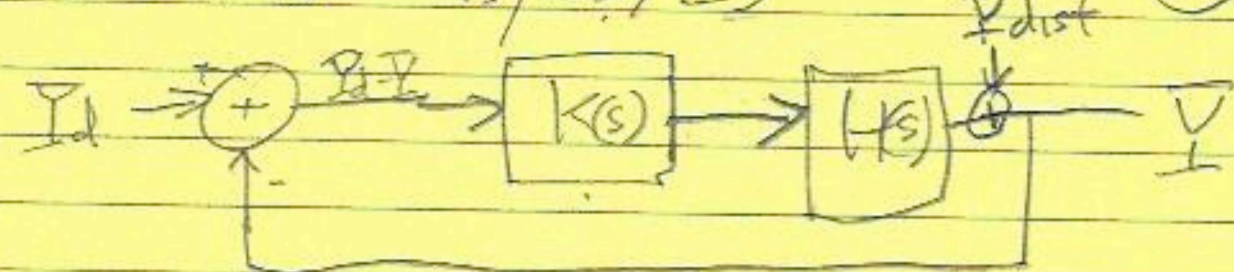
6.3100 10/22/25 ①

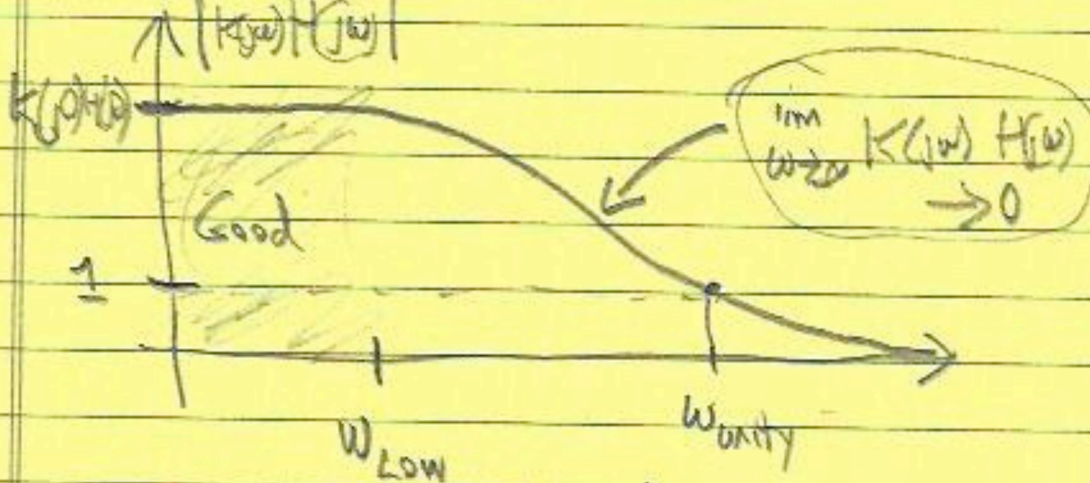


In sinusoidal steady-state (so system assumed stable)

$$\underline{V} = \underbrace{\frac{K(j\omega)H(j\omega)}{1 + K(j\omega)H(j\omega)}}_{G(j\omega)} \underline{V}_d + \underbrace{\frac{1}{1 + K(j\omega)H(j\omega)}}_{G_{dist}(j\omega)} \underline{V}_{dist}$$

if $|K(j\omega)H(j\omega)| >> 1$ $\quad \underbrace{G(j\omega) \simeq 1}_{\text{good tracking}}$ $\quad \underbrace{G_{dist}(j\omega) \cong 0}_{\text{good dist rejection}}$

So design $K(s)$ so $|K(j\omega)H(j\omega)| >> 1$ $\quad \underbrace{\omega < \omega_{Low}}_{\text{freqs you care about}}$



1) physical sys don't respond instantly
2) Practical controllers can't increase $K(j\omega)$ with ω fast enough

