

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.3100: Intro to Modeling and Control—Fall 2025

Marginal Maglev: Lead Design

In our fourth lab (Marginal Maglev), we designed a lead controller for a magnetic levitation system. In this post lab exercise, we will implement a double lead controller in simulation. Assume that the open loop transfer function of the magnetic levitation system is given by

$$H(s) = \frac{\gamma(-\lambda_E)}{(s - \lambda_E)(s^2 - \gamma_{ay})}$$

and (for ease of grading) please use the following parameter values:

$$\begin{aligned}\gamma &= 1900 \\ \gamma_{ay} &= 1100 \\ \lambda_E &= -132\end{aligned}$$

Problem One: Start by implementing a PD controller that can stabilize this system. What values of K_p and K_d work in your simulation?

Following examples in your lab, use Matlab functions to draw bode plots of transfer functions of $H(s)$, $K(s)$ and $H(s)K(s)$. Report the phase margin from your simulation. In addition, show the closed loop system step response.

Your plots:

Problem Two: The main issue with a PD controller is that it amplifies high frequency signals. If the physical system cannot dissipate high frequency disturbances effectively, then a PD controller may not perform well. Next, we will implement a lead controller. A lead controller can increase a system's phase margin without significantly amplifying high frequency signals.

First, let's implement a lead controller in the form:

$$K_{lead}(s) = K_0 \left(\frac{s_p}{s_z} \right) \left(\frac{s - s_z}{s - s_p} \right)$$

This requires 3 parameters: K_0 , s_p , and s_z . For this problem, please use

$$K_0 = 2.5 \quad s_p = -800 \quad s_z = -20$$

What are the low-frequency and high-frequency gains for this lead controller?

What is the maximum phase gain (and its corresponding frequency) of this lead controller?

On the same bode plot, please show the transfer function of $H(s)$, $K_{lead}(s)$, and $H(s)K_{lead}(s)$.

What is the phase margin of the open-loop system $K_{lead}H(s)$? Is this a stable system?

Please show the step response of the system with this lead controller.

Problem Three: While a lead controller can be used to stabilize the maglev system, we would like to explore ways to improve the system's phase margin. Let's try a double-lead controller:

$$K_{dl} = \left(\sqrt{K_0} \left(\frac{s_p}{s_z} \right) \left(\frac{s - s_z}{s - s_p} \right) \right)^2$$

What are the low-frequency and high-frequency gains of this controller?

Implement this double-lead controller with the same numbers as in Problem 2 and show the bode plots for the transfer functions $H(s)$, $K_{dl}(s)$, and $H(s)K_{dl}(s)$.

What is the new phase margin? Has it improved? Why? What are the pros and cons of this double lead controller?

Problem Four: Now design a double-lead controller that satisfies the following two constraints:

$$|K_{new}(s \rightarrow 0)| = K_0 = 2.5 \quad (1)$$

$$|K_{new}(s \rightarrow \infty)| = 100 \quad (2)$$

Your double-lead controller should have the form:

$$K_{new} = K_0 \frac{s_{p1}(s - s_{z1})s_{p2}(s - s_{z2})}{s_{z1}(s - s_{p1})s_{z2}(s - s_{p2})}$$

Your goal is to choose the two pairs of poles and zeros so that you can improve the phase margin without increasing the high frequency controller gain. This design process is iterative. Please show the bode plot of the new system $H(s)K_{new}(s)$.

What is the new phase margin and what are your values of s_{z1} , s_{z2} , s_{p1} , and s_{p2} ?

Problem Five: In this problem we will implement the double-lead controller on a micro-controller (like the Teensy), following the example from our lab. We need discrete-time equations to approximate its behavior. Recall the form of the single-lead controller

$$e(t) \rightarrow K_0 \left(\frac{s_p}{s_z} \right) \left(\frac{s - s_z}{s - s_p} \right) \rightarrow c(t)$$

The corresponding differential equation is:

$$\dot{c}(t) - s_p c(t) = K_0 \left(\frac{s_p}{s_z} \right) (\dot{e}(t) - s_z e(t)).$$

We develop a discrete-time difference equation approximation:

$$\frac{c[n] - c[n-1]}{\Delta T} - s_p c[n-1] = K_0 \left(\frac{s_p}{s_z} \right) \left(\frac{e[n] - e[n-1]}{\Delta T} - s_z e[n-1] \right)$$

By re-arranging the terms, we can find an update equation for $c[n]$ in terms of the previous controller output $c[n-1]$ and the current and previous error inputs $e[n]$ and $e[n-1]$:

$$c[n] = (1 + s_p \Delta T) c[n-1] + K_0 \left(\frac{s_p}{s_z} \right) (e[n] - (1 + s_z \Delta T) e[n-1])$$

Now let's develop a similar update equation for the output $c[n]$ of a discrete-time, double-lead controller:

$$e(t) \rightarrow \sqrt{K_0} \left(\frac{s_{p1}}{s_{z1}} \right) \left(\frac{s - s_{z1}}{s - s_{p1}} \right) \xrightarrow{g(t)} \sqrt{K_0} \left(\frac{s_{p2}}{s_{z2}} \right) \left(\frac{s - s_{z2}}{s - s_{p2}} \right) \rightarrow c(t)$$

Here an intermediate signal $g(t)$ is introduced to simplify your expression. In your controller, you will have access not only to the previous value of the controller output ($c[n-1]$) and the current and previous values of the error signal ($e[n]$ and $e[n-1]$), but also the previous value of the intermediate signal ($g[n-1]$). Enter your equations below (you don't need to substitute with numerical values for this question).