9/29/25 6.3100 1

System Description

N "states"

$$\underbrace{x[n]}_{N} = \underbrace{A}_{N \times N}\ \underbrace{x[n-1]}_{\text{States}} + \underbrace{B}_{\text{Single input}}\ u[n-1] + \underbrace{B_d}_{\text{Single Disturbance}}\ u_{d\,\text{dist}}[n-1]$$

$eig(A) = \lambda(A) \equiv$ natural freqs

$|eig(A)| < 1$ for stability

I) A & B matrices easy to construct, Model is easy to simulate

II) We can learn <u>some</u> things from structure

i.e. $(I - A)^{-1} B\ u[\infty] = x_{\text{no dist}}[\infty]$

$(I - A)^{-1} B_d\ u_d[\infty] = x_{\text{dist}}[\infty] - x[\infty]$

if $|\lambda(A)| < 1$

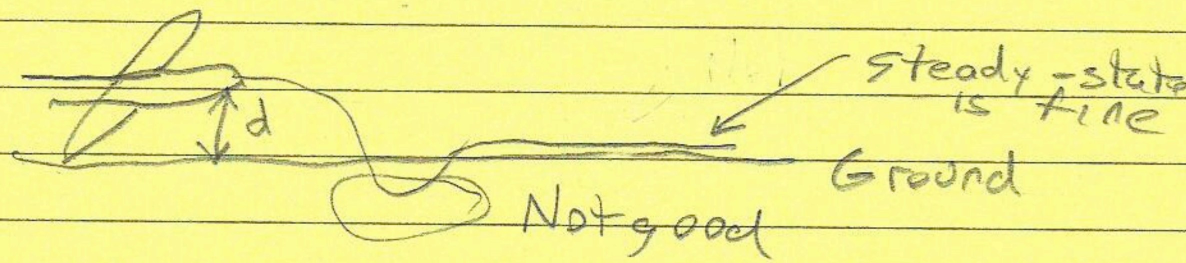
III) Easy ~~to~~ for computer Compute λ's versus K_p, K_i, K_d

~~Design~~ by examining graphs

pick parameters!

But

- What about before steady state



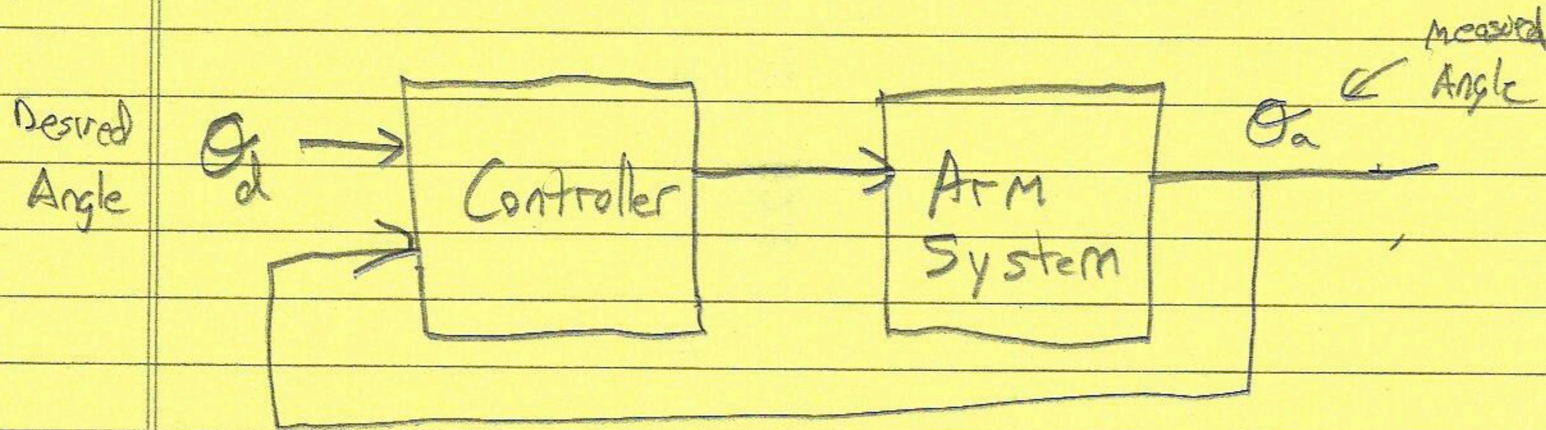
- PID control was given, how do we generate "new control ideas"

- The matrix approach is all-at-once
How do we build up detail in the model, in the disturbance, in control?

