = \gamma \, \Xi \qquad \Phi = \frac{\gamma}{s} \Xi$$

$$\Xi = K_p (D_d - D) \qquad \Xi = K_p (D_d - D)$$



$$K_p \frac{\gamma}{s} \frac{\eta}{s} = \frac{K_p \gamma \eta}{s^2} = K_p H(s)$$

Transfer operater

$$D = \frac{K_p \gamma \eta}{s^2} (D_d - D)$$

$$\left(1 + \frac{K_p \gamma \eta}{s^2}\right) D = \frac{K_p \gamma \eta}{s^2} D_d$$

K(s) H(s)

$$D = \frac{\frac{K_p \gamma \eta}{s^2}}{1 + \frac{K_p \gamma \eta}{s^2}} D_d$$

$$D = \frac{K_p \gamma \eta}{s^2 + K_p \gamma \eta} D_d$$

$$(s^2 + K_p \gamma \eta) D = K_p \gamma \eta D_d$$

$$\frac{d^2}{dt} d(t) + K_p \gamma \eta\, d(t) = K_p \gamma \eta\, d_d(t)$$

Zero Input Response $d_d(t) = 0$ $d(0) \neq 0$

$$\frac{d^2}{dt^2} d(t) + K_p \gamma \eta\, d(t) = 0$$

from e^{At}

Suppose $d(t) = Ae^{\lambda t}$ ← intial condition

$$\frac{d^2}{dt^2} d(t) = A\lambda^2 e^{\lambda t}$$

$$\lambda^2 Ae^{\lambda t} + K_p \gamma \eta A e^{\lambda t} = 0$$

find roots $\left\{ \lambda^2 + K_p \gamma \eta = 0 \right.$ Note

$$\lambda = \pm j\sqrt{K_p \gamma \eta}$$

$$A_1 e^{j\sqrt{K_p \gamma \eta}\, t} + A_2 e^{-j\sqrt{K_p \gamma \eta}\, t}$$

Is a ZIR for any A_1 & A_2

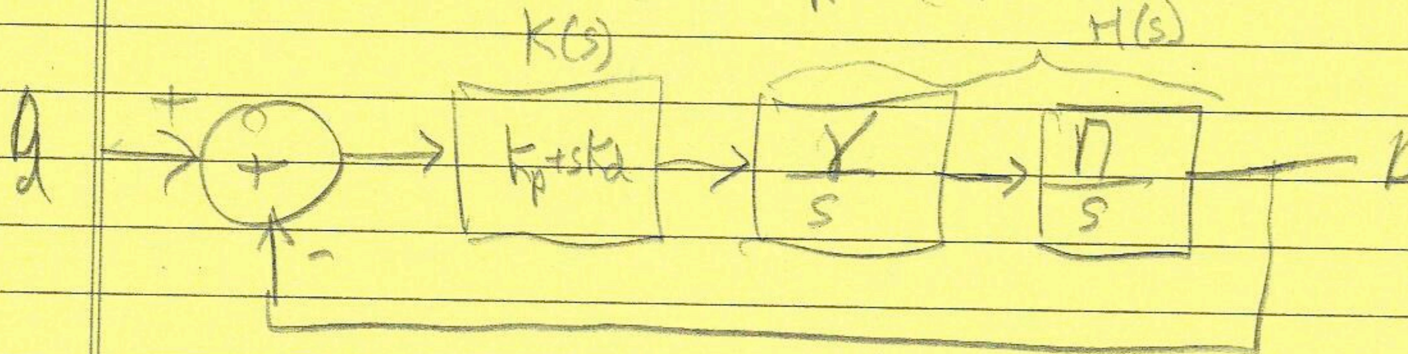
Pick them to match $d(0)$!

real part $(\lambda) < 0 \rightarrow$ ZIR $\rightarrow 0$ as $t \rightarrow \infty$

P. D. controller

$$c(t) = K_p\left(d_d(t) - d(t)\right) + K_d s\left(d_d(t) - d(t)\right)$$

$$\underline{C} = \left(K_p + sK_d\right)\left(D_d - D\right)$$



$$K(s)H(s) = \left(K_p + sK_d\right)\left(\frac{\gamma}{s}\frac{\eta}{s}\right)$$

$$D = \frac{K(s)H(s)}{1 + K(s)H(s)} D_d$$

$$= \frac{\left(K_p + sK_d\right)\left(\gamma\eta\right)}{s^2 + \left(K_p + sK_d\right)\gamma\eta}$$

Find roots

$$\left(s^2 + sK_d\gamma\eta + K_p\gamma\eta\right)D = \left(K_p + sK_d\right)D_d$$

~~Sub $j\omega$ for s~~

$$\underbrace{s^2 + sK_d\gamma\eta + K_p\gamma\eta = 0}_{\text{roots are natural frequencies}}$$

$i\mathbb{R} \not\subset \mathbb{R}$