$|K(\omega_{unity})H(\omega_{unity})| = 1$ by definition

But

Avoid $\measuredangle = -180$ at ω_{unity}

If $\measuredangle\left(K(j\omega_{unity})H(j\omega_{unity})\right) = -180°$

then $K(j\omega_{unity})H(j\omega_{unity}) = -1$

and $\underbrace{G(j\omega_{unity}) = \infty \quad G_{dist}(j\omega_{unity}) = \infty}$

Bad Bad Bad

If $\measuredangle K(j\omega_{unity})H(j\omega_{unity})$ is near $-180 \Rightarrow G(j\omega_{unity}) \gg 1$, $G_{dist}(j\omega_{unity}) \gg 1$

Calculating Angle

$$\measuredangle\left(\underbrace{K(j\omega)}_{M_K e^{j\phi_K}}\underbrace{H(j\omega)}_{M_H e^{j\phi_H}}\right) = \measuredangle K(j\omega) + \measuredangle H(j\omega)$$

$$= M_K M_H e^{j(\phi_K + \phi_H)}$$

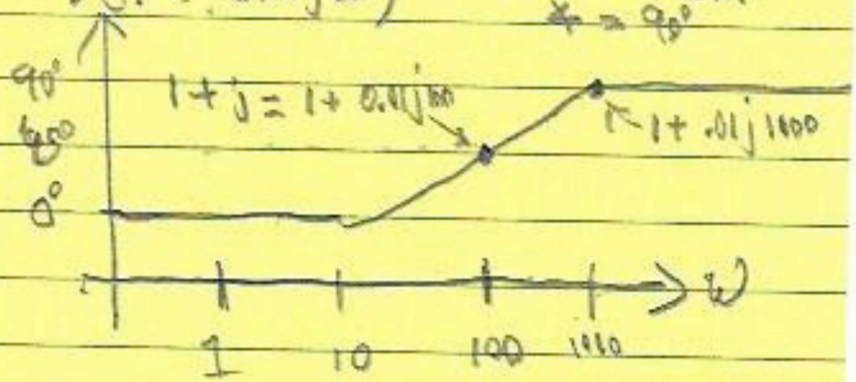
Examples

$$\measuredangle\left(\underbrace{A}_{1} + j\omega \underbrace{B}_{0.01}\right)$$

$\measuredangle(1 + 0.01 j\omega)$

For $\omega \ll \frac{1}{0.01}$, $\measuredangle = 0$

For $\omega \gg \frac{1}{0.01}$, $\measuredangle = 90°$

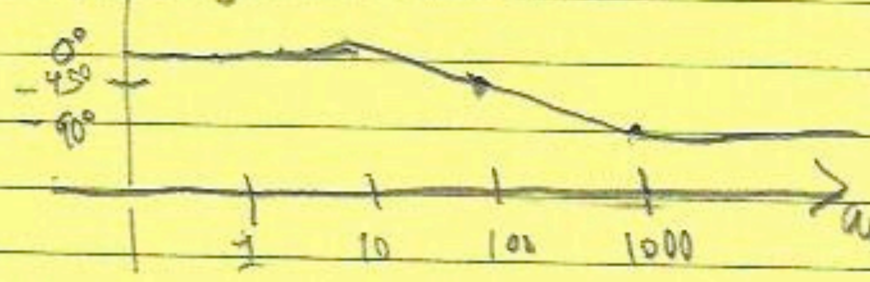


log spacing

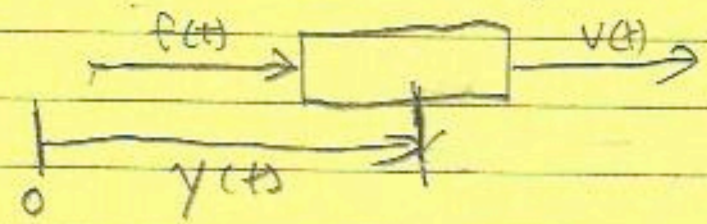
$\measuredangle \frac{1}{1+0.01j\omega}$

