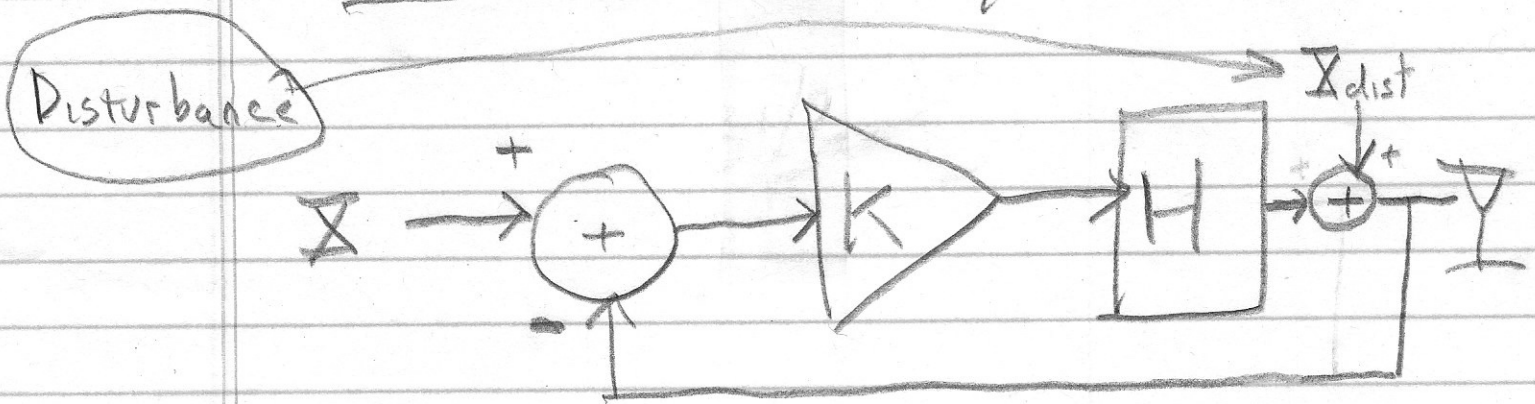


(1)

Feed Back Systems



$$\underline{X_{dist} = 0}$$

$$Y = G X$$

$$G(s) = \frac{K(s)H(s)}{1 + K(s)H(s)}$$

$$\underline{X = 0}$$

$$Y = KH(X - Y) + X_{dist}$$

$$Y = \frac{1}{1 + KH} X_{dist} \quad G_{dist}$$

If $|K(j\omega)H(j\omega)| \gg 1$

$$G(j\omega) \approx 1 \quad \leftarrow \text{Good Tracking}$$

$$G_{dist}(j\omega) \approx 0 \quad \leftarrow \text{Good Disturbance Rejection}$$

(2)

IF $K(j\omega)H(j\omega) \approx -1$

$$G(j\omega) = \frac{\overset{KH}{\textcircled{-1}}}{1 + (-1)} \Rightarrow \infty$$

$$G_{\text{dist}}(j\omega) = \frac{1}{1 + (-1)} \Rightarrow \infty$$

So if $|K(j\omega)H(j\omega)| = 1$ and $\angle K(j\omega)H(j\omega) = \pm 180^\circ$

Tracking & Disturbance rejection are terrible!

And the closer $K(j\omega)H(j\omega)$ is to -1
the worse the tracking and disturbance rejection

But Just because $K(j\omega)H(j\omega)$ is always far from -1 , no guarantee of stability for G

Sinusoidal
Steady
State
exists
only
if
system is
stable

Why? Given $x(t) = e^{j\omega t}$

$y(t) = G(j\omega)e^{j\omega t}$ for large t
only if G is stable

$G(j\omega)$ is physically meaningless if G is unstable

Example

Good phase
margin necessary
but not sufficient

3

$$H_{prop} = \frac{200}{(s+10)(s)(s)} \quad K=1$$

$$G = \frac{KH}{1+KH}$$

$$= \frac{200}{(s+10)(s)(s) + 200}$$

$$= \frac{200}{s^3 + 10s^2 + 200}$$

At $\omega = 4.3 \text{ rad/sec}$

$$|K H(j\omega)| = \left| \frac{200}{(j\omega+10)(j\omega)(j\omega)} \right|$$

$$= \left| \frac{200}{-j\omega^3 - 10\omega^2} \right| = \left| \frac{200}{-185 - 80j} \right|$$
$$\approx 1$$

(4B)

$$\angle KH(j\omega) = \angle \frac{200}{(-185 - 80j)} = -203^\circ \neq -180^\circ$$

$$\omega = 4.3 \text{ rad/sec} \\ (\approx \omega_{\text{unity}})$$

$$\text{at } \omega = 4.3 \quad |KH(j\omega)| \approx 1 \quad \angle KH(j\omega) = -203^\circ$$

