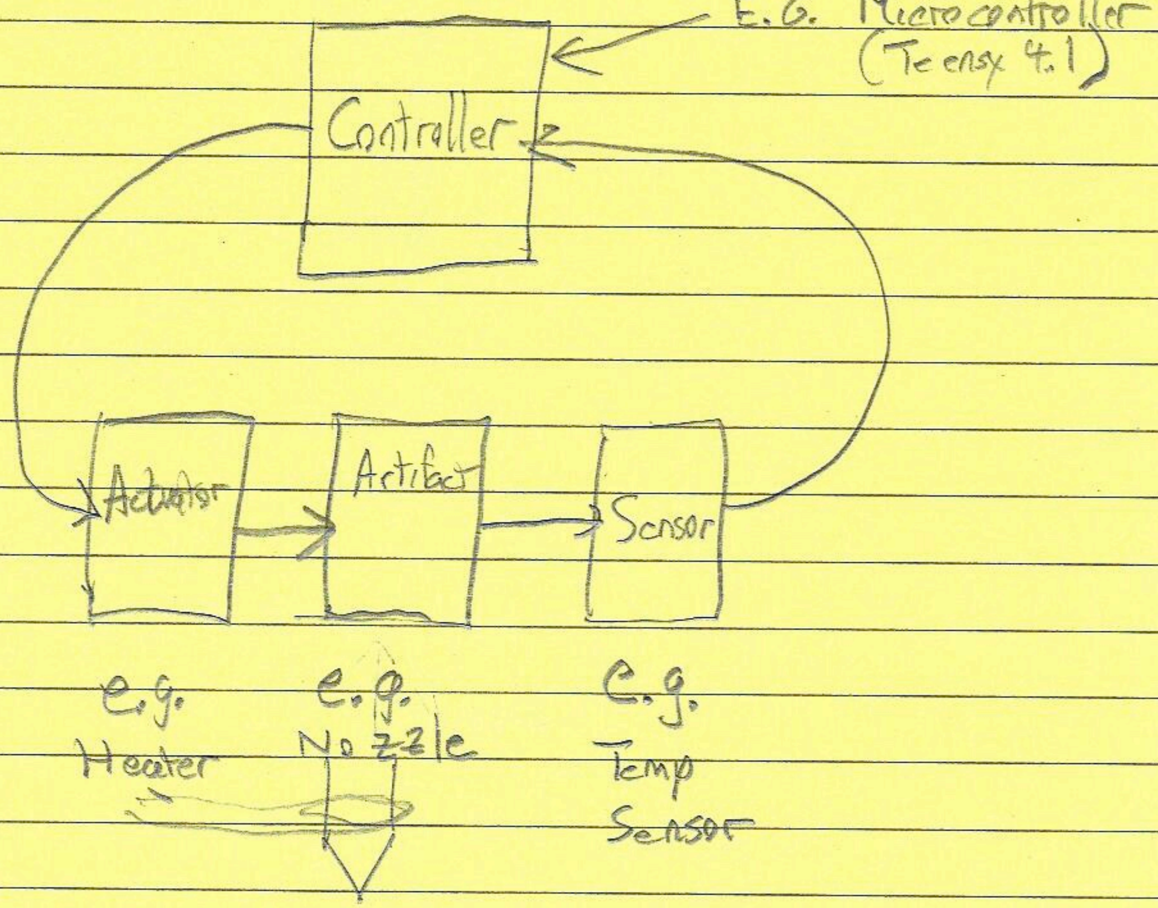
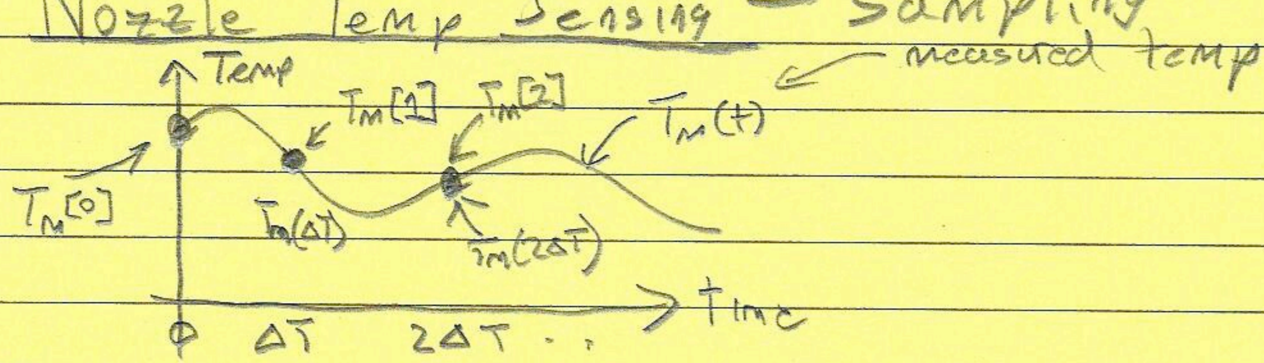
Generic Control System



