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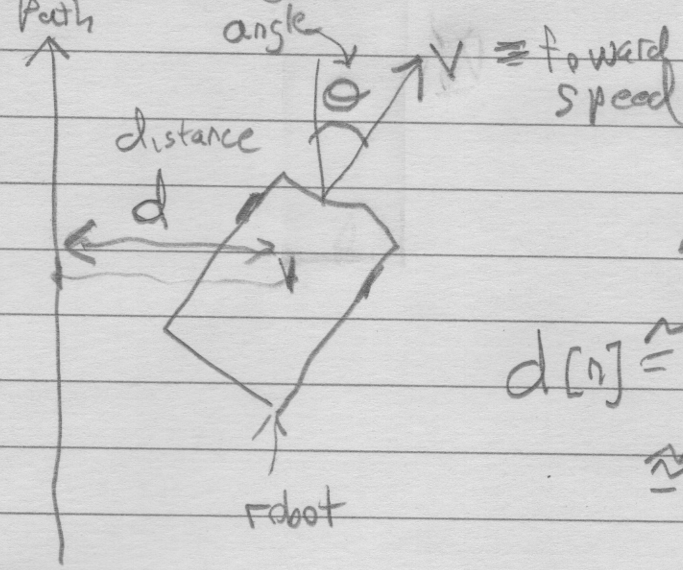
Path-following Robot



ΔT = sample period

$$d[n] \cong d[n-1] + \Delta T V \sin(\theta[n-1])$$

$$\cong d[n-1] + \Delta T V \theta[n-1]$$

if θ is small (and in radians!)

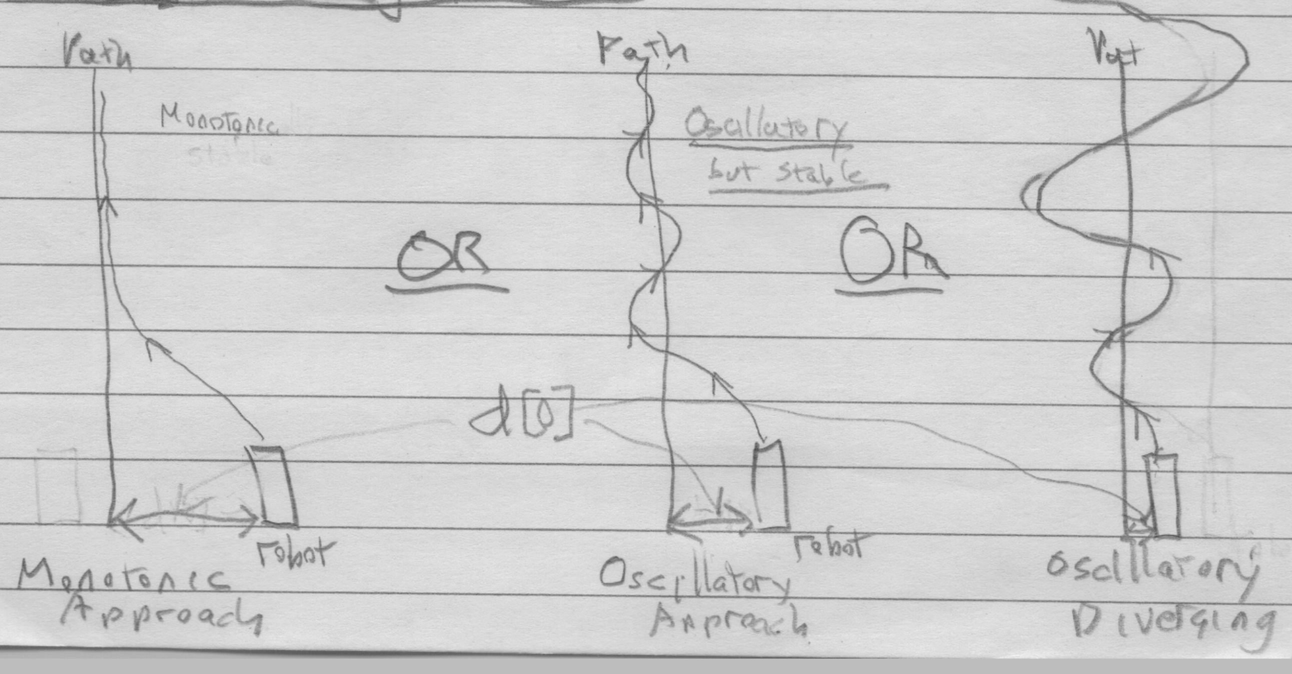
ω = robot rotational speed (we control)

$$\theta[n] = \theta[n-1] + \Delta T \omega[n-1]$$

Spose $\omega[n] = K_p(-d[n])$
(further from the path, faster we rotate back)