Satisfies
phase
margin
conditions

$$\rightarrow KH \neq -1$$

But poles($G(s)$) are unstable!

$$\text{poles}(G) = \underbrace{0.75}_{\text{Real}} \pm 4.1j, -11.5$$

Real
positive

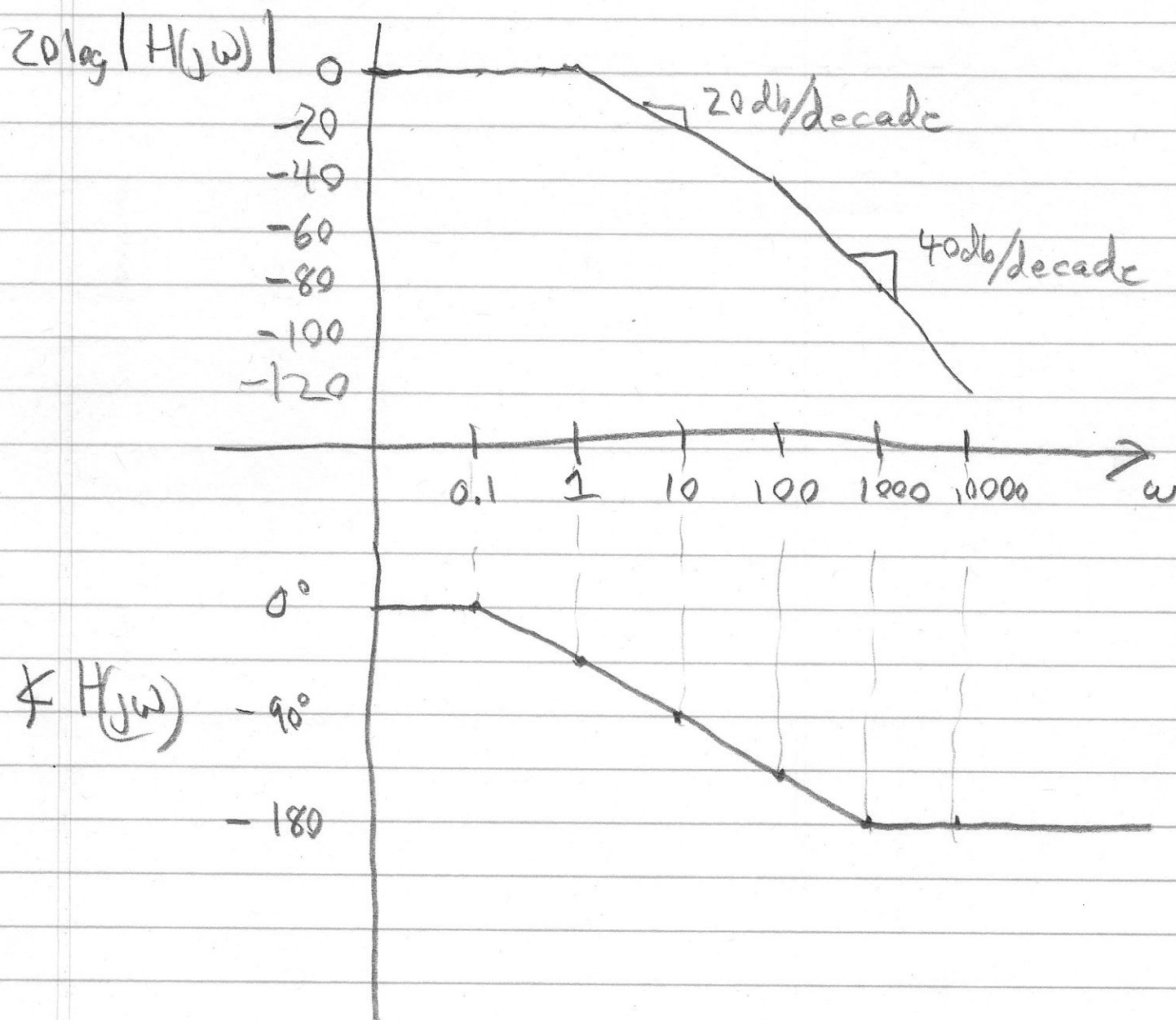
$G(s)$ is unstable

Good phase margin is a necessary
condition for good control
not a sufficient condition!

