

6.302 3/30/20 (Preliminary)

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Lead and Lag Compensation

Sketching frequency response Reminder

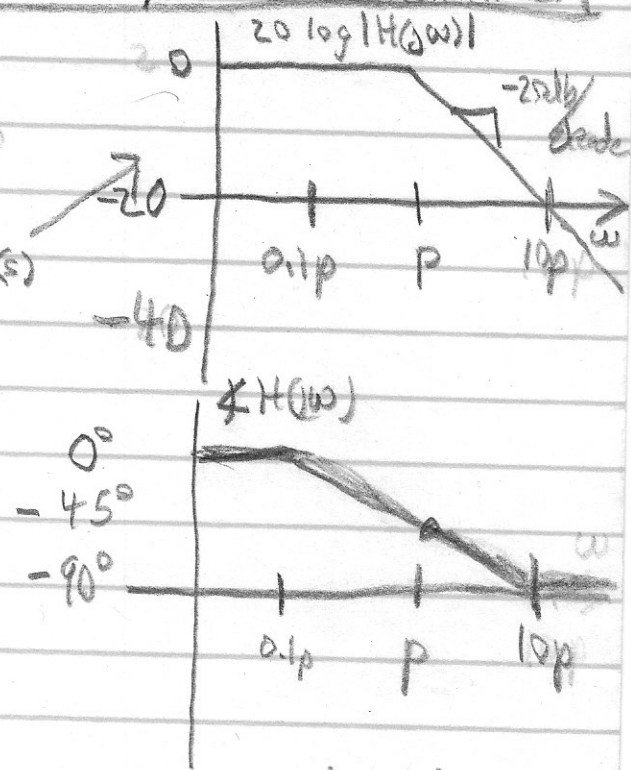
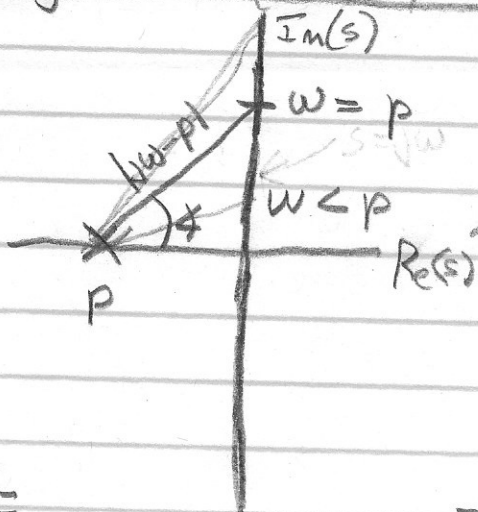
Real Pole

$$H(s) = \frac{-P}{s-p}$$

$$20 \log |H(j\omega)| =$$

$$20 \log |p| - 20 \log |j\omega - p|$$

$$\angle H(j\omega) = -\arctan\left(\frac{\omega}{p}\right)$$



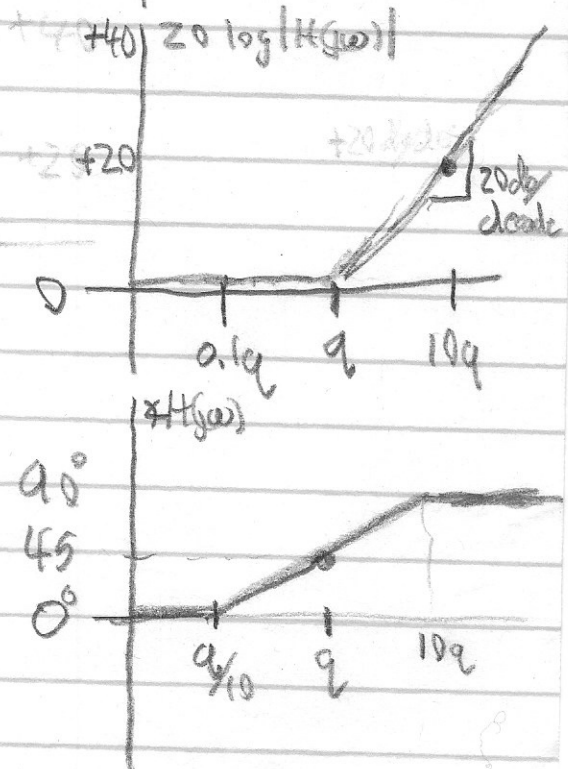
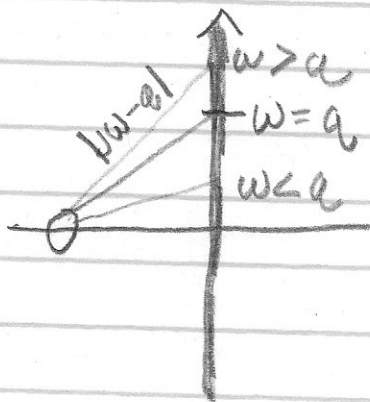
Real Zero