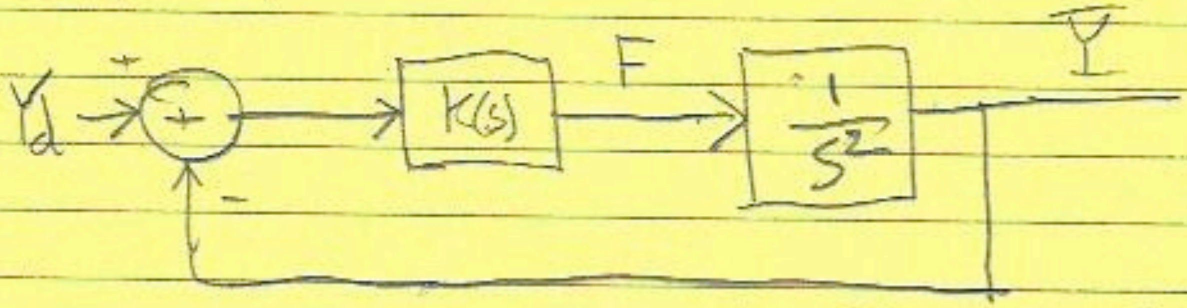
$$\measuredangle \frac{1}{A + j\omega B}$$

$$\frac{1}{M} e^{-j\phi}$$

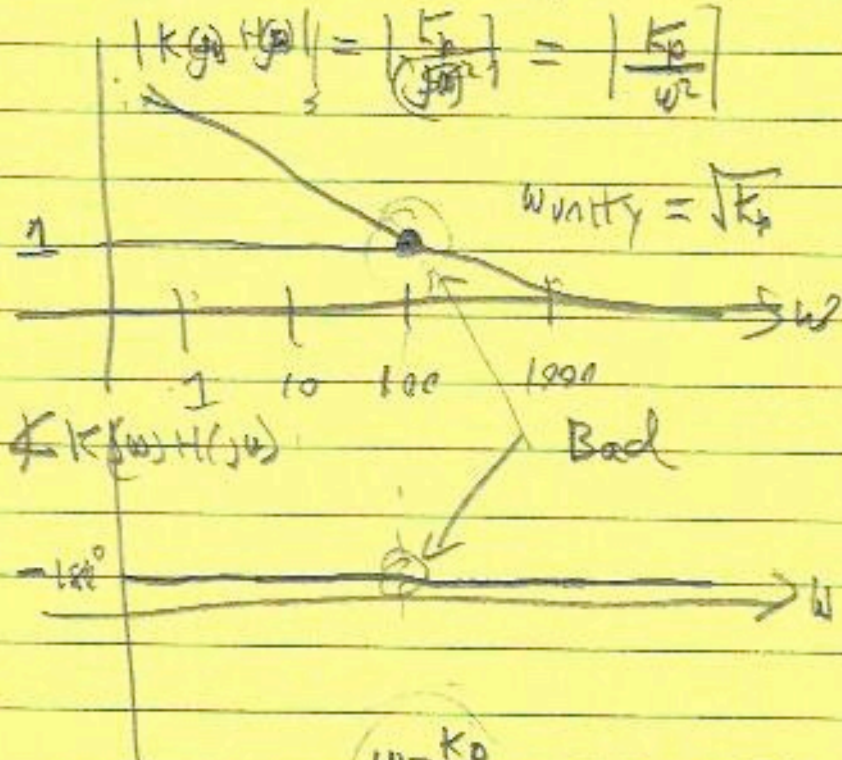