5

Example

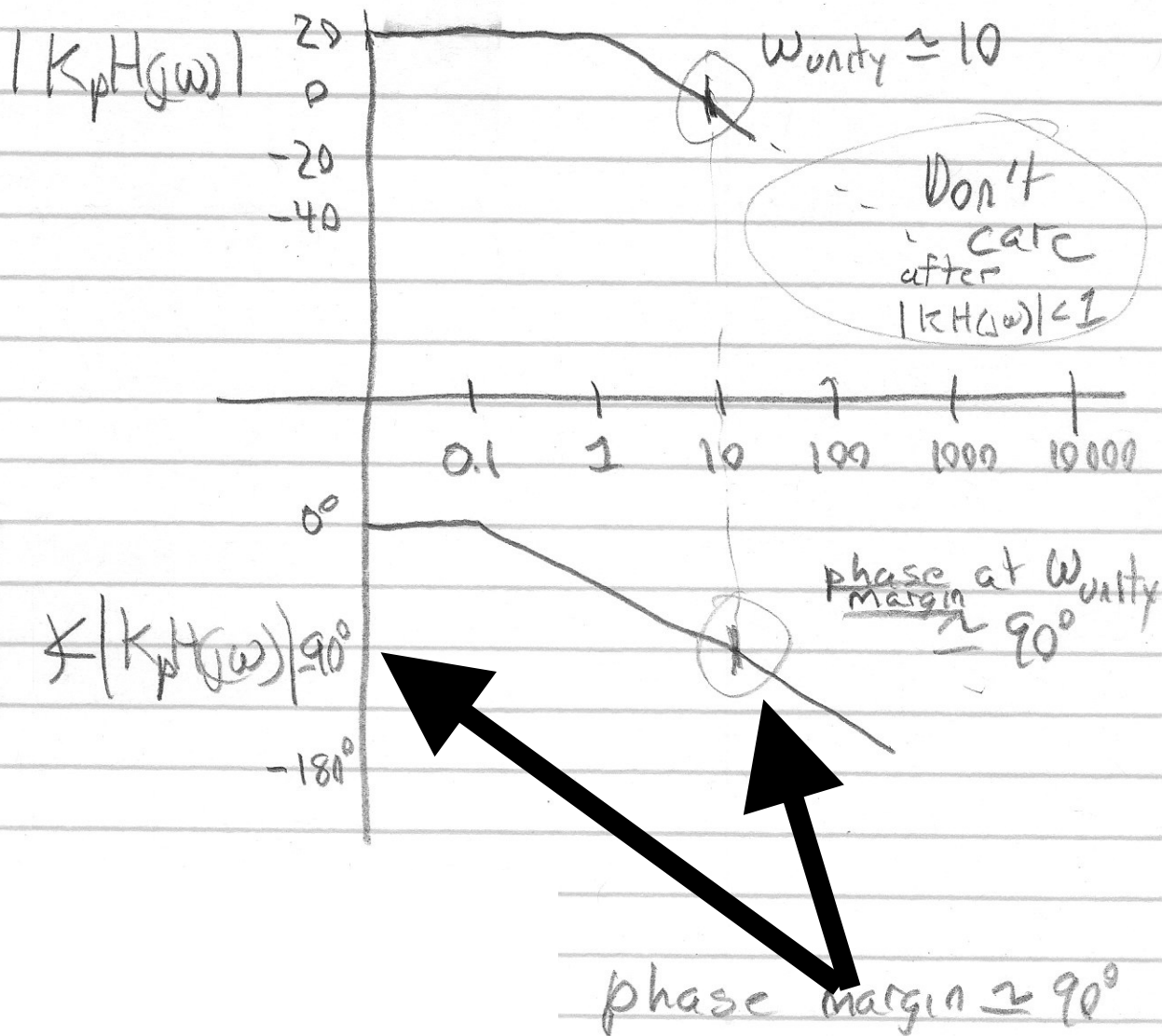
$$H(s) = \frac{100}{(s+1)(s+100)}$$



Design controller using $K_p = 10, 100 \text{ \& } 10,000$
What is the phase margin?