$$H(s) = \frac{s-q}{-q}$$

$$20 \log |H(j\omega)| =$$

$$20 \log |j\omega - q| - 20 \log |q|$$

$$\angle H(j\omega) = \arctan\left(\frac{\omega}{q}\right)$$



Straight Lines in Approximate Bode

A zero Example

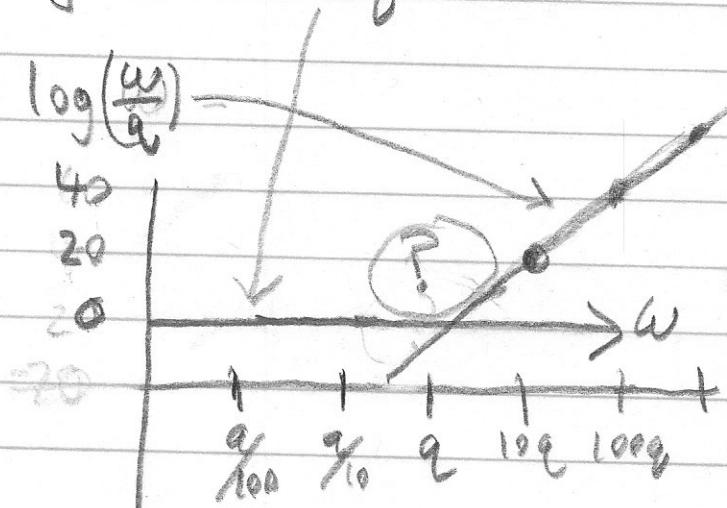
$$H(s) = \frac{s - q}{-q} \Big|_{s=j\omega} = 1 - \frac{s}{q} \Big|_{s=j\omega} = \underbrace{\left| 1 - j \frac{\omega}{q} \right|}_{= \sqrt{1 + \left(\frac{\omega}{q}\right)^2}}$$

$$20 \log |H(j\omega)| = 20 \log \sqrt{1 + \left(\frac{\omega}{q}\right)^2}$$

$$\approx 20 \log 1 \quad \omega \ll q = 0$$

$$\approx 20 \log \left(\frac{\omega}{q}\right)$$

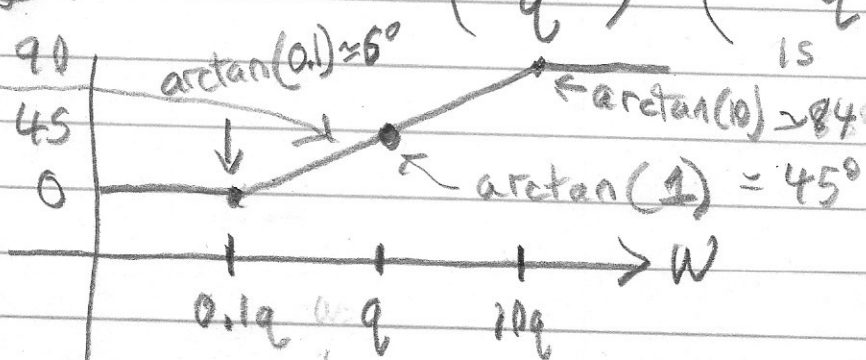
20 log |H(jω)|



In ? region: not so important

log-spaced ω's

$$\angle H(j\omega) = \arctan\left(\frac{\omega}{q}\right) \quad \left(\begin{array}{l} \text{positive since} \\ q \text{ (zero)} \\ \text{is negative} \end{array} \right)$$