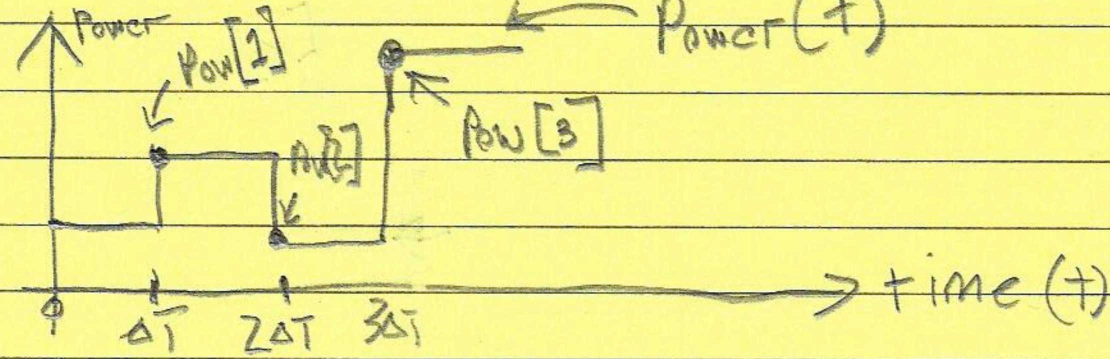
Clocked System

Every ΔT:
- Read Senor
- Compute actuator upate
- Update the actuator

Nozzle Temp Sensing — Sampling



Nozzle Heater (actuator)



Control Proportional Control (Simplest!)

$$Pow[n] = \underbrace{K_p}_{\text{Proportional Gain}} \underbrace{\left(T_d[n] - \underbrace{T_m[n]}_{\text{Read sensor}}\right)}_{\text{compute actuator update}}$$

Updated power

Artifact Model (Evolution Eqn)

$$T_M[n] = T_M[n-1] + \Delta T\, \gamma_{th}\, Pow[n-1]$$

New Nozzle Temp

Old Nozzle Model

summarize a lot! (e.g. thermal mass ...)

or

$$\frac{dT}{dt} \simeq \frac{T_M[n] - T_M[n-1]}{\Delta T} = \gamma_{th}\, Pow[n-1]$$

Now combine Model & Control

$$T_M[n] = T_M[n-1] + \Delta T\, \gamma_{th} K_p \left(T_d[n-1] - T_M[n-1]\right)$$

or

$$T_M[n] = \left(1 - \Delta T\, \gamma_{th} K_p\right) T_M[n-1] + \Delta T\, \gamma_{th} K_p\, T_d[n-1]$$

1st-Order diff eqn!

General 1st order Diff Eq

"Input" (Usually known) $u[0], u[1], u[2]$ ...

"Output" (Usually to be computed) $y[0], y[1], \ldots$

initial condition

FODE

$$\forall n \quad y[n] = \lambda y[n-1] + \gamma u[n-1]$$

$$y[1] = \lambda y[0] + \gamma u[0]$$

$$y[2] = \lambda y[1] + \gamma u[1]$$

$$= \lambda^2 y[0] + \lambda \gamma u[0] + \gamma u[1]$$

$$y[3] = \ldots$$

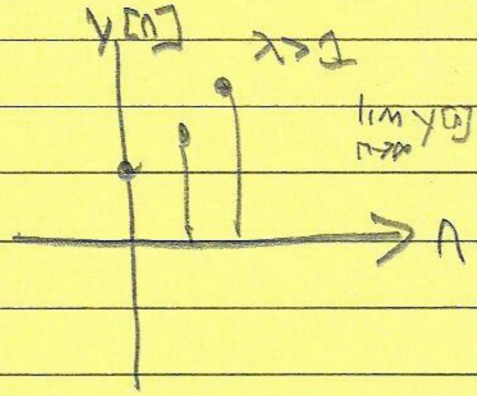
For arb n $\quad y[n] = \lambda^n y[0] + \gamma \sum_{m=0}^{n-1} \lambda^{(n-m)-1} u[m]$

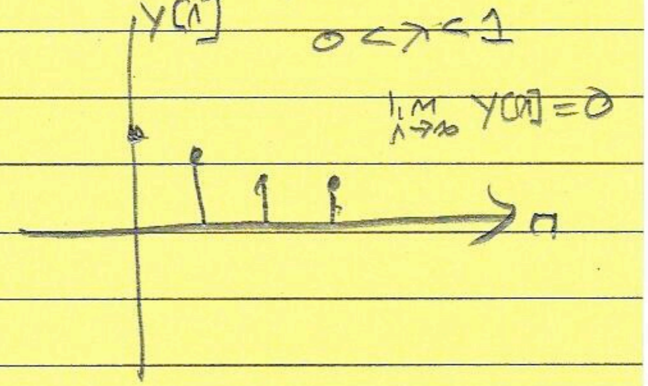
If $u[n] = 0 \ \forall n$ $\quad$ zero input response ZIR $\quad y[n] = \lambda^n y[0]$

