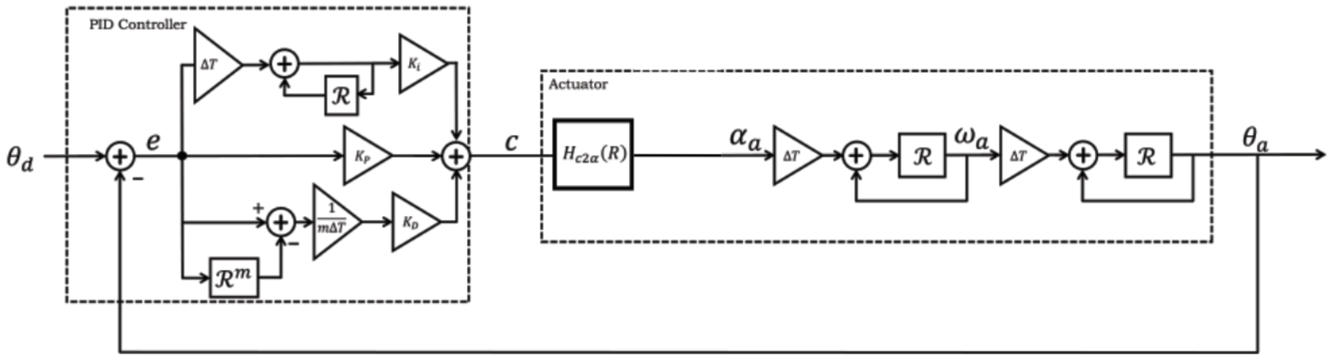
Code of Arms: Post-Lab – Solutions

This post-lab is due on Thursday, March 9, 2023, at 11:59pm EST.

Problems

1. System (Transfer) Function

In Lab 2 (Code of Arms), we studied a propellor arm system that is controlled by a PD or PID controller. The system block diagram is shown below:



Determine the system function (aka transfer function) for this block diagram as follows. Express your answers in terms of the operator $\mathcal{R}$.

1a. Let $C = K(\mathcal{R})E$. Determine an expression for $K(\mathcal{R})$ in terms of the controller constants K_p, K_d, K_i, and m as well as the step size ΔT.

$$K(\mathcal{R}) = K_p + K_d\left(\frac{1-\mathcal{R}^m}{m\Delta T}\right) + K_i\left(\frac{\Delta T}{1-\mathcal{R}}\right)$$

1b. Let $A_a = H_{c2a}(\mathcal{R})C$. Determine an expression for H_{c2a} for the "Non-Instant" motor model from part 7 of the lab.

$$H_{c2a}(\mathcal{R}) = \frac{-\gamma\beta\Delta T\mathcal{R}}{1-(1+\beta\Delta T)\mathcal{R}}$$

1c. Let $\Theta_a = H_{a2\theta}(\mathcal{R})A$. Determine $H_{a2\theta}$.

$$H_{a2\theta}(\mathcal{R}) = \frac{(\Delta T\mathcal{R})^2}{(1-\mathcal{R})^2}$$

1d. Let $\Theta_a = H_c(\mathcal{R})\Theta_d$, where $H_c(\mathcal{R})$ represents the “closed-loop” transfer function. Determine $H_c(\mathcal{R})$ in terms of $K(\mathcal{R})$, $H_{c2a}(\mathcal{R})$, and $H_{a2\theta}(\mathcal{R})$.

$$H_c(\mathcal{R}) = \frac{K(\mathcal{R}) \times H_{c2a}(\mathcal{R}) \times H_{a2\theta}(\mathcal{R})}{1 + K(\mathcal{R}) \times H_{c2a}(\mathcal{R}) \times H_{a2\theta}(\mathcal{R})}$$