Complex systems are designed by composing simpler blocks

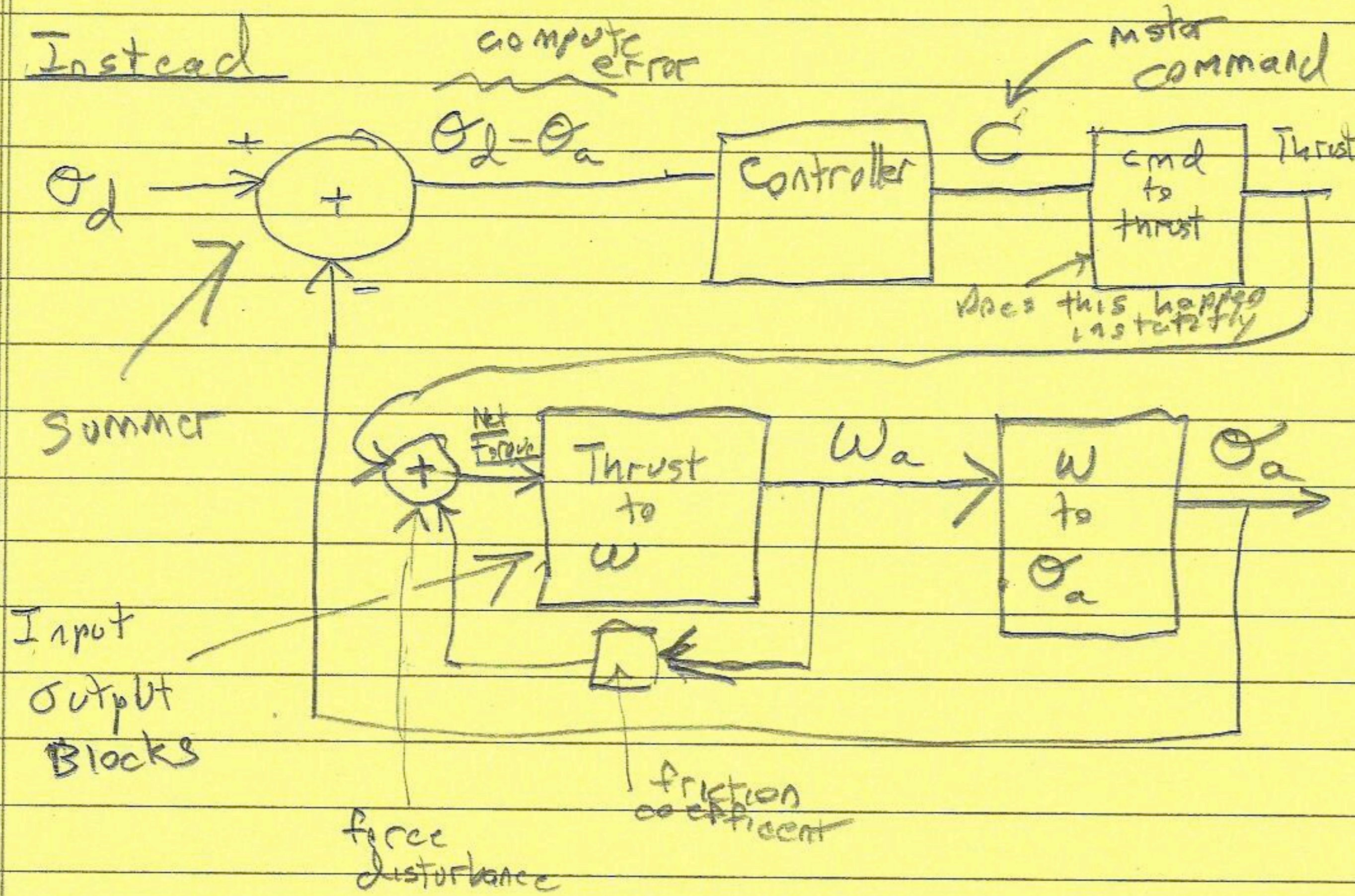
Where are the blocks?

Propeller - Arm Example



Almost No Information here

Instead



What goes in the Blocks

Cmd → Thrust $\omega \to \theta$ thrust or accel $\to \omega$

Z-transform → Difference Eqns? How do we compose? Laplace transform

Easier with Differential Equations!
We Think

Back to First-Order Scalar

DT

$y[n] = \lambda y[n-1] + \gamma u[n-1]$

Gen Sol

$$y[n] = \lambda^n y[0] + \sum_{m=0}^{n-1} \lambda^{n-m-1} u[m]$$

(Proof by induction)

CT

$\frac{d}{dt} y(t) = \lambda y(t) + \gamma u(t)$

Gen Sol

$$y(t) = e^{\lambda t} y(0) + \gamma \int_0^t e^{\lambda(t-\tau)} u(\tau) d\tau$$

(Proof uses integration by parts)