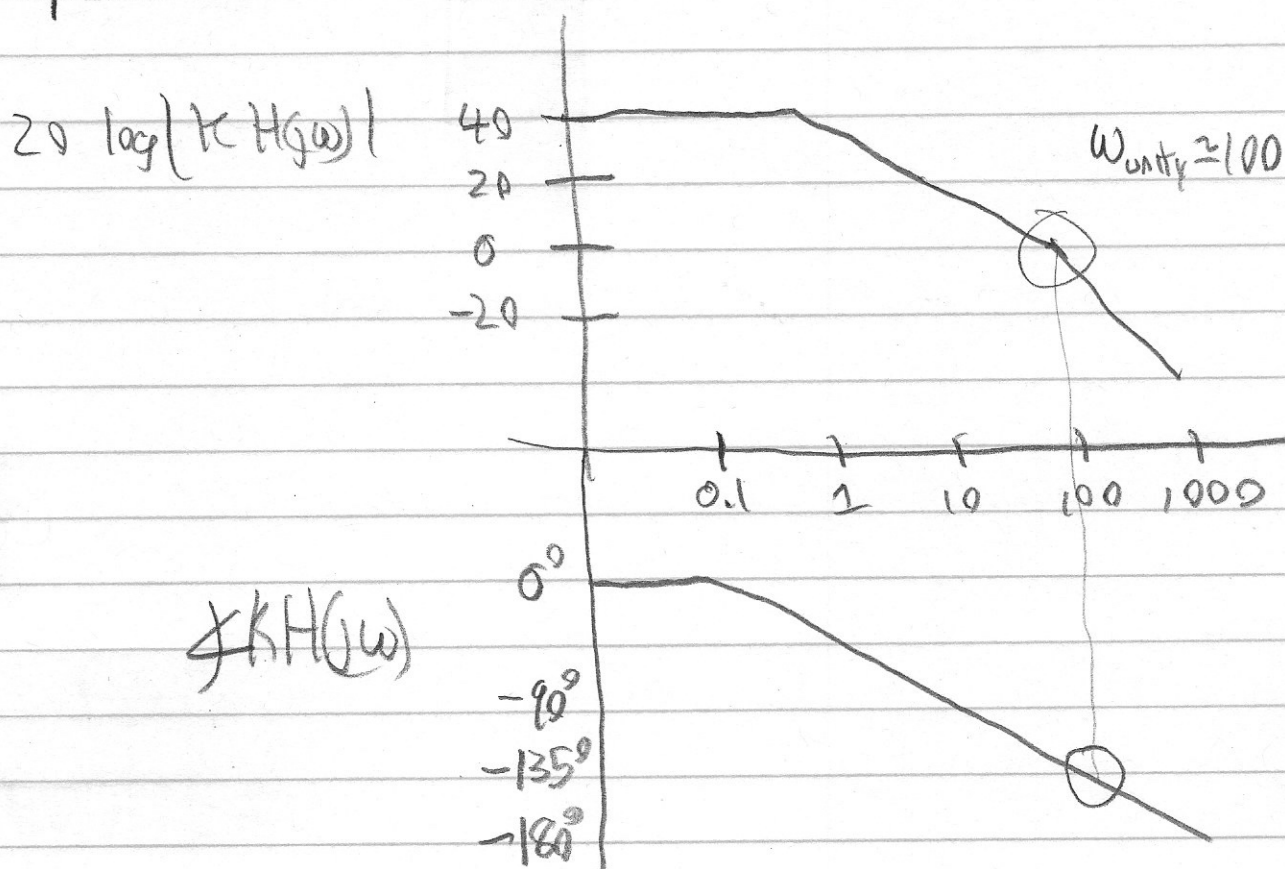
6

$K_p = 10$



7

$K_p = 100$



For $K_p = 100$ phase margin $\approx 4.5^\circ$
 okay but could be better. Lead? Lag?

Idea 1

Place a lead compensator
 with $sz = q = -100$, $sp = p = -1000$

$$K(s) = K_p \frac{p}{a} \left(\frac{s-q}{s-p} \right) = K_p 10 \frac{s+100}{s+1000}$$

$$KH(s)H(s) = 100 \cdot \frac{100}{(s+1)(s+100)} \cdot \frac{(s+100)}{s+1000} = \frac{10000}{(s+1)(s+1000)}$$

→ But Pole-Zero Cancellation Problematic!

Return to Later!