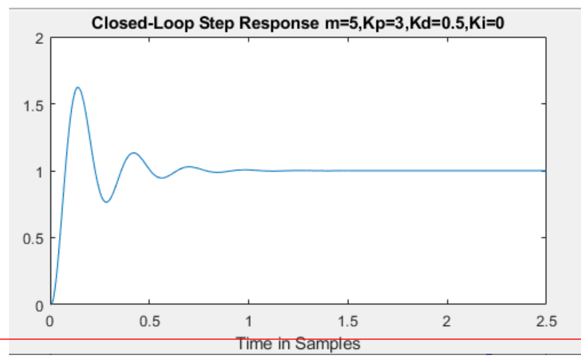
2. Step Responses

Compare the step responses of the PD and PID systems as follows. Set the input $\theta_d[n]$ to a unit step signal

$$\theta_d[n] = \begin{cases} 1 & \text{if } n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Then use your favorite computer language to compute the step response under the following conditions.

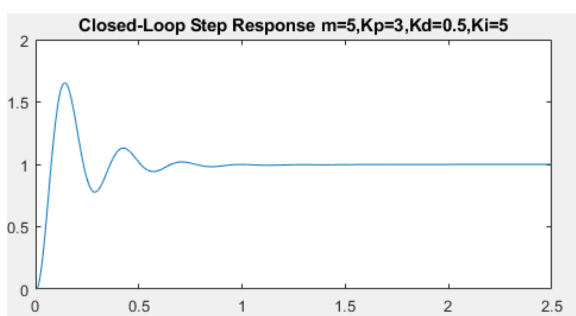
2a. PD controller. Set $K_i = 0$ and use the control parameters that you used in lab. Paste a plot of the step response in the box below.



What is the steady-state error of this system?

steady-state error = 0

2b. PID controller. Now set K_i to the value you used in lab and calculate the response with a PID controller.



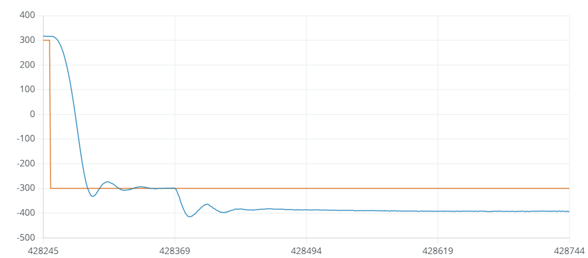
What is the steady-state error of this system?

steady-state error = 0

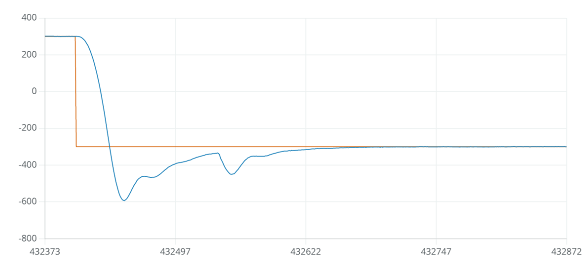
3. Disturbance Measurements

From a modeling point of view, adding the integral term doesn't help much for this system. However, the integral term helps significantly when dealing with disturbances. You have collected data when a weight is dropped onto the propellor arm. Let's compare the disturbance rejection properties for the PD and PID controllers.

3a. Show the experimental data of your PD controller when a weight is added.



3b. Show the experimental data of your PID controller when a weight is added.