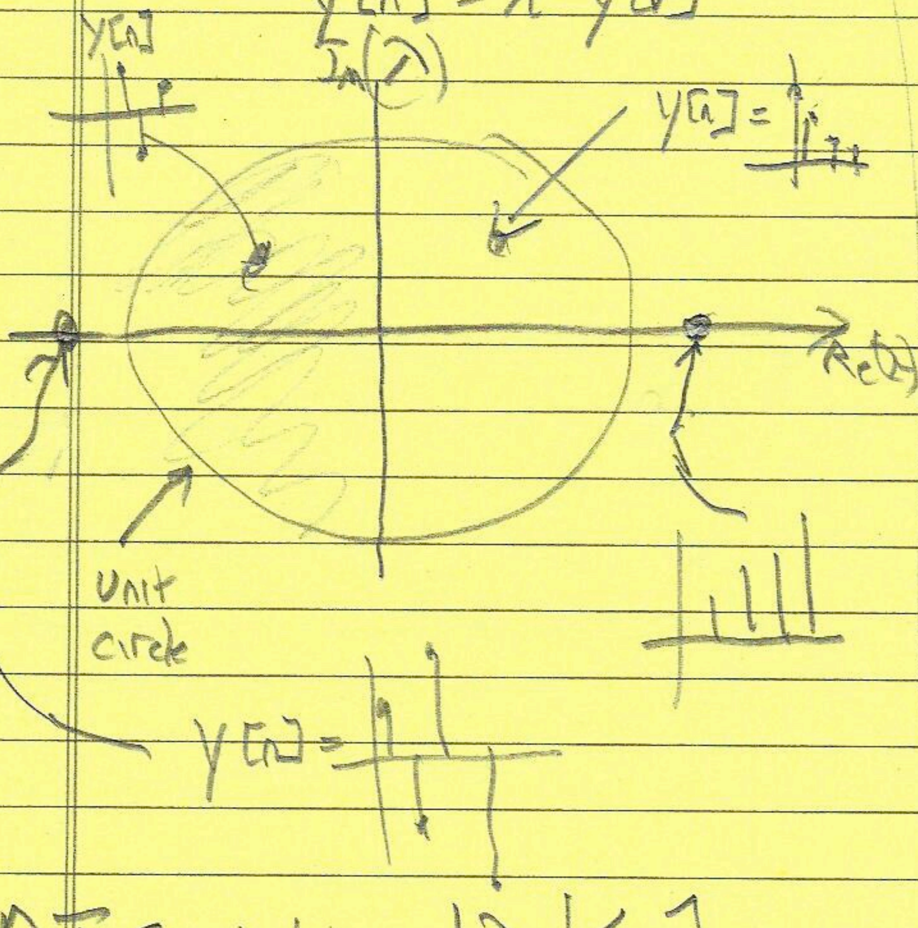
Same Linearity properties

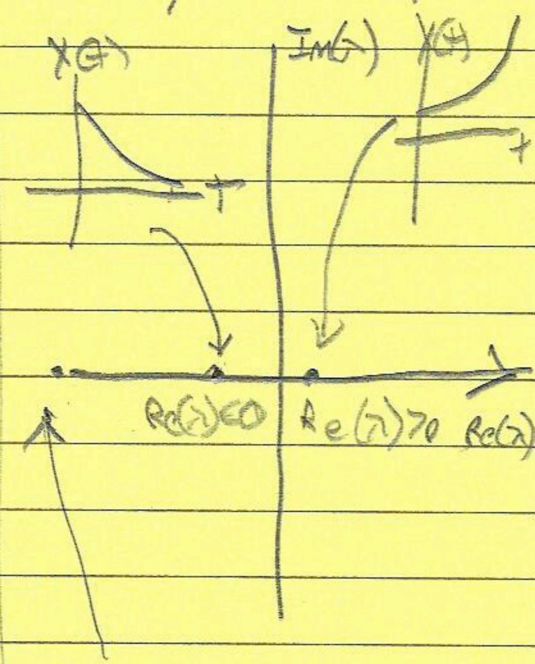
Z.I.R. $y[n] = \lambda y[n-1]$

$y[n] = \lambda^n y[0]$

Z.I.R $\dot{y}(t) = \lambda y(t)$

$y(t) = e^{\lambda t} y(0)$

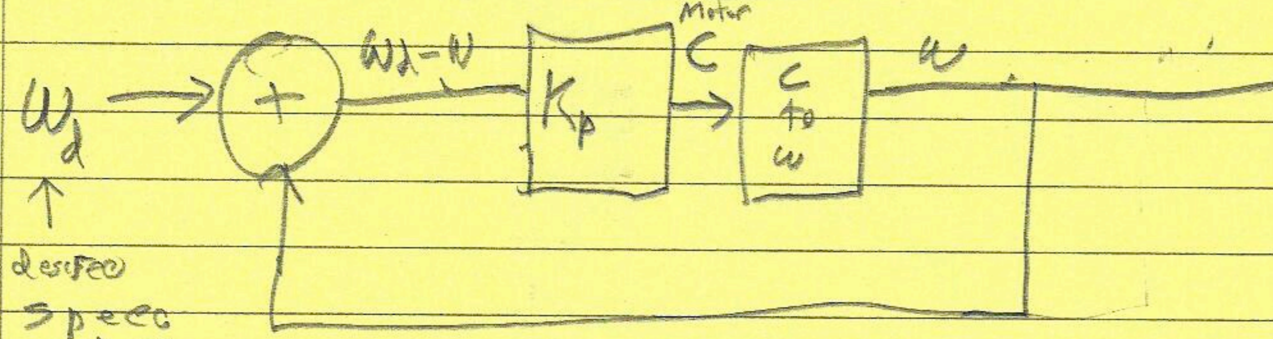




D.T. Stability $|\lambda| < 1$

C.T. Stability $Re(\lambda) < 0$

Speed Control



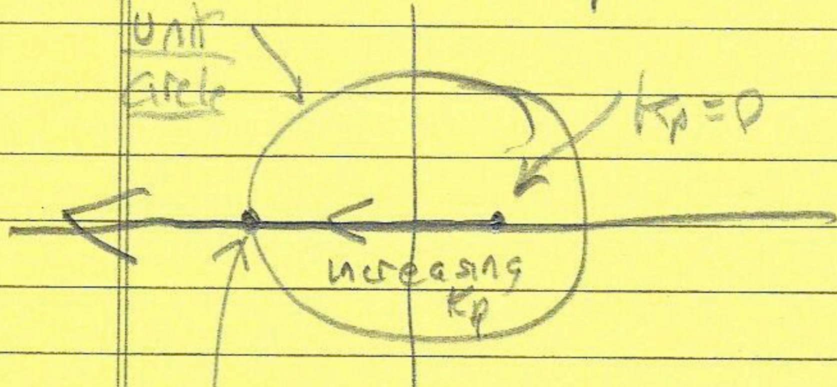
D.T.

$$\omega[n] = \omega[n-1] - \Delta T\left[\beta\omega[n-1] - \gamma C[n-1]\right]$$

$$C[n-1] = K_p(\underbrace{\omega_d[n]}_{=0} - \omega[n])$$

$$\Downarrow$$

$$\omega[n] = \left(1 - \Delta T(\beta + \gamma K_p)\right)\omega[n-1]$$



