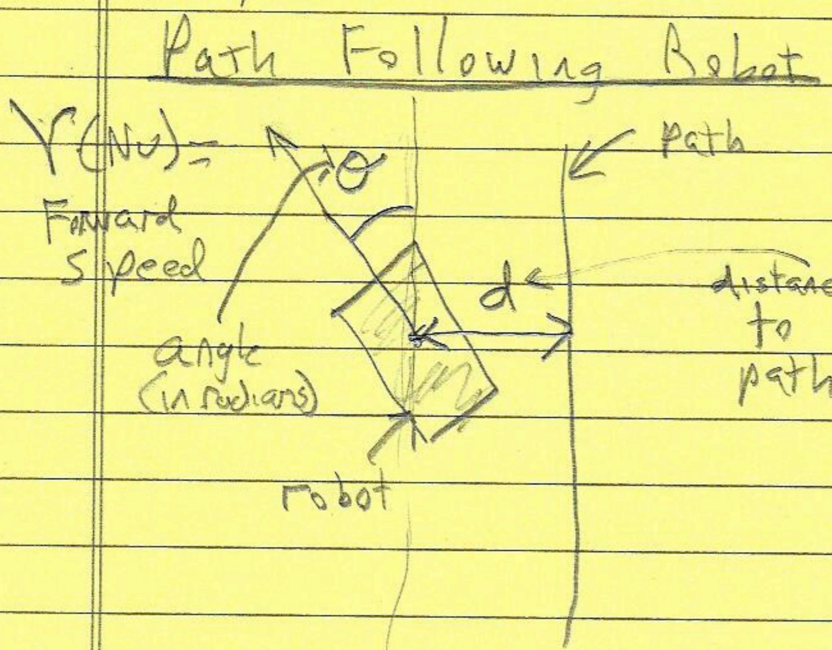
9/15/25 6.3100/2 (1)

Path Following Robot



γ = Robot Forward Speed
θ = robot angle
d = dist. to path
ΔT = time between updates
ω = robot rotational velocity

the control!

Eqns θ is Not piecewise constant

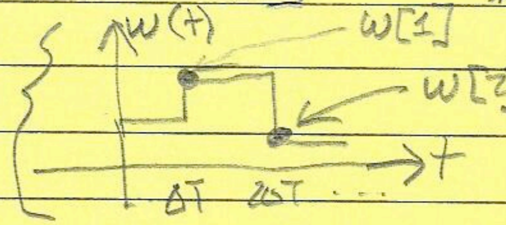
Nu ⇒ Forward Sp.

$$d[n] \overset{\approx}{=} d[n-1] + \Delta T \, \gamma \underbrace{\sin\theta[n-1]}_{\simeq \theta[n-1] \text{ (if } \theta \text{ is small)}}$$

