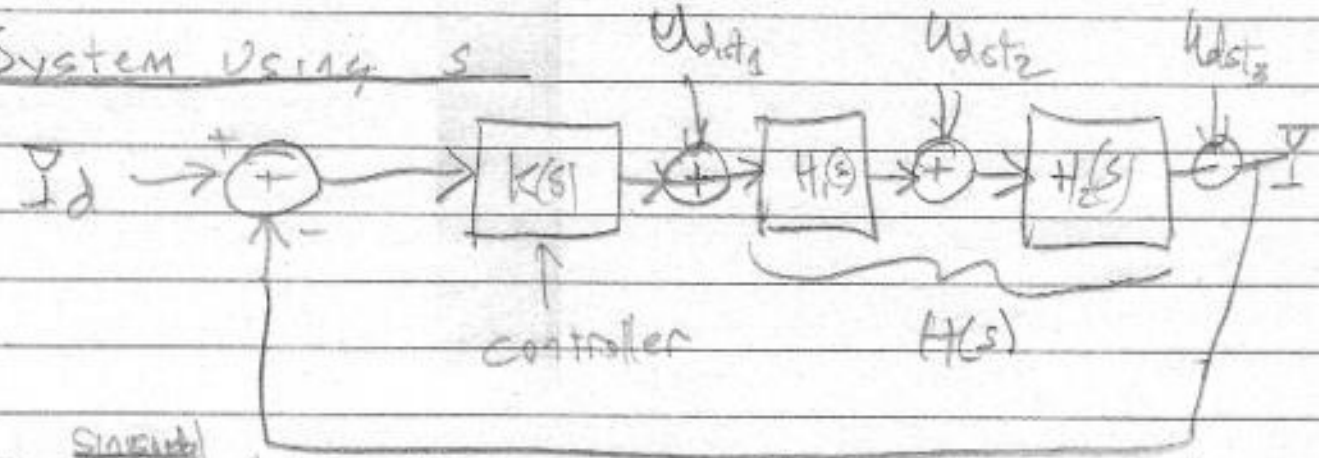
10/20/25 6.310 (1)

System Using s



In ~~steady-state~~ sinusoidal $s \to j\omega$

$$\hat{Y} = \underbrace{\frac{K(j\omega)H(j\omega)}{1+K(j\omega)H(j\omega)}}_{G(j\omega)}\hat{Y}_d + \frac{H(j\omega)}{1+K(j\omega)H(j\omega)}u_{dist_1} + \frac{H_2(j\omega)}{1+K(j\omega)H(j\omega)}u_{dist_2} + \frac{1}{1+K(j\omega)H(j\omega)}u_{dist_3}$$

$G_{dist_{1,2,3}}(j\omega)$

Notice same denom for all

If $|K(j\omega)H(j\omega)| \gg 1$ then

$$G(j\omega) \simeq 1$$

$$G_{1,2,3}(j\omega) \simeq 0$$

$\Rightarrow \hat{Y} = \hat{Y}_d$ and disturbances are rejected

So Let's design $K(s)$ so that

$$|K(j\omega)\, H(j\omega)| >> 1 \text{ for all } \omega$$

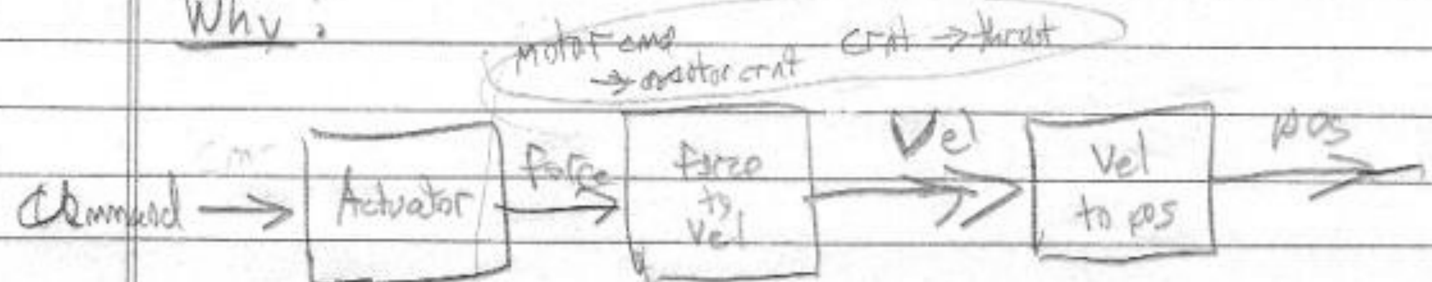
Yeah!