$$K_{p_{max}} = \frac{\frac{2}{\Delta T} - \beta}{\gamma}$$

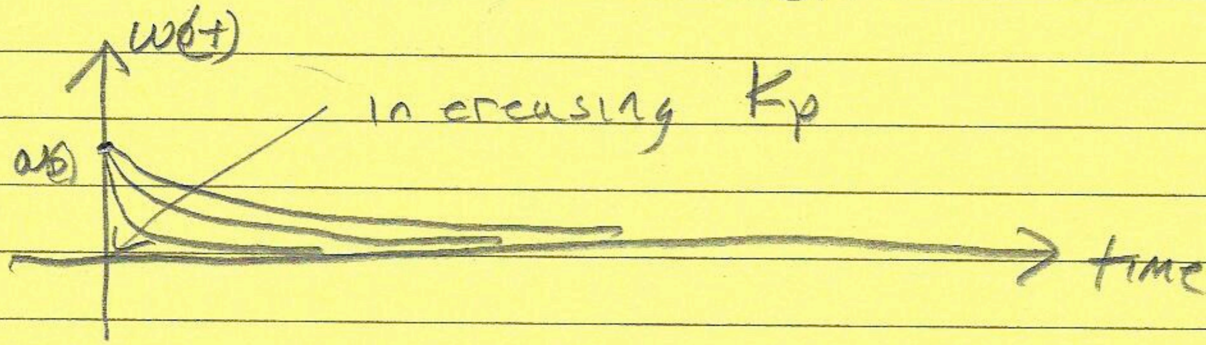
$$\uparrow$$

$$\left(1 - \Delta T(\beta + \gamma K_p)\right) = -1$$

$$\frac{d}{dt}\omega(t) = \overbrace{-\beta\omega(t)}^{\text{friction}} + \gamma K_p\left(\overset{0}{\omega_d}(t) - \omega(t)\right)$$

$$\frac{d}{dt}\omega(t) = -\left(\beta + \gamma K_p\right)\omega(t)$$

$$\omega(t) = e^{-(\beta + \gamma K_p)t}\,\omega(0)$$



<u>So</u> as $K_p \to \infty$ $\lambda \to -\infty$ $e^{\lambda t} \to 0$ ever faster!

<u>Why</u>

$$\frac{d}{dt}\omega(t) \simeq \lim_{\Delta T \to 0} \frac{\omega(t+\Delta T) - \omega(t)}{\Delta T}$$

<u>C.I.</u> is like the $\Delta T \to 0$ limit