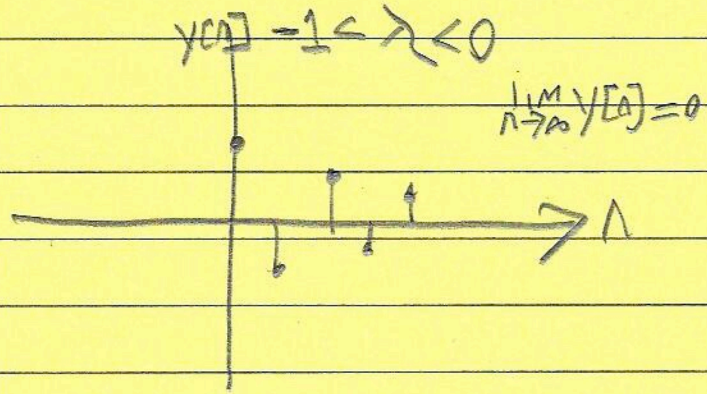
If $y[0] = 0$ $\quad$ ZSR (state) $\quad y[n] = \gamma \sum_{m=0}^{n-1} \lambda^{(n-m)-1} u[m]$

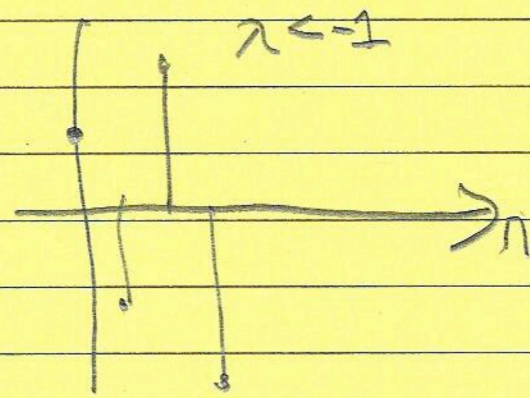
ZIR

$$y[n] = \lambda y[n-1] = \lambda^n y[0]$$









Steady-state response

if $u[n] = u_0 \ \forall n$ (Constant input)

& $|\lambda| < 1$

then

$$\lim_{n\to\infty} y[n] = \underbrace{\lim_{n\to\infty} \lambda^n y[0]}_{\to 0} + \lim_{n\to\infty} \gamma \cdot \underbrace{\sum_{m=0}^{n-1} \lambda^{(n-m)-1}}_{\text{sum}} u_0$$

$$y[\infty] = \frac{\gamma}{1-\lambda} u_0$$

Easier if $y[\infty]$ exists then

$$y[n] = \lambda y[n-1] + \gamma u_0$$

$$y[\infty] = \lambda y[\infty] + \gamma u_0$$

$$y[\infty] = \frac{\gamma}{1-\lambda} u_0$$

For Nozzle Example

$$\lambda = (1 - \Delta T \gamma_{th} K_p) \quad \gamma = \Delta T K_p \gamma_{th}$$

So $|\lambda| < 1$ iff $0 < \Delta T \gamma_{th} K_p < 2$

And if $0 < \Delta T \gamma_{th} K_p < 1$ $0 < \lambda < 1$

(No oscillations)

If $T_m[\infty]$ exists $\left(|1 - \Delta T \gamma K_p| < 1, T_d[n] = T_{d_0}\right)$

$$T_m[\infty] = \frac{\Delta T \gamma_{th} K_p}{1 - (1 - \Delta T \gamma_{th} K_p)} T_{d_0}$$

$$= 1 \cdot T_{d_0} = T_{d_0}$$

So using any $0 < K_p < \frac{2}{\Delta T \gamma_{th}}$

Steady-state temp matches desired temp!

&

$$K_p (T[\infty] - T_{d_0}) = 0 = pow[\infty]$$

Is that right?

Spose we include heat loss

$$T_m[n] = T_m[n-1] + \Delta T\, \gamma_{th}\, \overset{K_p(T_d[n] - T[n-1])}{P_{ow}[n-1]}$$

$$- \underbrace{\Delta T\, \beta\, T_m[n-1]}_{\text{Heat loss}}$$

$$T_m[n] = T_m[n-1] + \Delta T\, \gamma_{th} K_p \left(T_d[n-1] - T_m[n-1]\right)$$

$$- \Delta T\, \beta\, T_m[n-1]$$

$$= \underbrace{\left(1 - \Delta T\left(\gamma_{th} K_p + \beta\right)\right)}_{\lambda} T_m[n-1]$$

$$+ \underbrace{\Delta T\, \gamma_{th} K_p}_{\gamma}\, T_d[n-1]$$

if $|\lambda| < 1$ $T_d[n] = T_{d_0}$

$$T_m[\infty] = \left(1 - \Delta T\left(\gamma_{th} + \beta\right)\right) T_m[\infty]$$

$$+ \Delta T\, \gamma_{th} K_p\, T_{d_0}$$

$$T_m[\infty] = \underbrace{\frac{\cancel{\Delta T}\, \gamma_{th} K_p}{\cancel{\Delta T}\left(\gamma_{th} K_p + \beta\right)}}_{\text{is } \approx 1 \text{ if } |K_p \gamma_{th}| >> |\beta|} T_{d_0}$$

<u>So</u> Bigger K_p better accuracy