$$\theta[n] = \theta[n-1] + \Delta T \, \omega[n-1]$$

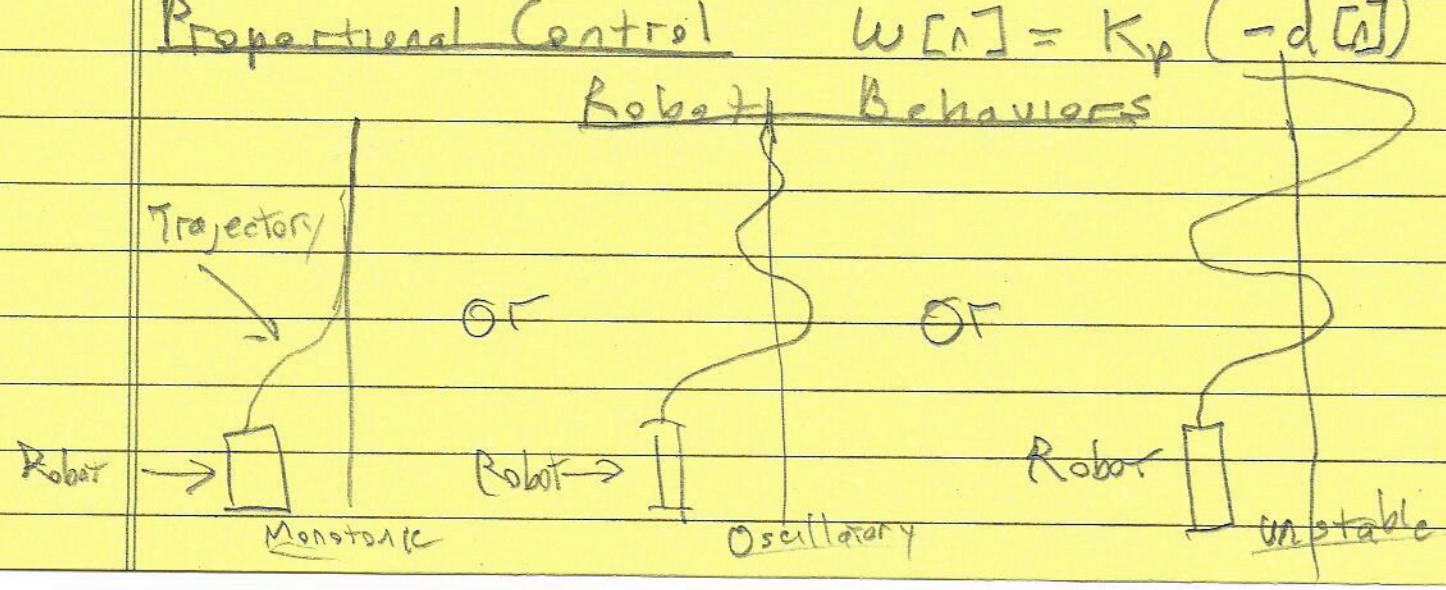
ω is piecewise constant

we set ω's



Proportional Control $\omega[n] = K_p(-d[n])$

Robot Behaviors



Matrix Form

$$\underbrace{\begin{bmatrix} d[n] \\ \theta[n] \end{bmatrix}}_{N \updownarrow \vec{x}[n]} = \underbrace{\begin{bmatrix} 1 & \Delta T\,\gamma \\ 0 & 1 \end{bmatrix}}_{N \updownarrow A \overleftrightarrow{N}} \underbrace{\begin{bmatrix} d[n-1] \\ \theta[n-1] \end{bmatrix}}_{N \updownarrow \vec{x}[n-1]} + \underbrace{\begin{bmatrix} 0 \\ \Delta T \end{bmatrix}}_{N \updownarrow B} w[n-1]$$

General Solution F.O.V.D.E.

Scalar $y[n] = \lambda y[n-1] + \gamma u[n-1]$

$$\Downarrow$$

$$y[n] = \lambda^n y[0] + \sum_{m=0}^{n-1} \lambda^{n-1-m} \gamma u[m]$$

Move outside

Vector $x[n] = A x[n-1] + B u[n-1]$

$$\Downarrow$$

$$x[n] = A^n x[0] + \sum A^{n-1-m} B u[m]$$

matrices don't commute

Assume A is diagonalizable

$A \in \mathbb{R}^{N \times N}$ is diag iff $\exists$

$\{\vec{V}_1, \ldots, \vec{V}_N\}$ lin indep s.t.

$A\vec{V}_i = \lambda_i \vec{V}_i \quad \forall\, i \in [1, \ldots N]$

or

$A\underline{V} = \underline{V}\begin{bmatrix}\lambda_1 & & \\ & \ddots & \\ & & \lambda_N\end{bmatrix}$ and $\underline{V}^{-1}$ exists

$\Rightarrow A = \underline{V}\begin{bmatrix}\lambda_1 & & \\ & \ddots & \\ & & \lambda_N\end{bmatrix}\underline{V}^{-1}$

Sufficient Conditions

$\left.\begin{array}{l} A = A^T \text{ (symmetry)} \\ A \text{ has } n \text{ unique evals} \end{array}\right\}$ Not Nec

ex. $\underbrace{A = \begin{bmatrix}1 & & \\ & 1 & \\ & & 1\end{bmatrix}}_{\text{lower triangular}}$ is diag. but has all λ's equal

&

is not symmetric

If A is diagonalizable

$A^k = A \cdot A \cdot A \cdots A = (\underline{V}\lambda\underline{V}^{-1})(\underline{V}\lambda\underline{V}^{-1})\ldots$

$= \underline{V}\lambda^k\underline{V}^{-1} = \underline{V}\begin{bmatrix}\lambda_1^k & & \\ & \ddots & \\ & & \lambda_N^k\end{bmatrix}\underline{V}^{-1}$

If $u[n] = K_p(-d[n])$

$$\begin{bmatrix} d[n] \\ \theta[n] \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & \Delta T V \\ -\Delta T K_p & 1 \end{bmatrix}}_{A} \begin{bmatrix} d[n-1] \\ \theta[n-1] \end{bmatrix}$$

$$\begin{bmatrix} d[n] \\ \theta[n] \end{bmatrix} = A^n \begin{bmatrix} d[0] \\ \theta[0] \end{bmatrix} = \underbrace{V}_{\begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}} \begin{bmatrix} \lambda_1^n & \\ & \lambda_2^n \end{bmatrix} \underbrace{V^{-1} \begin{bmatrix} d[0] \\ \theta[n] \end{bmatrix}}_{\begin{bmatrix} p_1 \\ p_2 \end{bmatrix}}$$

Note

$$d[n] = c_1 \lambda_1^n + c_2 \lambda_2^n$$

constants

Find c_1 c_2 without computing V

Spose know first two pts & λ's

$$\begin{cases} d[0] = c_1 \lambda_1^0 + c_2 \lambda_2^0 = c_1 + c_2 \\ d[1] = c_1 \lambda_1 + c_2 \lambda_2 \end{cases}$$

Solve for c_1, c_2!