Not possible in practice for a pair of reasons

Reason 1 $H(j\omega)$ goes to zero with ω too fast

as $\omega \to \infty$ $|H(j\omega)| \propto \frac{1}{\omega^2}$ or higher!

Why?



e.g. motor cmd to thrust

Physical Systems don't respond instantly

$$\underbrace{\frac{\gamma_a}{s-\beta_a}, \frac{\gamma_v}{s-\beta_p}, \frac{1}{s}}_{H(s)\big|_{s=j\omega}}$$

$$\frac{K_1}{s^3 + a_2 s^2 + a_1 s + a_0 s}\Bigg|_{s=j\omega}$$

$(j\omega)^3$

3

<u>Okay</u> so make $K(s)$ compensate

$$K(s) = \frac{K_i}{s} + K_p + K_d s + K_{dd} s^2 + K_{ddd} s^3$$

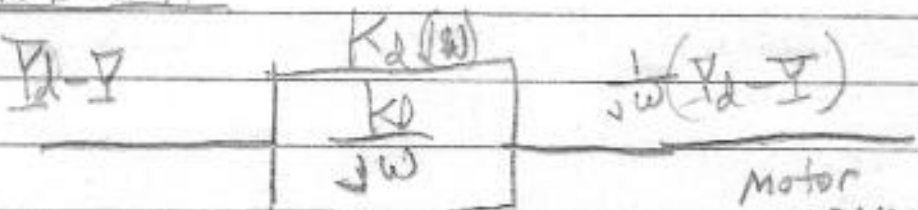
<u>then</u> $\lim_{\omega \to \infty} |K(j\omega) H(j\omega)|$ will not go to zero!

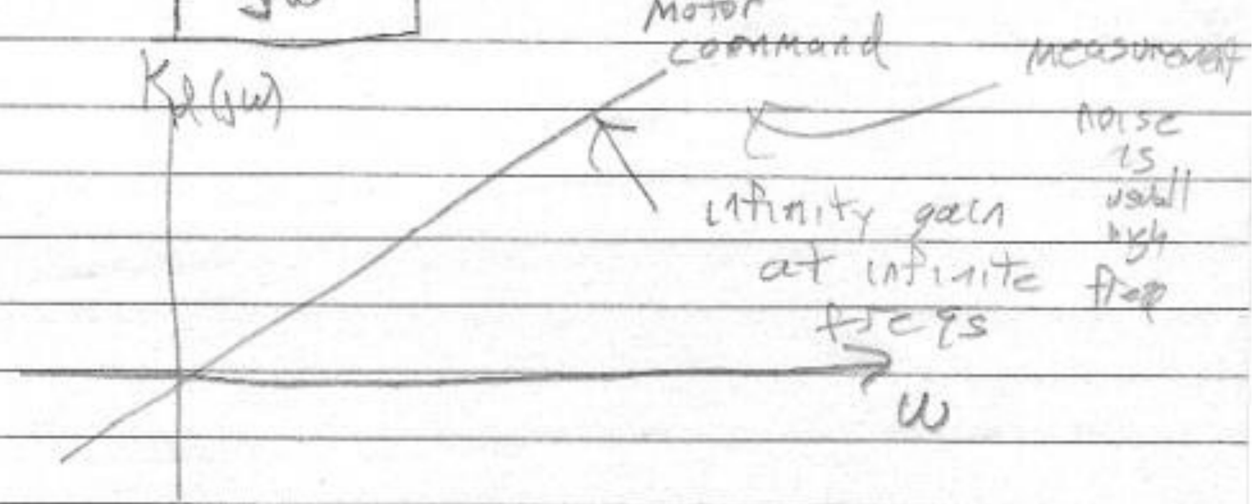
<u>Reason 2</u> High order $K(s)$ are impractical