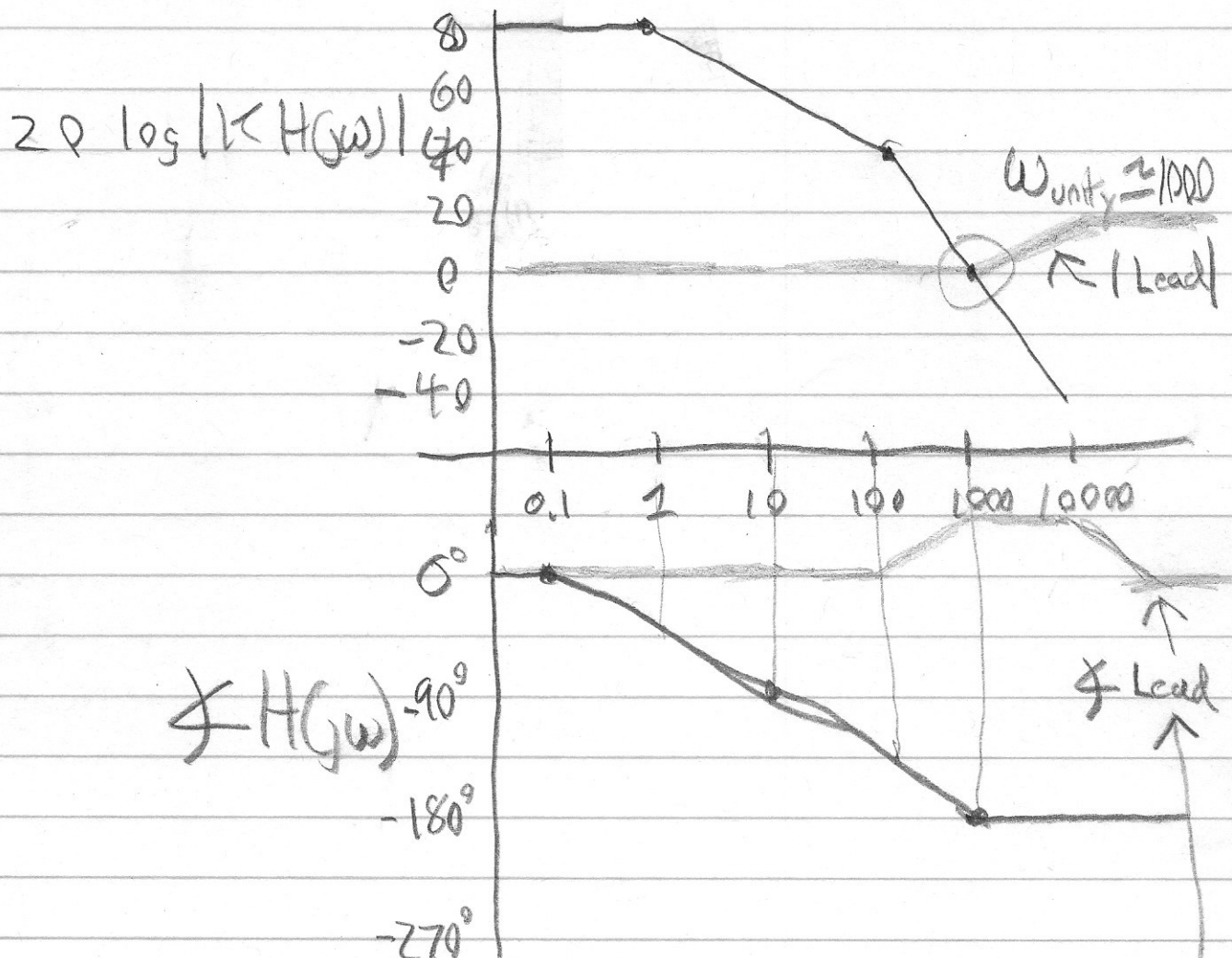
$$K_p = 10000$$

$$KH \gg 1!$$

Good Tracking!

Good dist Reject!

8



For $K=10000$

phase margin $\approx 0^\circ$

Bad, almost unstable

Idea 1

Place a lead with

$$q = -1000$$

$$p = -10000$$

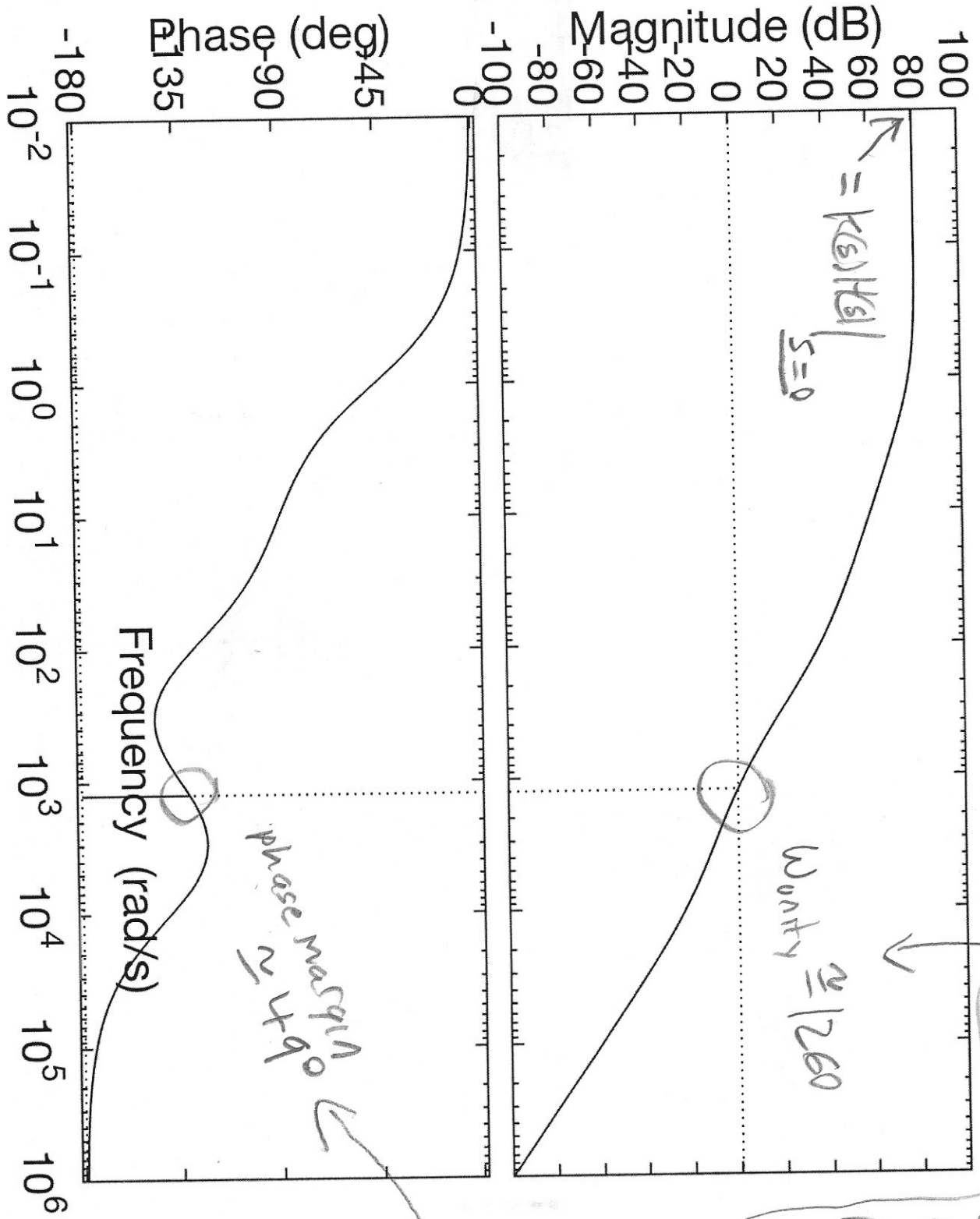
$$K(s)H(s) = 100000 \left(\frac{s+1000}{s+10000} \right) \left(\frac{100}{(s+1)(s+100)} \right)$$