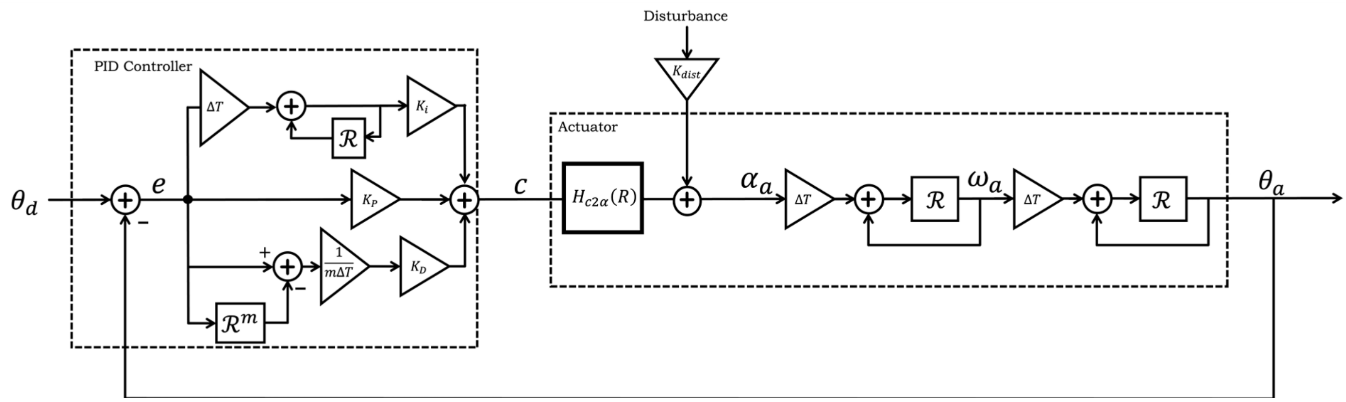


3c. Which controller provides better disturbance rejection?

PD or PID: It should be clear that the PID controller has better disturbance rejection.

4. Disturbance Model

We can model the effect of the disturbance by adding a new input pathway as shown below.



Determine the disturbance transfer function H_{dist} as follows

$$\Theta_a = H_{dist}(\mathcal{R})M_{dist}$$

where M_{dist} represents the disturbance input. When you write the disturbance transfer function, you can set the other input Θ_a to 0.

4a. Derive the disturbance transfer function.

$$H_{a2\theta}(\mathcal{R}) - H_{c2a}(\mathcal{R})K(\mathcal{R})\Theta_a + K_{dist}M_{dist} = \Theta_a$$

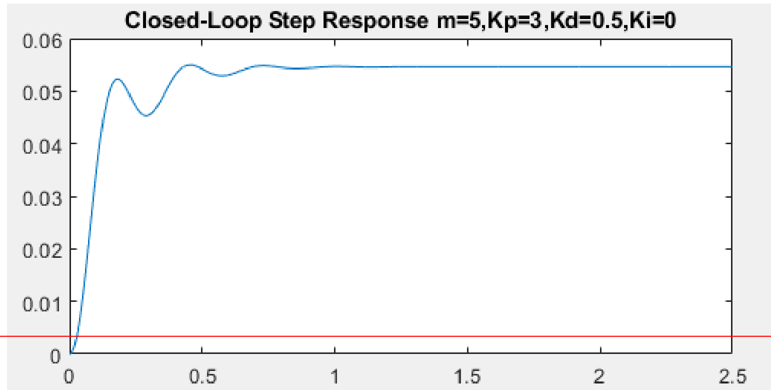
$$\Theta_a\left(-H_{a2\theta}(\mathcal{R})H_{c2\alpha}(\mathcal{R})K(\mathcal{R}) - 1\right) = -K_{dist}M_{dist}H_{a2\theta}(\mathcal{R})$$

$$\Theta_a = \left(\frac{K_{dist}H_{a2\theta}(\mathcal{R})}{H_{a2\theta}(\mathcal{R})H_{c2a}(\mathcal{R})K(\mathcal{R}) + 1}\right)M_{dist}$$

$$H_{dist}(\mathcal{R}) = \frac{K_{dist}H_{a2\theta}(\mathcal{R})}{H_{a2\theta}(\mathcal{R})H_{c2a}(\mathcal{R})K(\mathcal{R}) + 1}$$

Implement this disturbance function in your favorate programming language. Simulate the system disturbance rejection with a PD controller and PID controller. In the simulations, you can model dropping a weight as a step function. Use $K_{dist} = 10$.

4b. Disturbance rejection with a PD controller.



4c. Disturbance rejection with a PID controller.