Prop arm vel est was <u>noisy</u> $= \frac{\phi[n] - \phi[n-m]}{m \Delta T}$

<u>accel est would be worse</u>

<u>Prop arm</u>





Okay So

Design $K(s)$ so that

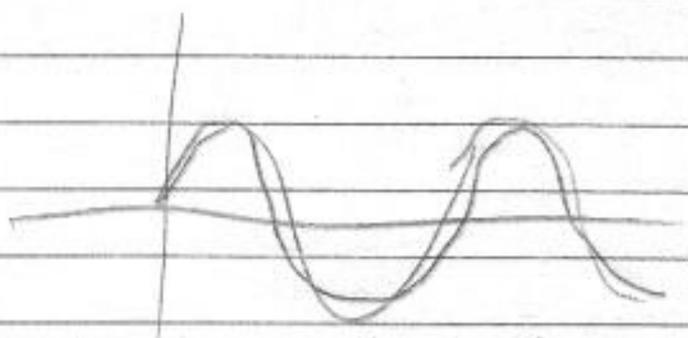
for lower freqs

$$|K(j\omega)\,H(j\omega)| >> 1 \quad \text{for } \omega\text{'s you care about}$$

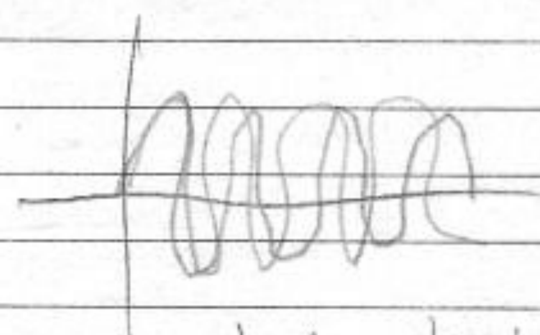
~~for higher freqs~~

$$\lim_{\omega \to \infty} |K(j\omega)\,H(j\omega)| \to 0$$

