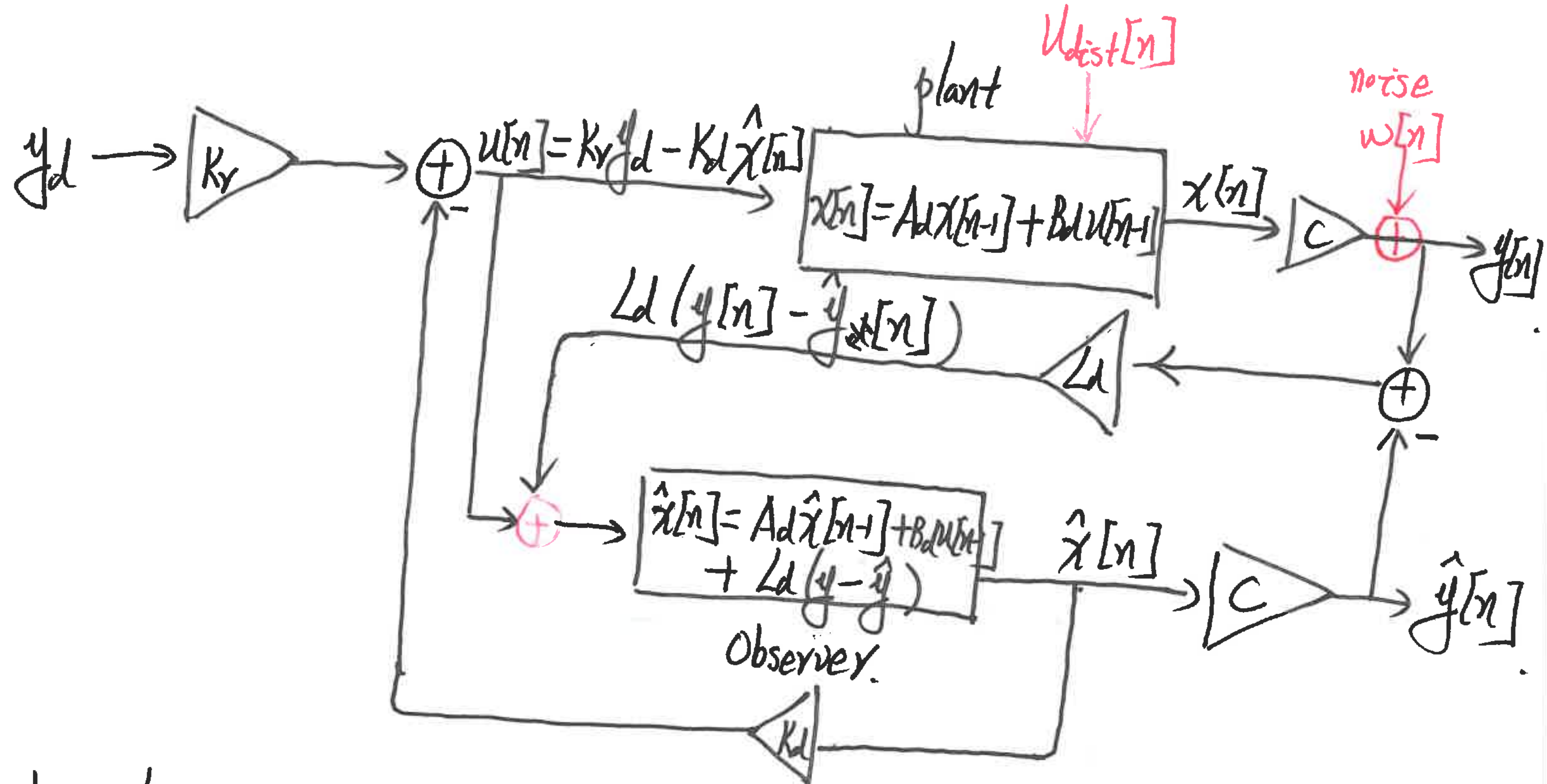
12/03/25.

* Observer based State Feedback (D.T.)