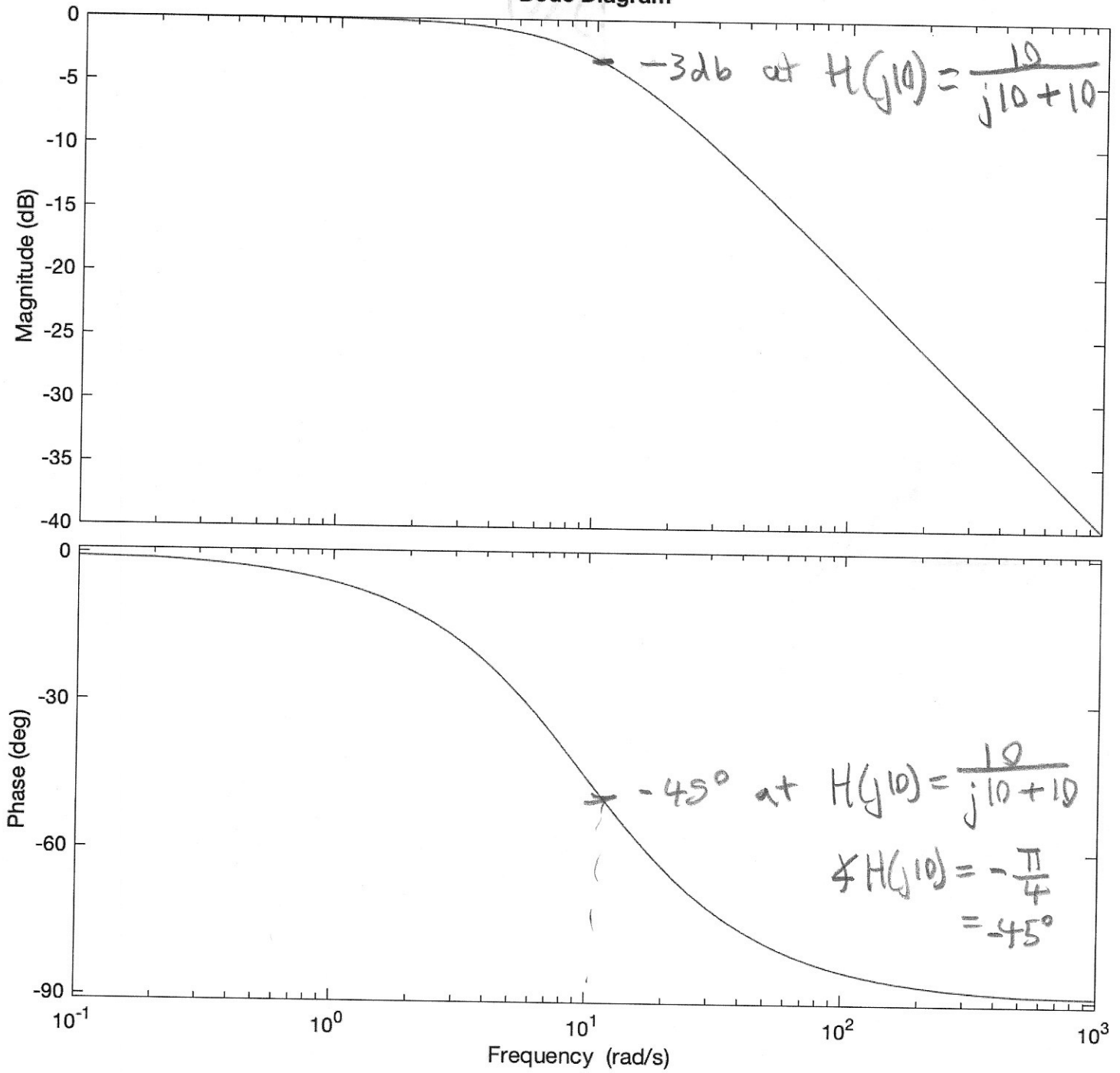


Straight line good enough

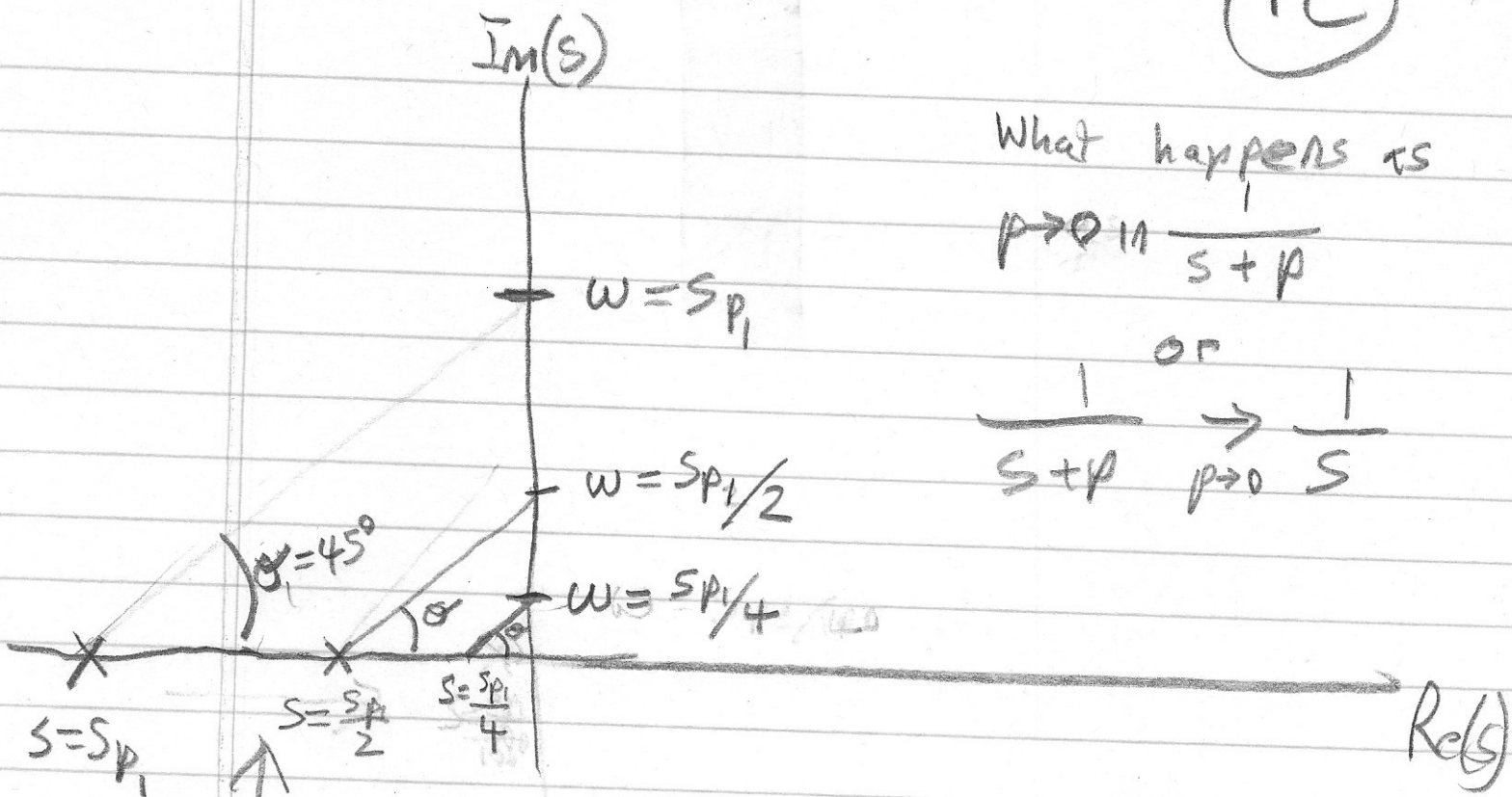
$s = tf(s)$
 $bode(10/(s+10))$ } matlab

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Bode Diagram



IC



What happens as

$$p \rightarrow 0 \Rightarrow \frac{1}{s+p}$$

or

$$\frac{1}{s+p} \xrightarrow{p \rightarrow 0} \frac{1}{s}$$