9

Actual Margin

$$K(s)H(s) = 10^5 \left(\frac{s + 10^3}{s + 10^4} \right) \left(\frac{100}{(s+1)(s+100)} \right)$$

compensator zeroes
little shift



$$= K(s)H(s) \Big|_{s=0}$$

$$\omega_{unity} \approx 1260$$

phase margin 45°

near 45° predicted

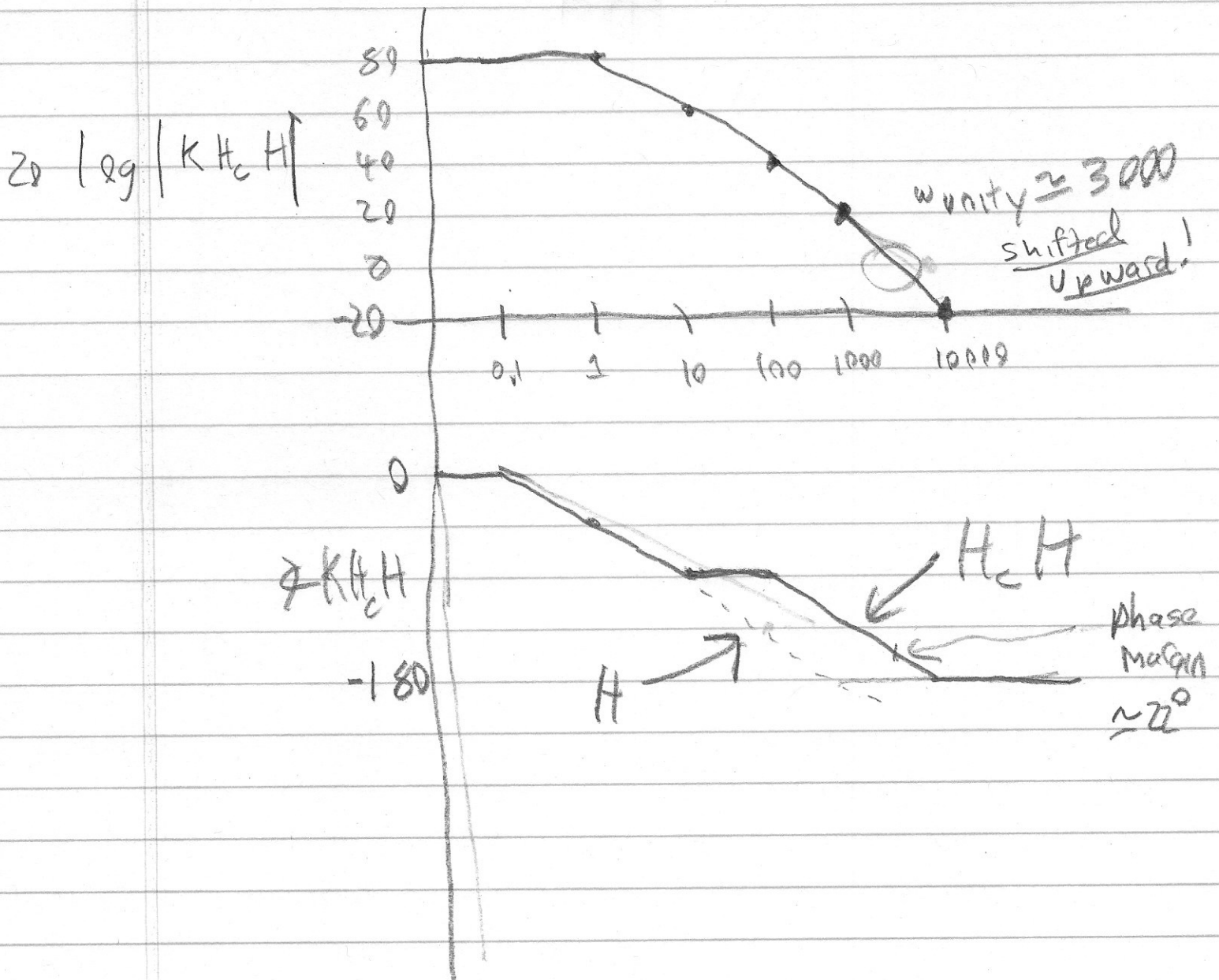
in unity (from ≈ 1000)
for $H(s) = \frac{10^5}{(s+1)(s+100)}$