No tracking for large ω



good tracking

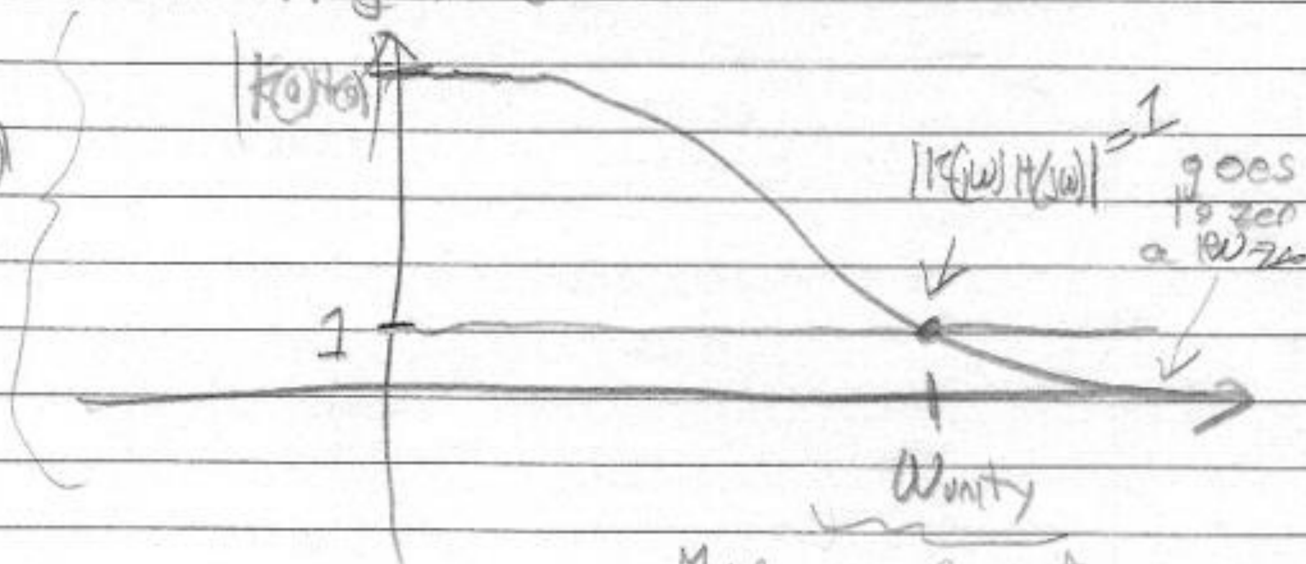


lousy tracking

But there is a subtle problem

$|K(j\omega)\,H(j\omega)|$

$|K(j\omega)\,H(j\omega)|$ will always have an ω_{unity}



Must exist if

$K(j\omega)\,H(j\omega) >> 1$ for ω small

$K(j\omega)\,H(j\omega) \to 0$ for ω large

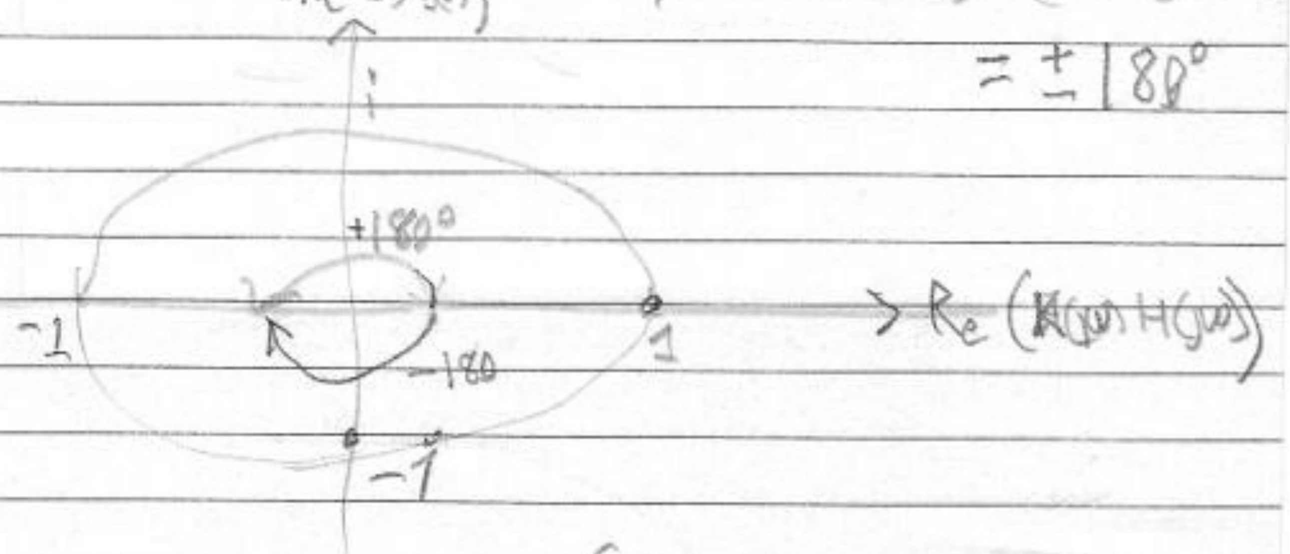
What happens if $K(j\omega)H(j\omega) = -1$ 5