1. Review:



Physical system: $x[n] = A_d x[n-1] + B_d u[n-1] + \underbrace{B_{dist} u_{dist}[n-1]}_{\text{disturbance}}$ ①

$y[n] = C_d x[n] + \underbrace{w[n]}_{\text{noise}}$

Estimator : $\hat{x}[n] = A_d \hat{x}[n-1] + B_d u[n-1] + L_d (y[n-1] - \hat{y}[n-1])$ ②

$\hat{y}[n] = C_d \hat{x}[n]$

Controller : $u[n] = K_{rd} y_d[n] - K_d \hat{x}[n]$

Think how the real feedback control system works:

a. The plant in real life is a physical existence, it does not explicitly contain any equation during its operation.

We model it with equations (Eq. ①) so that we can determine gains (K_d). Eq ① only exists in our note book or computer!

b. The estimator needs to be ~~so~~ implemented explicitly (on MCU), to accompany the plant. Eq ②. exists in the micro computer, i.e. Teensy.

c. Eq ① and Eq ② are used before real experiment to determine K_d, L_d (on a computer through simulation).

d. Real hardware implementation:

For $n = 1 : N$.

i) Measure $y[n]$, compare with $\hat{y}[n]$, get $L_d(y[n] - \hat{y}[n])$

ii). Use predicted $\hat{x}[n]$ in last step, determine $u[n] = K_r y_d - K_d \hat{x}[n]$

iii) Send $u[n]$ to $\begin{cases} \text{machine} \longrightarrow \text{physical system evolution} \\ \text{estimator} \end{cases}$

$$\hat{x}[n] = A_d \hat{x}[n-1] + B_d u[n-1] + L_d(y[n-1] - \hat{y}[n-1])$$

$$\hat{y}[n] = C_d \hat{x}[n].$$

End.

2. Understand the influence of disturbance, noise on gains' choice.

Track dynamics of error: $err[n] = x[n] - \hat{x}[n]$.

① − ②: $err[n] = A_d\, err[n-1] - L_d C_d\, err[n-1] + B_{dist} U_{dist}[n-1] - L_d\, w[n-1]$.

Or $err[n] = (A_d - L_d C_d)\, err[n-1] + B_{dist} U_{dist}[n-1] - L_d\, w[n-1]$ ③.

Rewrite Eq ① for $x[n]$ in closed-loop form by substituting

$$u[n] = K_r y_d[n] - K_d(x[n] - err[n])$$

$$x[n] = (A_d - B_d K_d)\, x[n-1] + B_d K_d\, err[n-1] + B_d K_r y_d[n-1] + B_{dist} U_{dist}[n-1] \quad ④.$$

Concatenate ③ and ④, consider the state space $\begin{pmatrix} x[n] \\ err[n] \end{pmatrix}$.

$$\begin{pmatrix} x[n] \\ err[n] \end{pmatrix} = \begin{pmatrix} A_d - B_d K_d & B_d K_d \\ 0 & A_d - L_d C_d \end{pmatrix} \begin{pmatrix} x[n-1] \\ err[n-1] \end{pmatrix} + \begin{pmatrix} B_d K_r \\ 0 \end{pmatrix} y_d[n-1] \quad ⑤.$$

$$+ \begin{pmatrix} B_{dist} \\ B_{dist} \end{pmatrix} \underbrace{U_{dist}[n-1]}_{\text{disturbance input}} - \begin{pmatrix} 0 \\ L_d \end{pmatrix} \underbrace{w[n-1]}_{\text{noise input}}.$$

a). $x[n]$ depends on $err[n]$, but $err[n]$ does not depend on $x[n]$,

Or $err[n]$ is only determined by matrix $A_d - L_d C_d$. (~~and~~ L_d gains).

Can consider $err[n]$ in the reduced space:

$$err[\infty] = (A_d - L_d C_d)\, err[\infty] + B_{dist} U_{dist}[\infty] - L_d w[\infty]$$

$$err[\infty] = (I - (A_d - L_d C_d))^{-1} B_{dist} U_{dist}[\infty] \leftarrow \text{perturbation response}$$

$$\mp (I - (A_d - L_d C_d))^{-1} L_d w[\infty] \leftarrow \text{noise response.}$$

How to choose L_d?

Large $L_d \rightarrow$ pole $(A_d - L_d C_d) \rightarrow 0, \rightarrow (I - (A_d - L_d C_d))^{-1} \sim I^{-1}$

$\rightarrow$ perturbation or initial state discrepancy gets suppressed!