as pole approaches 0

$$\angle H(j\omega) \rightarrow -90^\circ$$

or

$$\frac{1}{s} \Big|_{s=j\omega} = \frac{1}{j\omega} = -j \frac{1}{\omega}$$

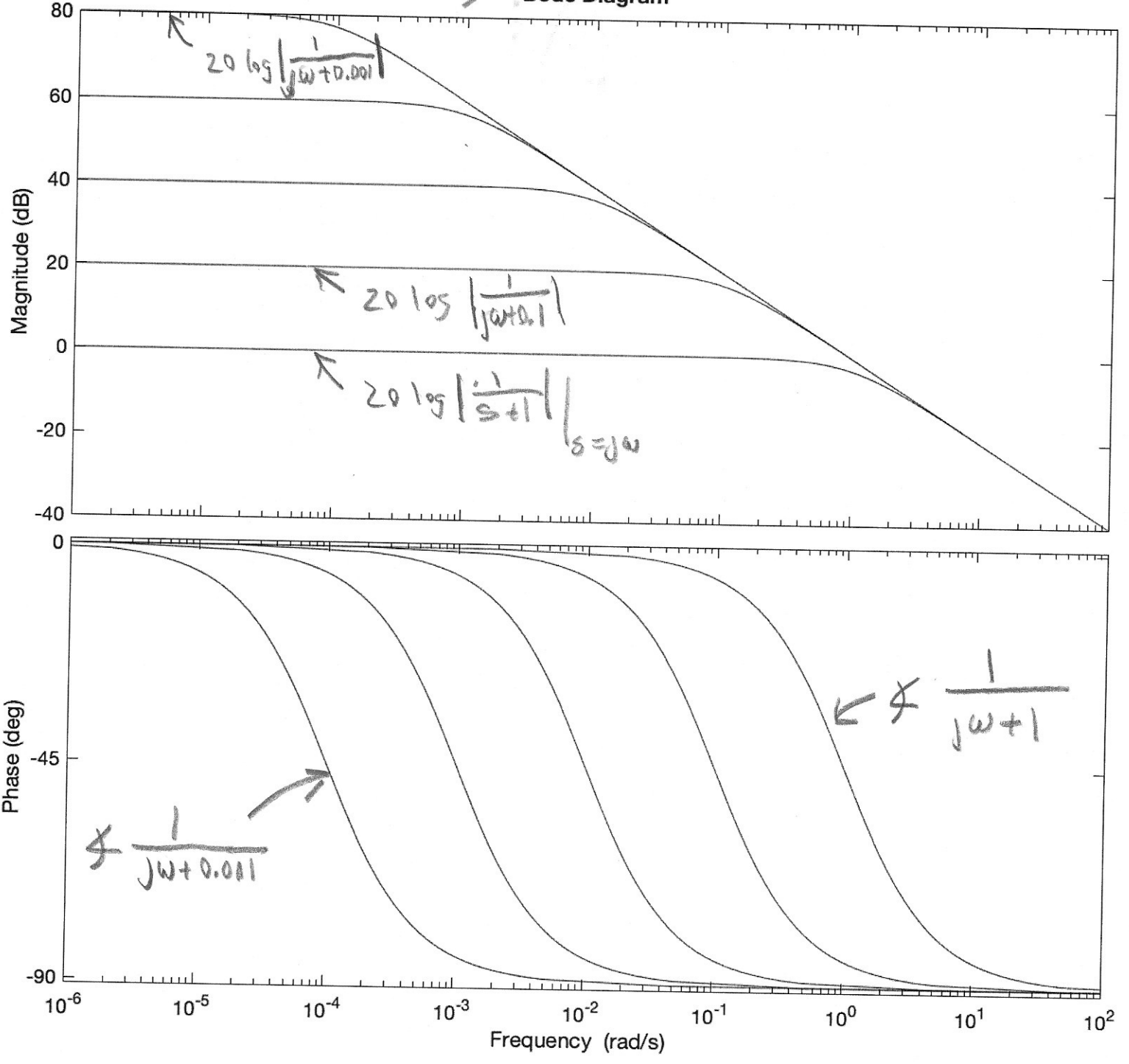
$$\angle H(j\omega) = -90^\circ$$

$\text{bode}(1/(s+1));$ hold on;
 $\text{bode}(1/(s+0.1));$
 \vdots
 $\text{bode}(1/(s+0.0001));$

matlab commands