Possible Trajectories

Unstable



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Matrix Form

$$\begin{bmatrix} d[n] \\ \theta[n] \end{bmatrix} = \begin{bmatrix} 1 & \Delta T V \\ -\Delta T K_p & 1 \end{bmatrix} \begin{bmatrix} d[n-1] \\ \theta[n-1] \end{bmatrix}$$

$$x[n] = A \; x[n-1]$$

ZIR $\quad x[n] = A^n \, x[0]$

$\underline{\text{if}}$ $A\underline{V} = \underline{V} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \leftarrow$ diagonalizable

then $$\begin{bmatrix} d[n] \\ \theta[n] \end{bmatrix} = \begin{bmatrix} \underline{V} \end{bmatrix} \begin{bmatrix} \lambda_1^n & \\ & \lambda_2^n \end{bmatrix} \begin{bmatrix} \underline{V} \end{bmatrix}^{-1} \begin{bmatrix} d[0] \\ \theta[0] \end{bmatrix}$$

or $$d[n] = c_1 \lambda_1^n + c_2 \lambda_2^n$$

2 eqns in 2 unknowns
$$\left\{ \begin{aligned} d[0] &= c_1 \overbrace{\lambda_1^0}^{=1} + c_2 \overbrace{\lambda_2^0}^{=1} \\ d[1] &= c_1 \lambda_1 + c_2 \lambda_2 \end{aligned} \right.$$

$$\begin{bmatrix} d[1] \\ \theta[1] \end{bmatrix} = \begin{bmatrix} A \end{bmatrix} \begin{bmatrix} d[0] \\ \theta[0] \end{bmatrix} \Leftarrow \text{\underline{easy} to compute}$$

Does $d[n] \underset{n \to \infty}{\to} 0$? $\quad$ Yes if $|\lambda_1| < 1$ & $|\lambda_2| < 1$

Find λ's

If $\det(\lambda I - A) = 0$ $\quad$ λ is an eval

2x2

$$\det\begin{pmatrix} \lambda - 1 & -\Delta T V \\ +\Delta T K_p & \lambda - 1 \end{pmatrix} = 0$$

$$(\lambda-1)(\lambda-1) + (\Delta T)^2 K_p V = 0$$
$$= \lambda^2 - 2\lambda + (1 + (\Delta T)^2 K_p V) = 0$$

$$\lambda_1, \lambda_2 = 1 \pm \sqrt{1 - (1 + (\Delta T)^2 K_p V)}$$
$$= 1 \pm \sqrt{-(\Delta T)^2 K_p V}$$