Sliding Brick w/o Friction

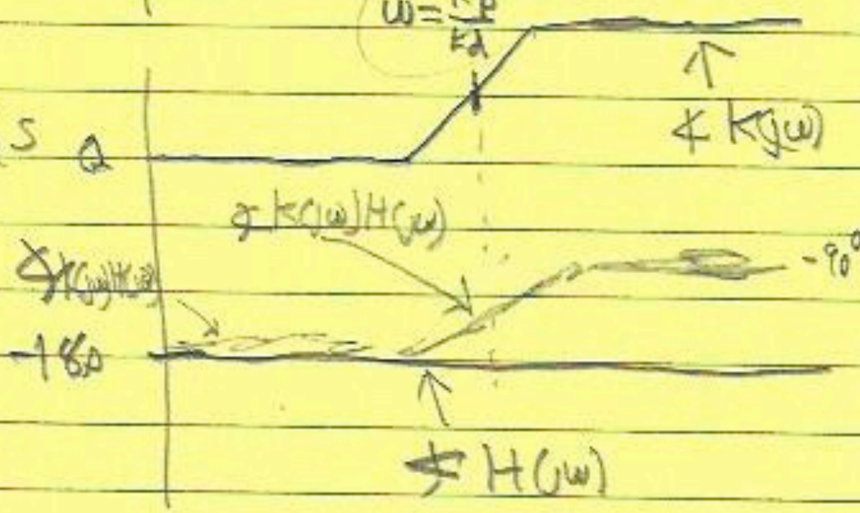




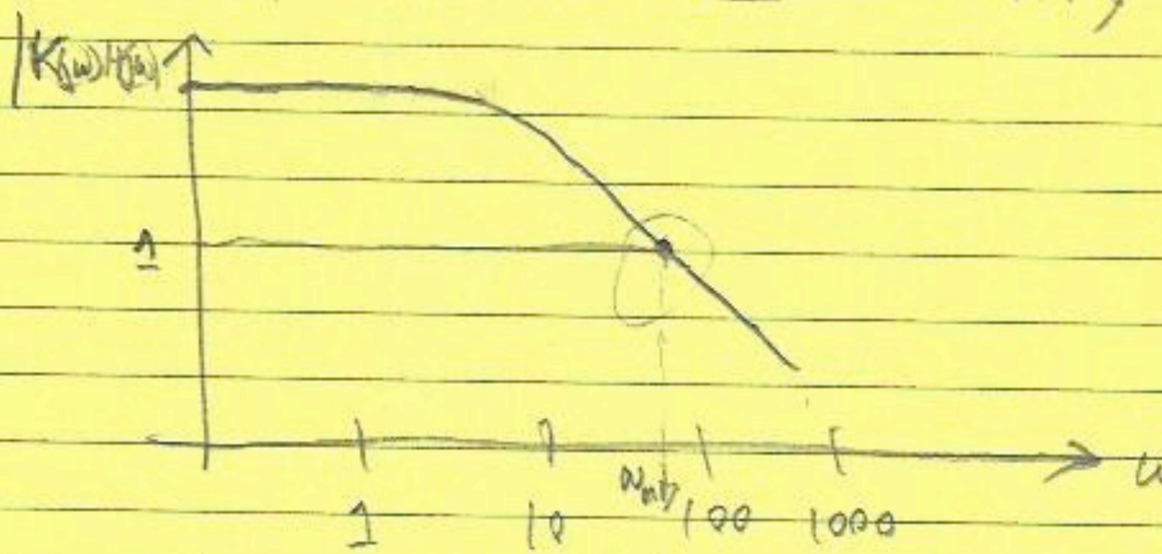
If $K(s) = K_p$