$$G(j\omega) = \frac{K(j\omega)H(j\omega)}{1 + \underbrace{K(j\omega)H(j\omega)}_{\text{spose } |K(j\omega_{unity})H(j\omega_{unity})| = 1}}$$

cancelling 1 depends on $\measuredangle K(j\omega)H(j\omega)$

Worst case $|K(j\omega_{unity})H(j\omega_{unity})| = 1$

& $\measuredangle K(j\omega_{unity})H(j\omega_{unity}) = \pm 180^\circ$



If $K(j\omega)H(j\omega) = -1$, $(|K(j\omega)H(j\omega)| = 1, \measuredangle = \pm 180°)$

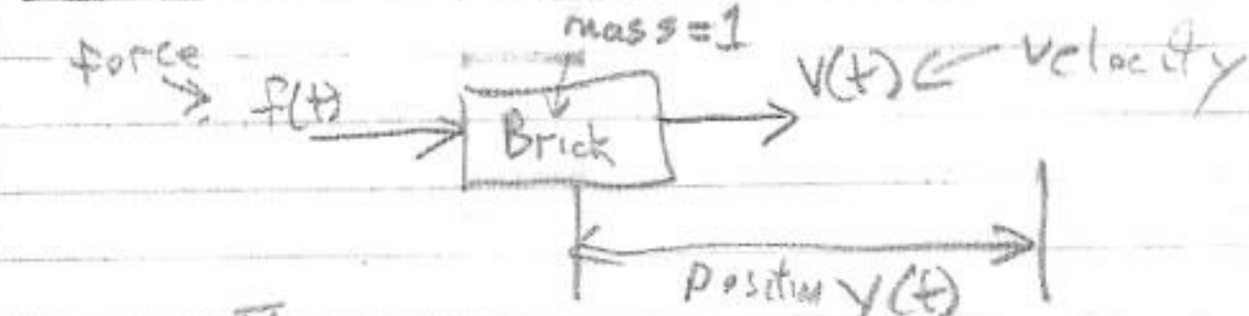
$G(j\omega) \rightarrow \infty \leftarrow$ want $G(j\omega) \cong 1$

$G_{dist}(j\omega) \rightarrow \infty$ for every disturbance $\leftarrow$ want $G_{dist}(j\omega) \cong 0$

Bad Bad Bad

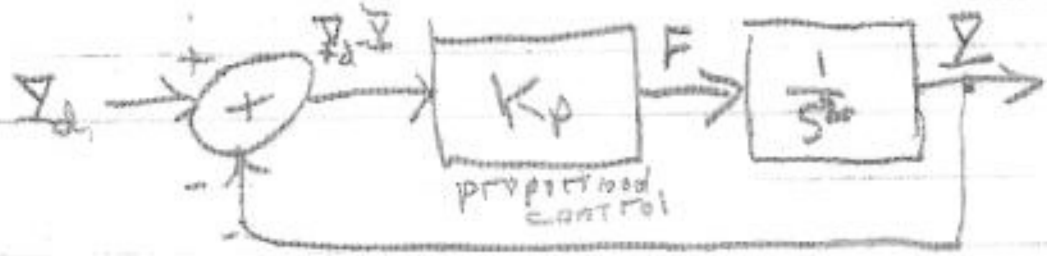
6

Frictionless Position Control



$$\underbrace{\frac{d}{dt} y(t) = V(t)}_{s\mathbb{Y} = \mathbb{V}} \qquad \underbrace{\frac{d}{dt} V(t) = f(t)}_{s\mathbb{V} = F} \quad \text{mass} = 1$$