$d[n] \to 0$ as $n \to \infty$ if $|\lambda_{1,2}| < 1$

Find λ's

If $\det(\lambda I - A) = 0 \Rightarrow \lambda$ is an eval

2x2 case

$$\det\left(\begin{bmatrix} \lambda-1 & \gamma\Delta T \\ -\Delta T K_p & \lambda-1 \end{bmatrix}\right) = 0$$

$$(\lambda-1)^2 + (\Delta T)^2 \gamma K_p = 0$$

$-\frac{b}{2} \pm \sqrt{\frac{b^2}{4} - ac}$ $\Leftrightarrow \lambda^2 - 2\lambda + (1 + \Delta T^2 \gamma K_p) = 0$

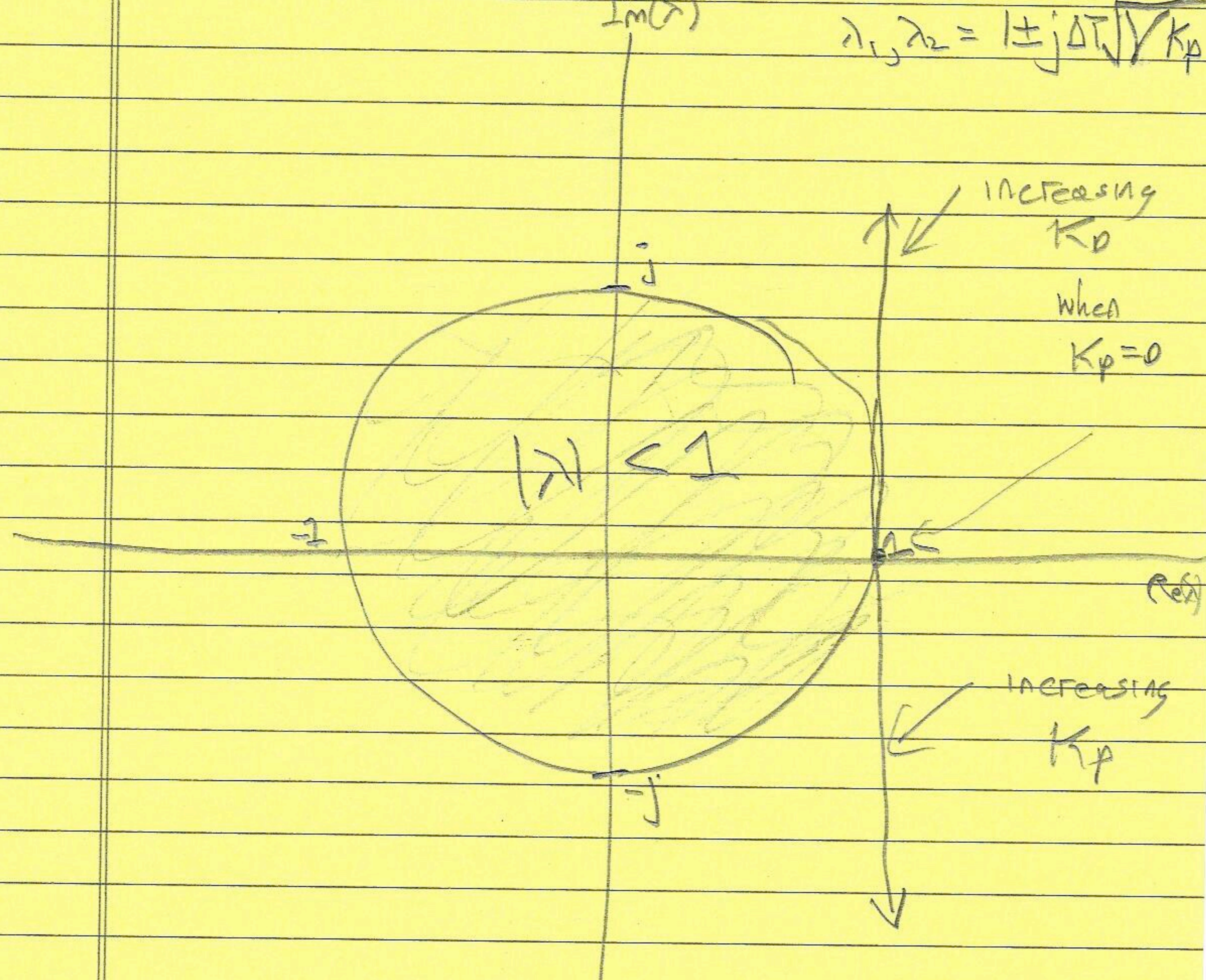
$$\lambda_1, \lambda_2 = 1 \pm \sqrt{1 - (1 + \Delta T^2 \gamma K_p)}$$

$$= 1 \pm \sqrt{-\Delta T^2 \underbrace{\gamma}_{>0} \underbrace{K_p}_{>0}}$$

$$= 1 \pm j\Delta T\sqrt{\gamma K_p}$$

λ's are complex!

Complex Plane



Is proportional control of path following robot ever stable?

No!