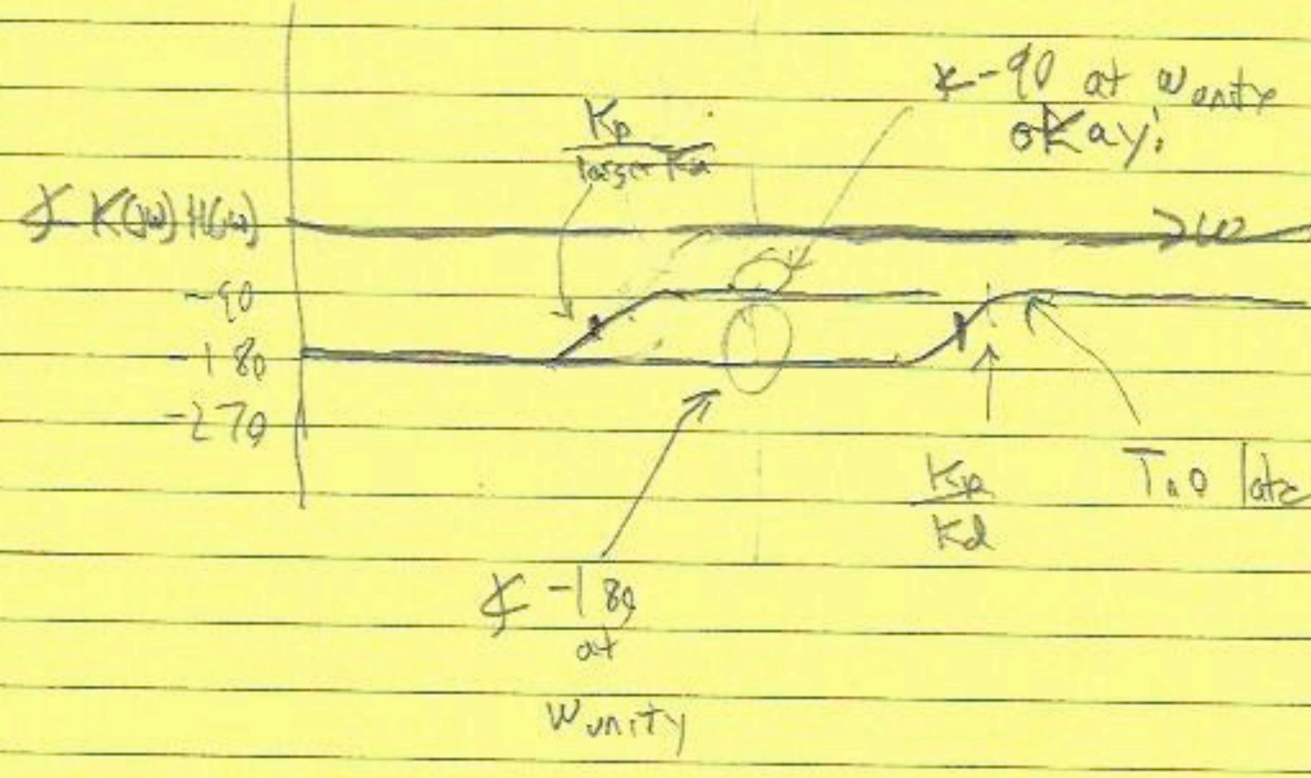


If $K(s) = K_p + K_d s$



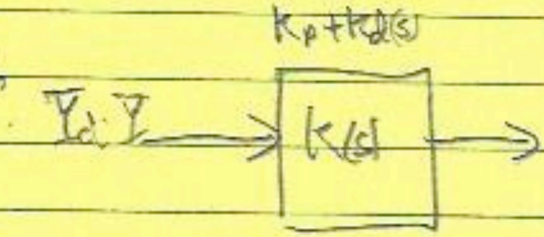
Make Sure $\measuredangle$ Rises before ω_{unity}