>0

imaginary

$$= 1 \pm j\,\Delta T \sqrt{K_p V}$$

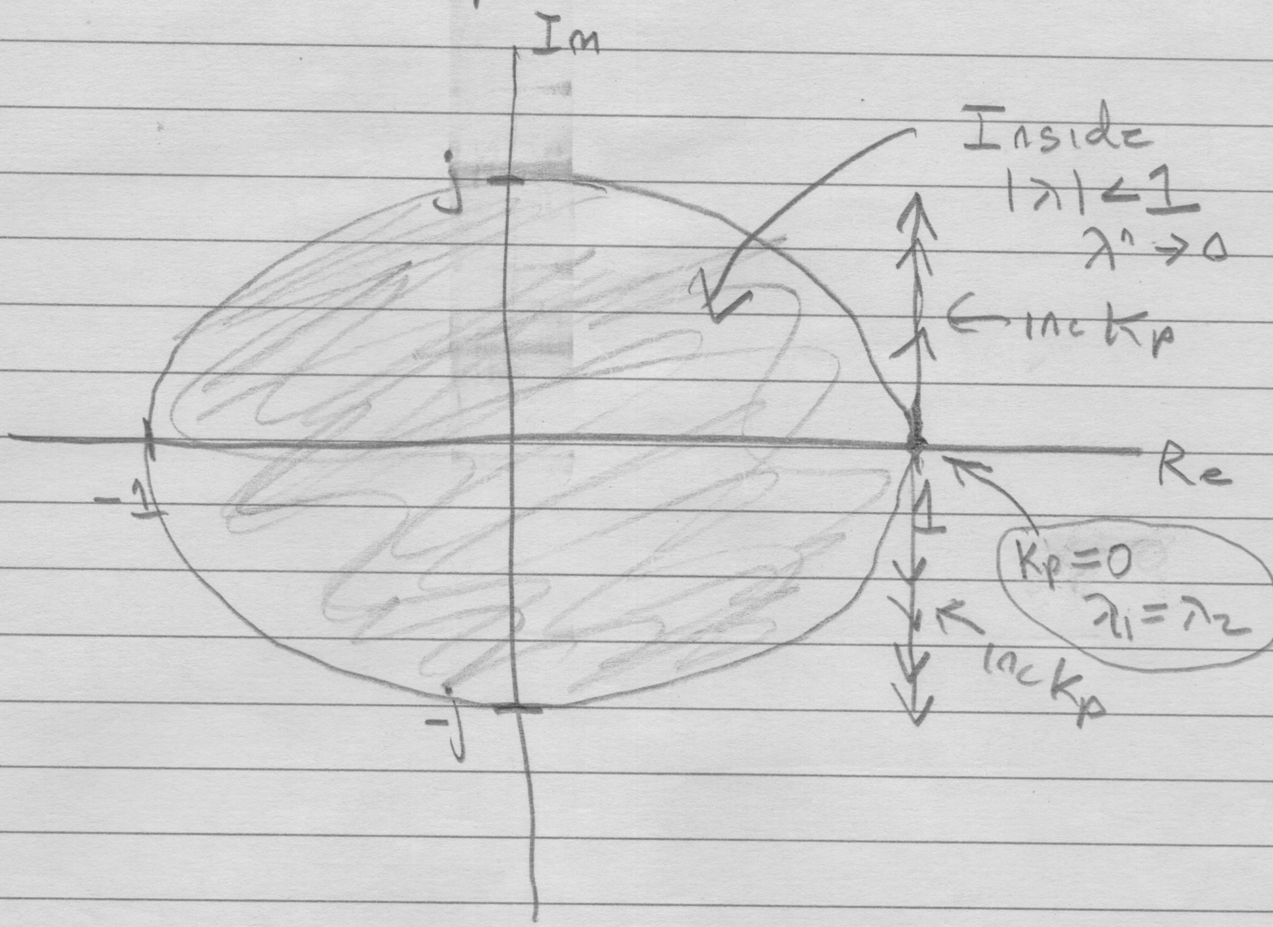
$j \uparrow \sqrt{-1}$

So

$$d[n] = C_1 \lambda_1^n + C_2 \lambda_2^n$$
$$= C_1 \left(1 + j\Delta T\sqrt{K_p V}\right)^n + C_2 \left(1 - j\Delta T\sqrt{K_p V}\right)^n$$

$\lim_{n\to\infty} d[n] \to 0$?