In the limit $err[n] \rightarrow 0$, Eq ⑤ reduces to

$$x_{ob}[n] = (A_d - B_d K_d)\, x_{ob}[n-1] + B_d K_r y_d[n-1] \quad (\text{no error any more}).$$

The same as in the measured state feedback.

However, $(I - (A_d - L_d C_d))^{-1} L_d$ gets too large $\longrightarrow$ noise amplified.

General Guideline: ① Choose L_d such that poles $(A_d - L_d C_d)$ are smaller than those of $(A_d - B_d K_d)$

② Avoid too high L_d which leads to noisy controller.

A simplified Mag-lev model (no electric pole)

$$\frac{d}{dt}\begin{pmatrix}\Delta y\\ vel\end{pmatrix} = \begin{pmatrix}0 & 1\\ \gamma_{dv/dy} & 0\end{pmatrix}\begin{pmatrix}\Delta y\\ vel\end{pmatrix} + \begin{pmatrix}0\\ \gamma\end{pmatrix}u(t) + B_{dist}u_{dist}(t)$$

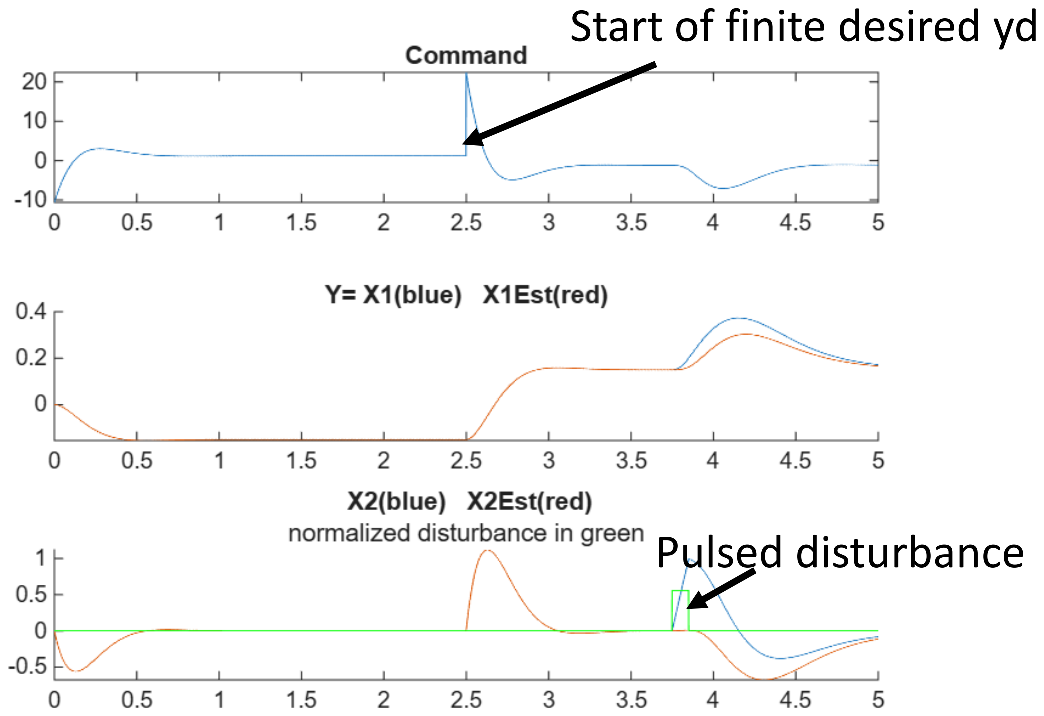
(Dummy) numerical values for parameters:

$\gamma_{dv/dy} = 8,\ \gamma = 1, B_{dist} = \begin{pmatrix}0\\ 0.2\end{pmatrix}, y_d = 0.15,$