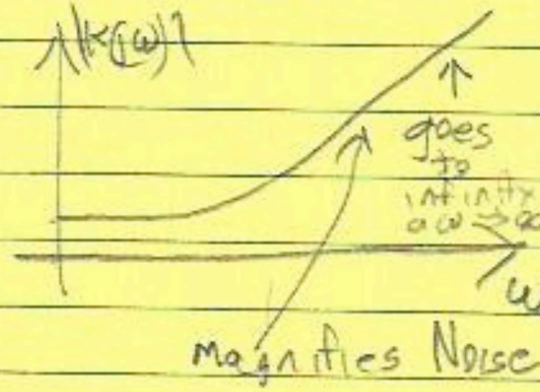




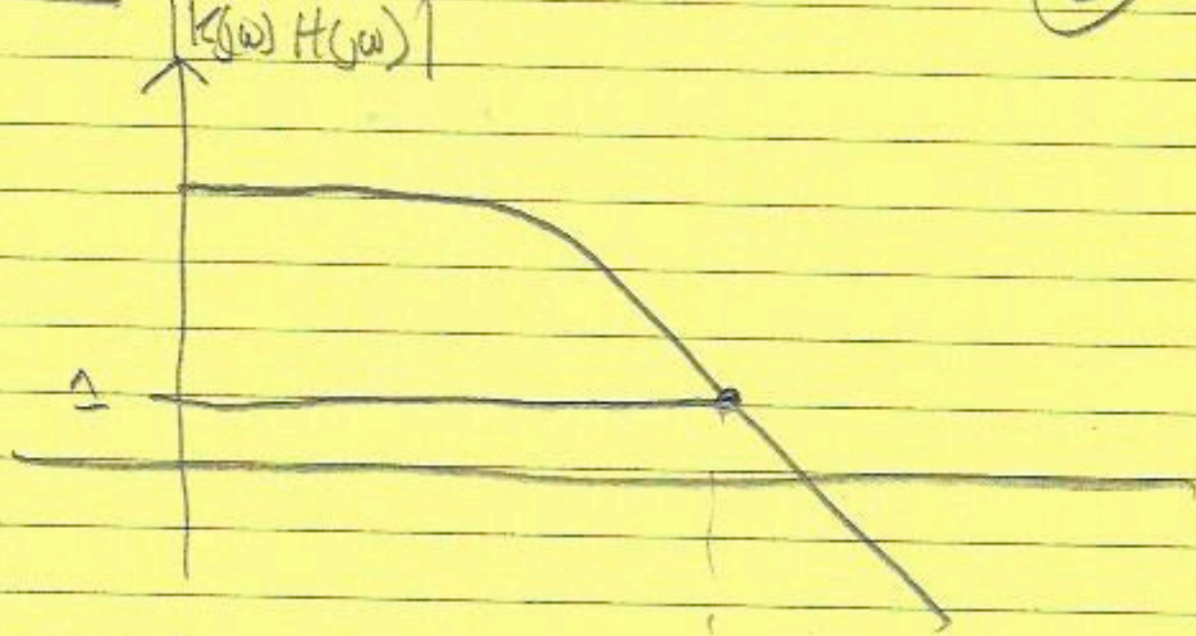
So make Kd bigger we knew that!

And





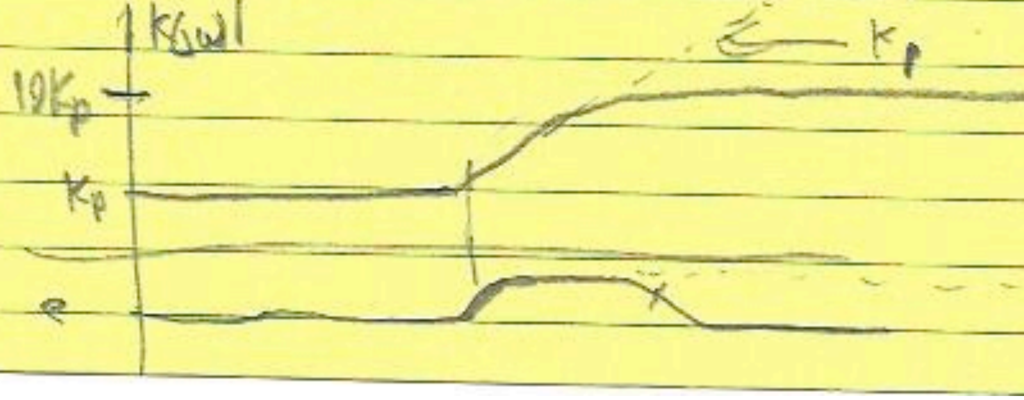
Spose



$$K(j\omega) = K_p \frac{1 + \frac{K_d}{K_p}(j\omega)}{1 + 0.1\frac{K_d}{K_p}(j\omega)}$$

Lead

$\lim_{\omega \to 0} = K_p$ $\lim_{\omega \to \infty} = 10 K_p$



Some Terminology