The Complex Plane



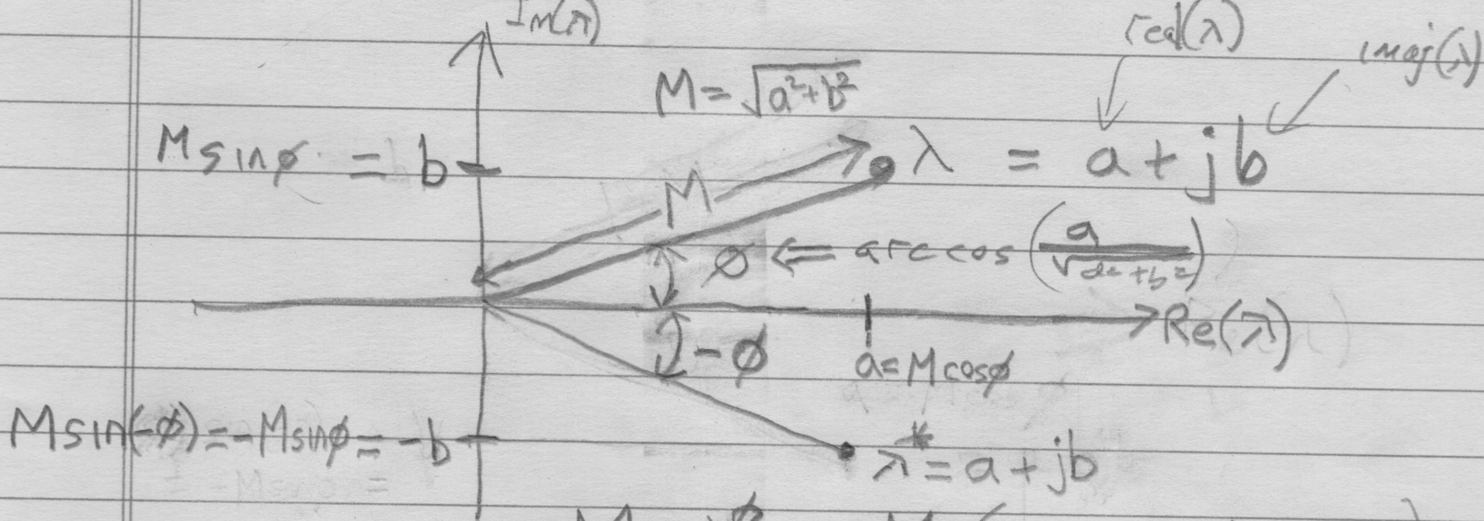
$\lambda_1, \lambda_2 = 1 \pm j \Delta T \sqrt{K_p V}$

<u>So</u> $|d[n]| \to \infty$ as $n \to \infty$
<u>for all $K_p > 0$</u> !

<u>But why does it oscillate?</u>

<u>Is</u> $d[n] = C_1 (1 + j\Delta T \sqrt{K_p V}) + C_2 (1 - j\Delta T \sqrt{K_p V})$
always real?
<u>Yes</u> real because $C_1 = C_2^*$ (complex conj)

Polar Form



$$\underbrace{Me^{j\phi}}_{\text{euler}} = M\underbrace{(\cos\phi + j\sin\phi)}_{\text{euler}}$$

$$\lambda_1^n = (Me^{j\phi})^n = M^n e^{jn\phi} = M^n(\cos n\phi + j\sin n\phi)$$

$$\lambda_2^n = (\lambda_1^*)^n = (Me^{-j\phi})^n = M^n e^{-jn\phi} = M^n(\cos n\phi - j\sin n\phi)$$

$$d[n] = C_1 M^n e^{jn\phi} + C_2 M^n e^{-jn\phi}$$

$$C_1 = M_c e^{j\phi_c} \quad C_2 = C_1^* = M_c e^{-j\phi_c}$$

$$d[n] = M_c M^n e^{j(n\phi + \phi_c)} + M_c M^n e^{-j(n\phi + \phi_c)}$$

$$= M_c M^n \cos(n\phi + \phi_c)$$

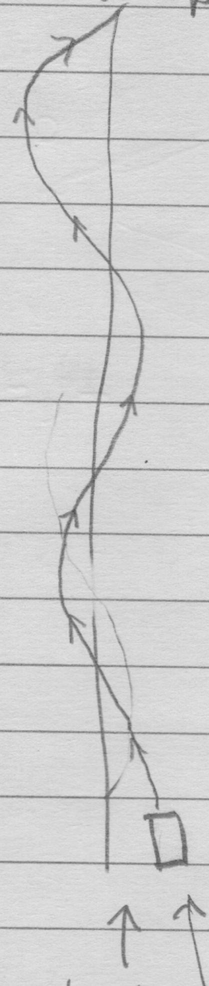
<u>BTW</u> $n_{period}\,\phi = 2\pi$ (one period)