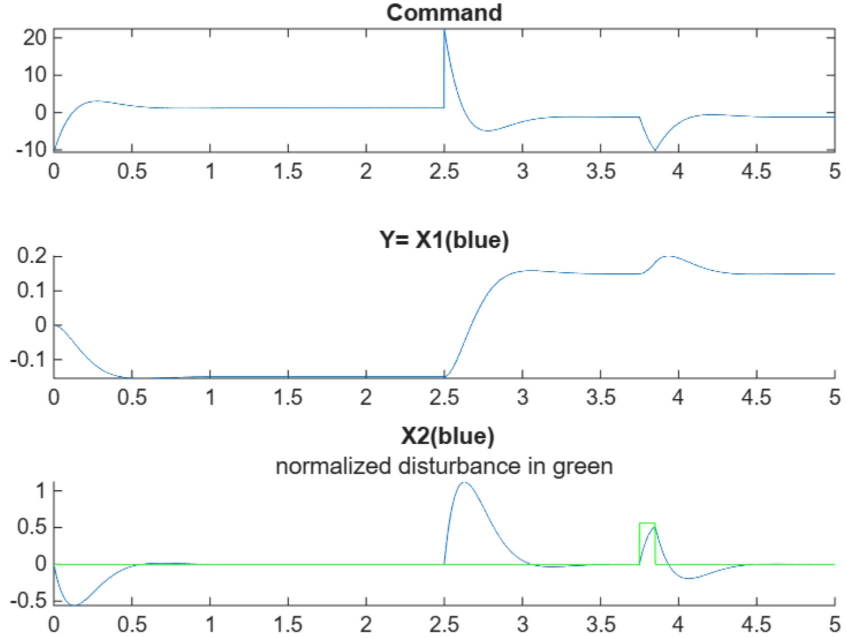
Convert into DT model, set up DT estimator, and simulate in a similar way as what you will see in Friday's lab

Large Poles for err[n] (closer to unit circle)

DT Controller and Observer Gains*****
Kd Gains: 78.7154 12.5471 Krd:70.7154
Ld gains: 0.009008 0.028036
*****||DT Poles||*****
controller poles: 0.99373 0.99373
estimator poles: 0.995 0.996



In comparison with measured state feedback



Medium Poles for err[n]

DT Controller and Observer Gains*****

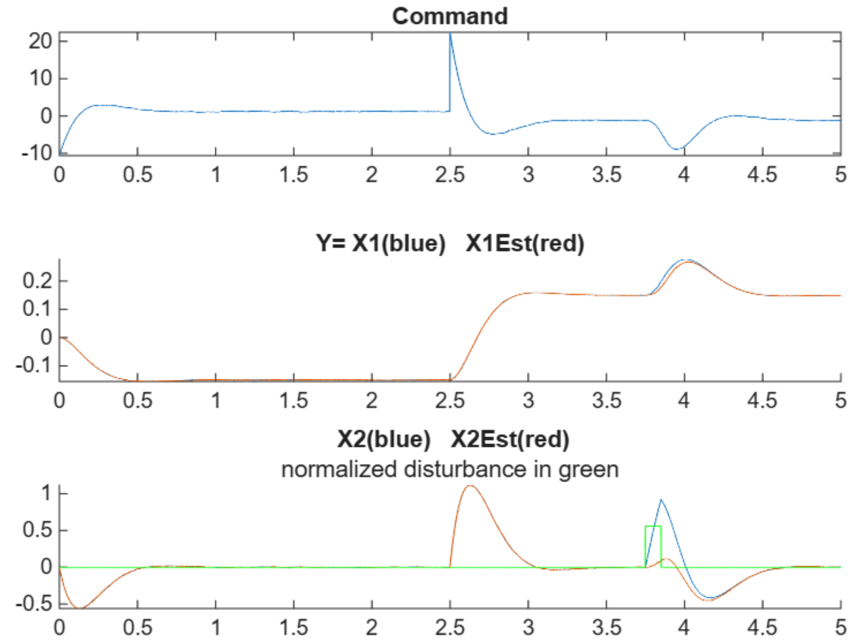
Kd Gains: 78.7154 12.5471 Krd:70.7154

Ld gains: 0.029008 0.21812

*****||DT Poles||*****

controller poles: 0.99373 0.99373

estimator poles: 0.985 0.986



Very small Poles for err[n]

DT Controller and Observer Gains*****

Kd Gains: 78.7154 12.5471 Krd:70.7154

Ld gains: 0.970008 235.2116

*****||DT Poles||*****

controller poles: 0.99373 0.99373

estimator poles: 0.51 0.52