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Bode Diagram



Demonstration

$$\frac{1}{s+p} \xrightarrow{p \rightarrow 0} \frac{1}{s} \xrightarrow{s=j\omega} \frac{1}{j\omega+p} \xrightarrow{p \rightarrow 0} \frac{1}{j\omega} = -j \frac{1}{\omega}$$

$$\text{If } H = H_a H_b$$

$$\text{then } \angle H = \angle H_a + \angle H_b$$

$$\text{and } 20 \log |H| = 20 \log |H_a| + 20 \log |H_b|$$

Example

$$H(s) = \frac{1000}{(s+10)(s+100)}$$

$$\angle H(j\omega) = \angle \frac{1}{j\omega+10} + \angle \frac{1}{j\omega+100}$$

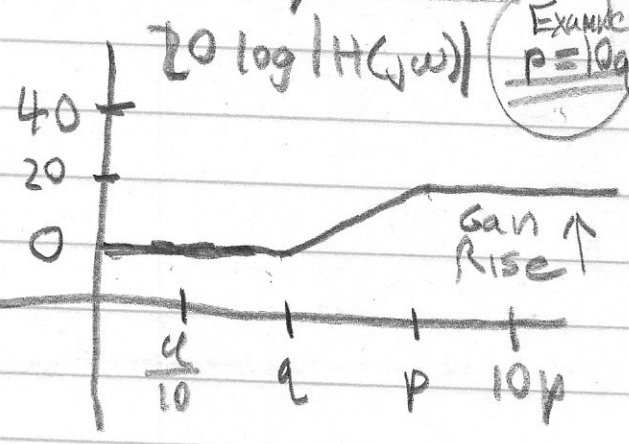
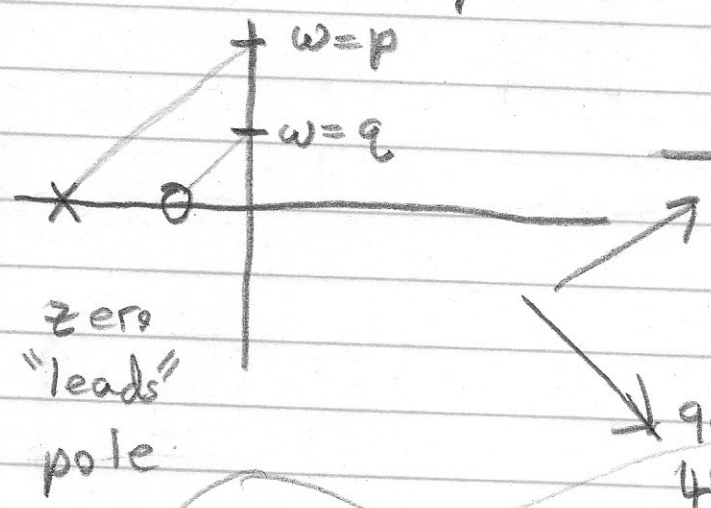
$$20 \log |H(j\omega)| = 20 \log(1000) - 20 \log |j\omega+10| - 20 \log |j\omega+100|$$