$$n_{period} = \frac{2\pi}{\phi}$$

Robot Trajectories

5A

small K_p



$|\lambda|$

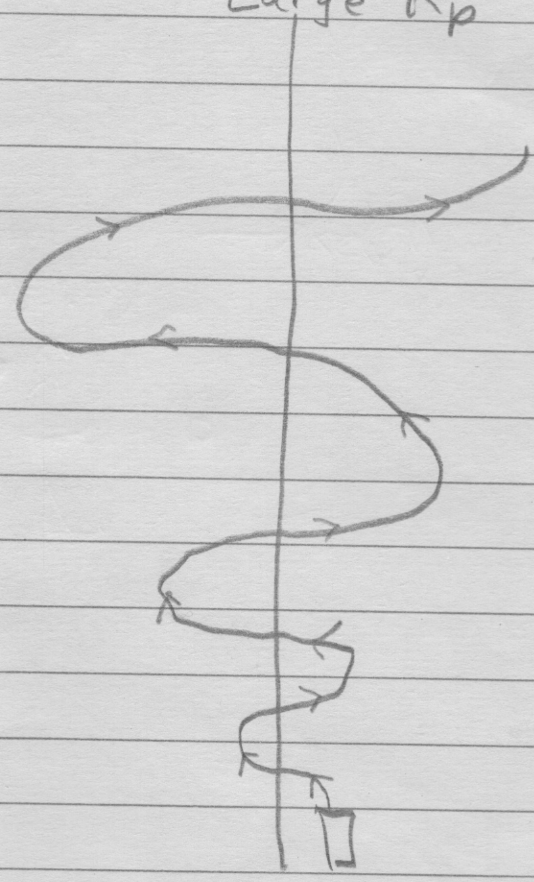
slightly ≥ 1

imaginary part is small

ϕ is small

$\frac{2\pi}{\phi}$ large

Large K_p



$|\lambda|$

much larger than 1

imaginary part is large

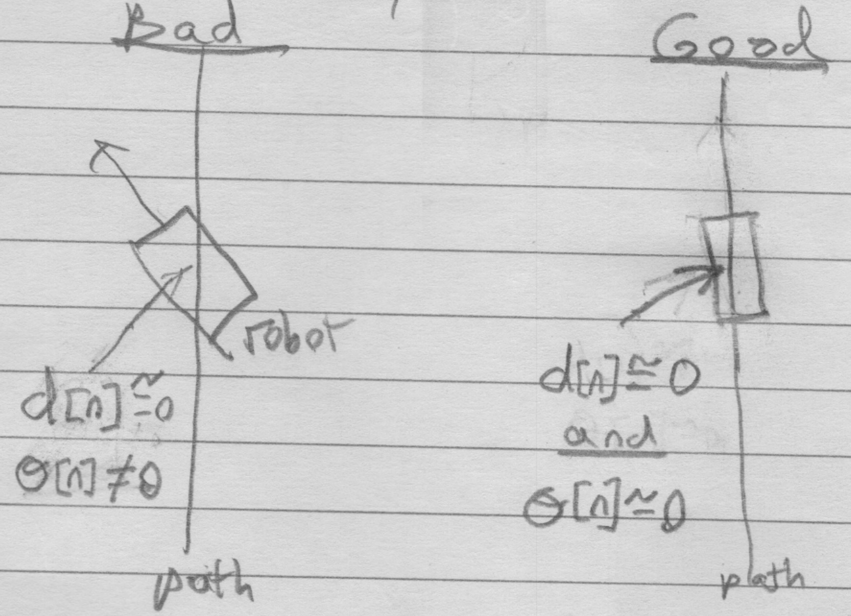
ϕ is large

$\frac{2\pi}{\phi}$ is small

Suppose

$$\omega[n] = K_p(-d[n]) + K_\theta(-\theta[n])$$

Drive distance and angle to zero simultaneously



Note

$$\begin{bmatrix} d[n] \\ \theta[n] \end{bmatrix} = \begin{bmatrix} 1 & \Delta T V \\ -\Delta T K_p & 1-\Delta T K_\theta \end{bmatrix} \begin{bmatrix} d[n-1] \\ \theta[n-1] \end{bmatrix}$$

$$\lambda^2 - (2 + \Delta T K_\theta)\lambda + 1 + \Delta T K_\theta + \Delta T^2 V K_p = 0$$

$$\lambda_1, \lambda_2 = 1 - \frac{\Delta T K_\theta}{2} \pm \Delta T \sqrt{\frac{K_\theta^2}{4} - K_p V}$$

Need Plots of λ_1, λ_2 verses K_θ, K_p