$K = 10,000$
Too Soon

(10)

$$H_c(s) = 10 \left(\frac{s+100}{s+1000} \right)$$

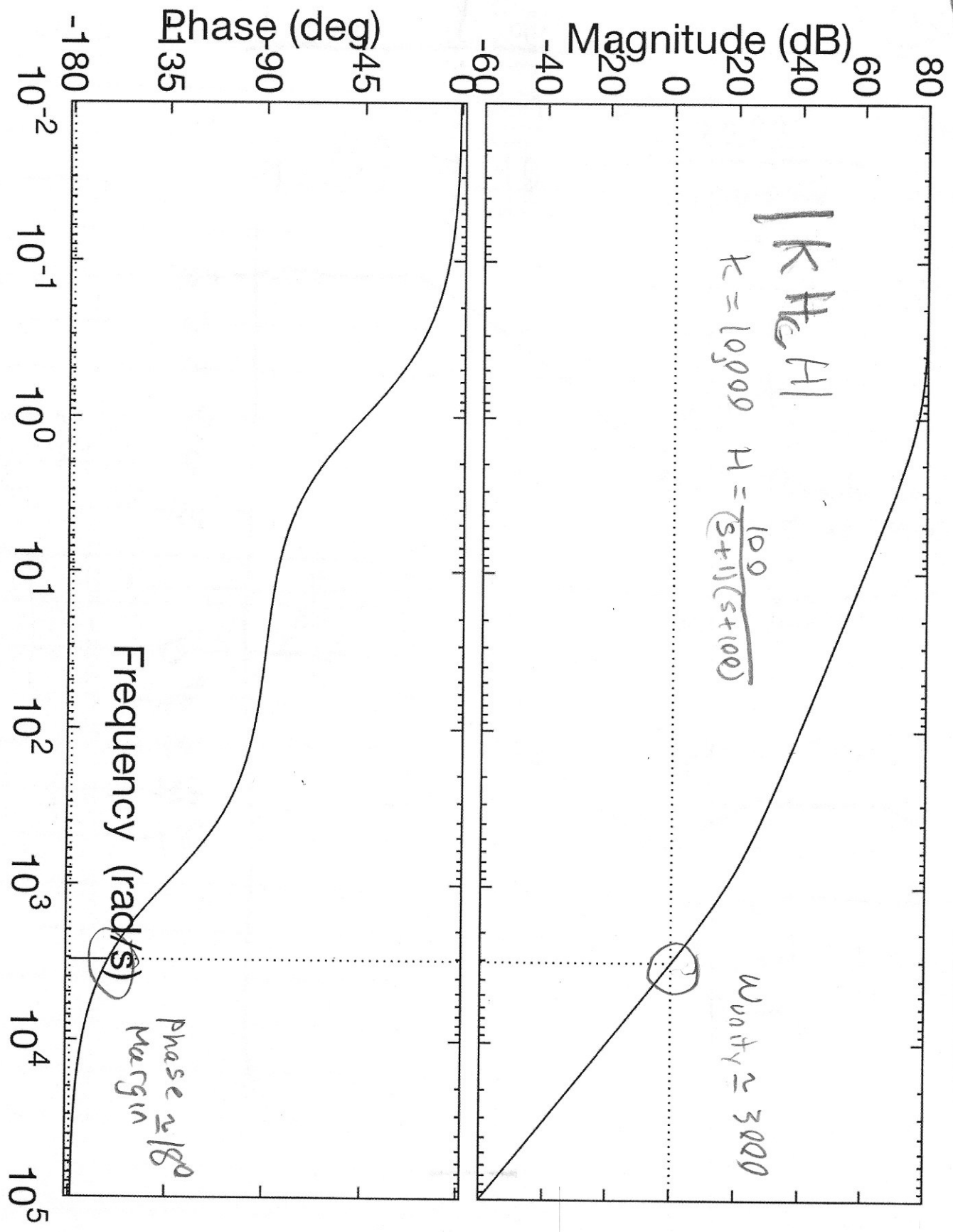
Zero at too
low a frequency



11

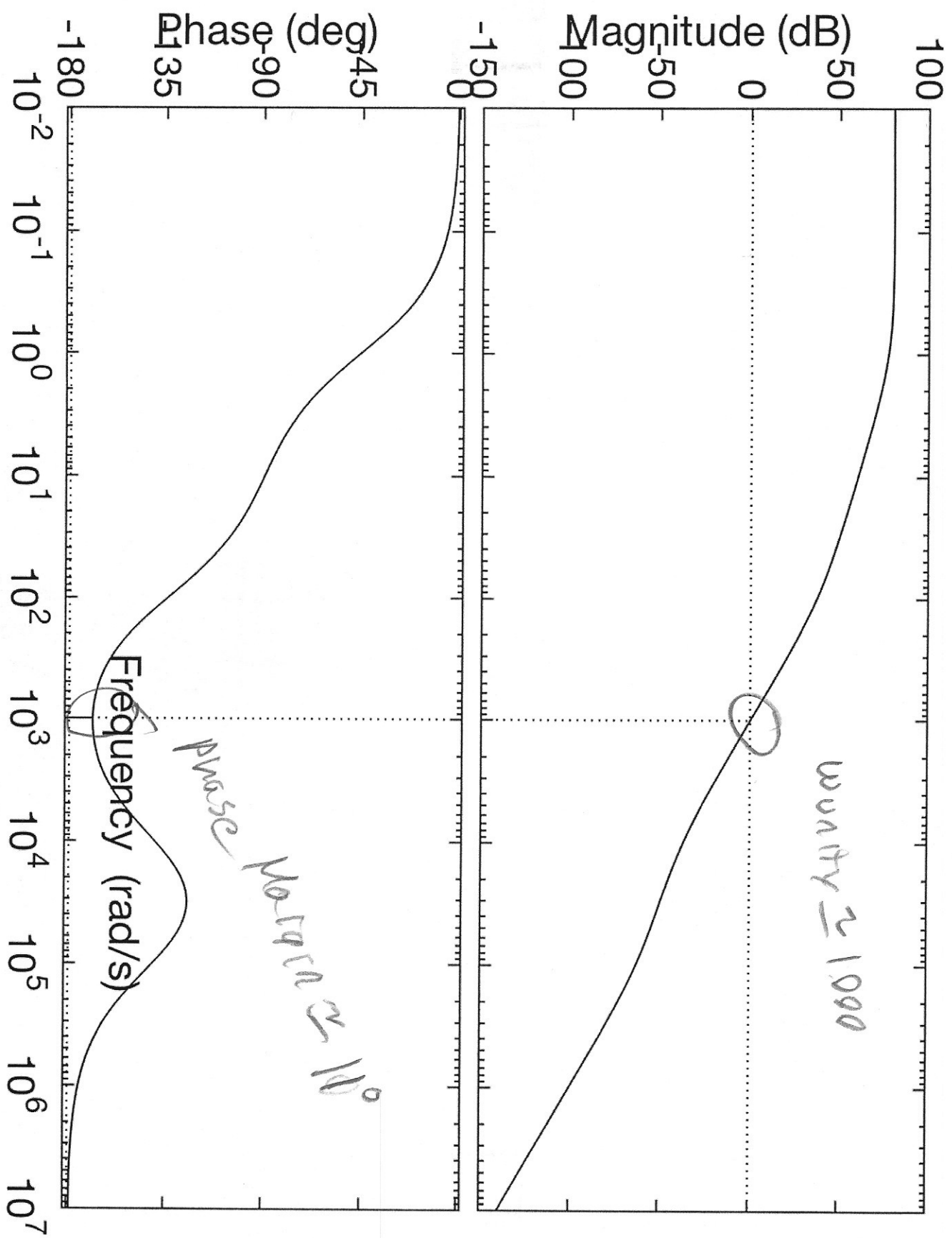
Actual Too Soon

$$H_e(s) = 10 \frac{s+100}{s+1000}$$



13

Actual Too Late $H_c(s) = 10 \frac{s+10000}{s+100000}$



14

Lead a little late $\rightarrow H_c(s) = 10 \frac{(s + 3900)}{(s + 39000)}$