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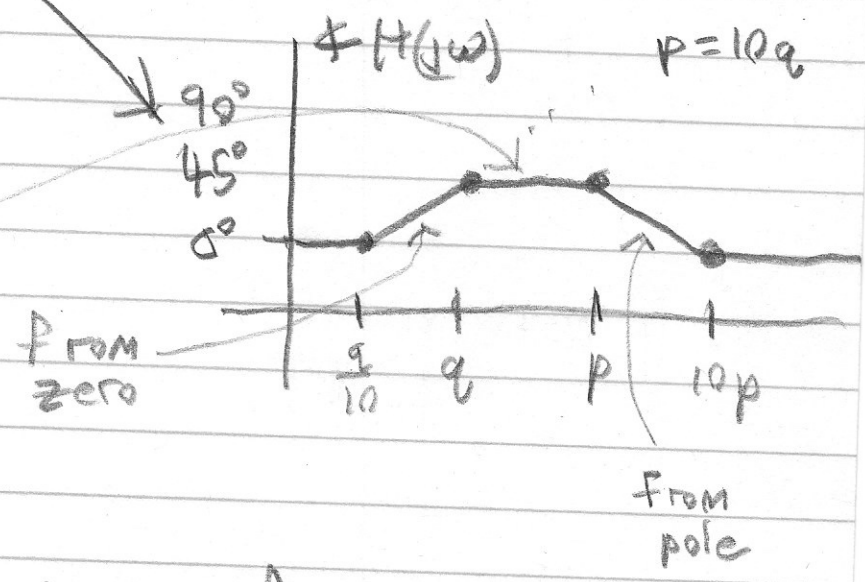
Lead Frequency Response

$$H(s) = \frac{p}{q} \left(\frac{s-q}{s-p} \right) \quad |q| < |p|$$

Example:
 $p = 10q$



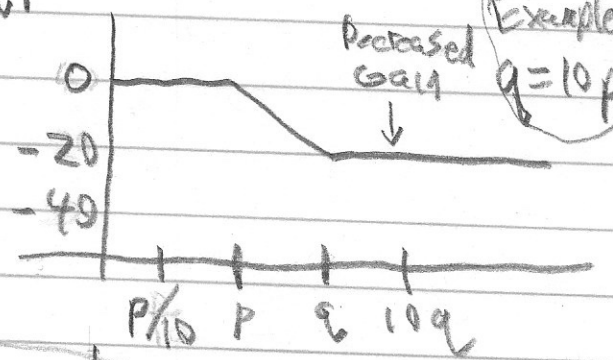
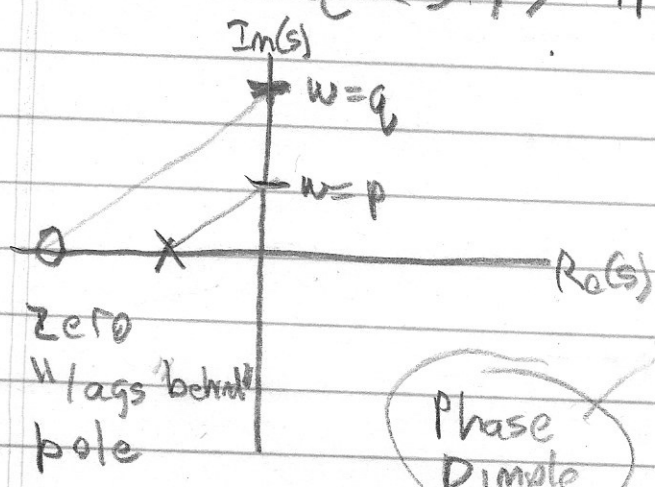
Phase Bump



Lag Frequency Response

$$H(s) = \frac{p}{q} \left(\frac{s-q}{s-p} \right) \quad |p| < |q|$$

Example:
 $q = 10p$



Phase Dimple