$$= \underbrace{K_p\left(y_d(t) - y(t)\right)}_{\text{proportional control}}$$



$$\sqrt{K_p} = \omega_{unity}$$

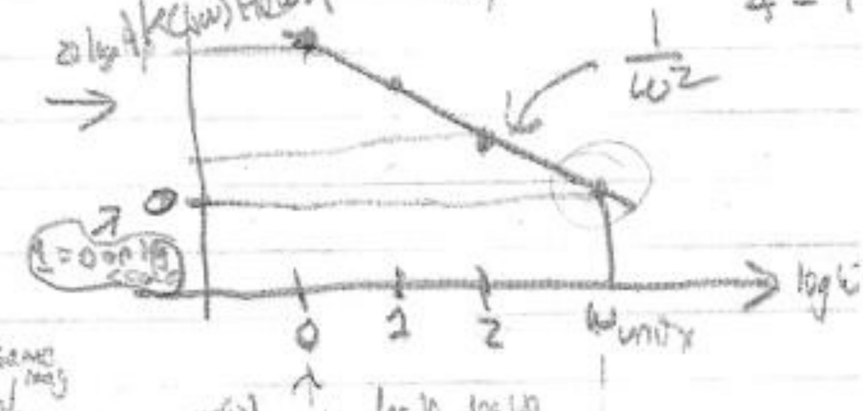
$$K(j\omega)H(j\omega) = \frac{K_p}{(j\omega)^2}\Bigg|_{\omega = \omega_{unity}} = -\frac{K_p}{\omega_{unity}^2} = -1$$

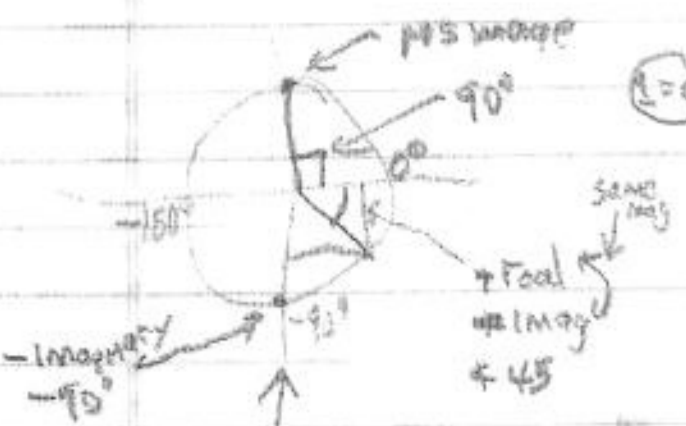
$(j\omega)^2 \to -\omega^2$

Mag = 1

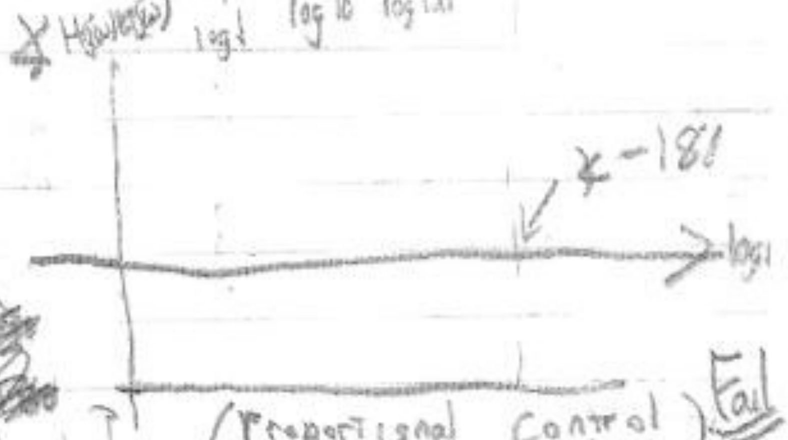
$\measuredangle = -1$

$20 \log |K(j\omega) H(j\omega)| \rightarrow$





angle



How Can we keep $\measuredangle K(j\omega)H(j\omega)$ away from -180° when near ω_{unity}?

(Proportional Control of Frictionless Brick) Fail