6.3100: Dynamic System Modeling and Control Design

State-Space and Observer-Based Control

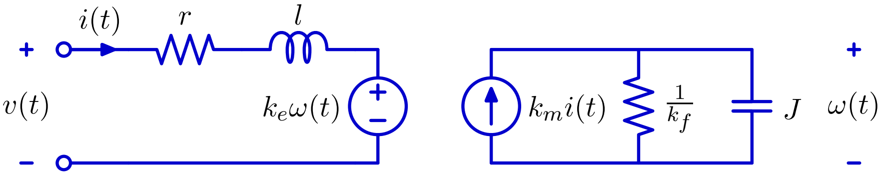
May 1, 2024

Motor Speed Control

Last time: control two-spring system using state-space/observer methods.

Today: apply same approach to control the speed of a motor.

Model of the plant:



The voltage $v(t)$ represents the electrical input to the motor.

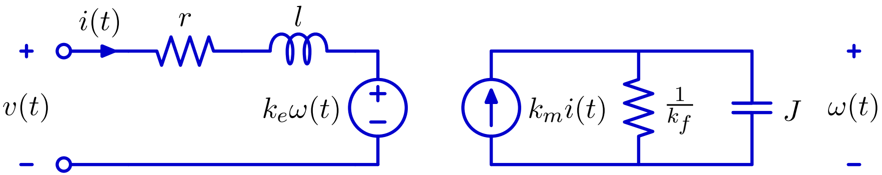
It excites a current $i(t)$, which generates a torque $k_m i(t)$ that tends to rotate the motor shaft.

The torque is resisted by the moment of inertia J and by friction (k_f).

As the motor spins, it generates a back emf ($k_e\omega(t)$) that tends to reduce the electrical current $i(t)$ drawn by the motor.

Motor Speed Control: State-Space Model

Mathematical description of the model:



Electrical port:

$$v(t) = ri(t) + l\frac{di(t)}{dt} + k_e\omega(t)$$

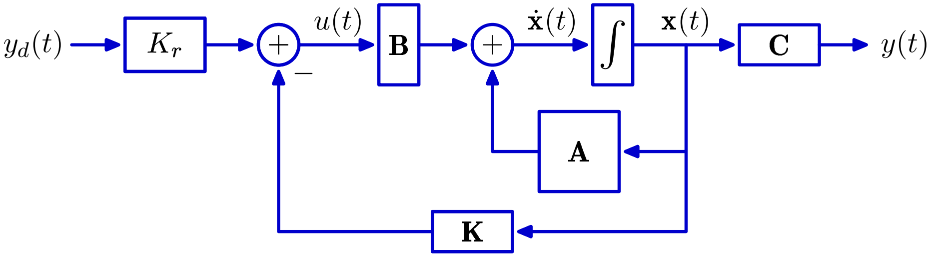
Mechanical port:

$$k_m i(t) = k_f\omega(t) + J\frac{d\omega(t)}{dt}$$

Determine a state-space description of this system.

State-Space Controller

A state-space controller can then be expressed as follows.



Find $\mathbf{K}$ with **pole placement**:

```
K = place(A,B,[pole1,pole2])
```

or **LQR**:

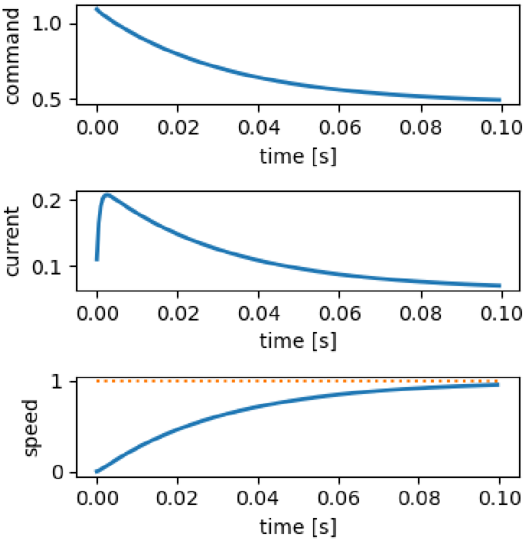
`K = lqr(A,B,Q,R)` where `Q = diag([penalty1,penalty2])` and `R = 1`

Then

```
Kr = -1/(C*inv(A-B*K)*B)
```

Choosing the Feedback Matrix K

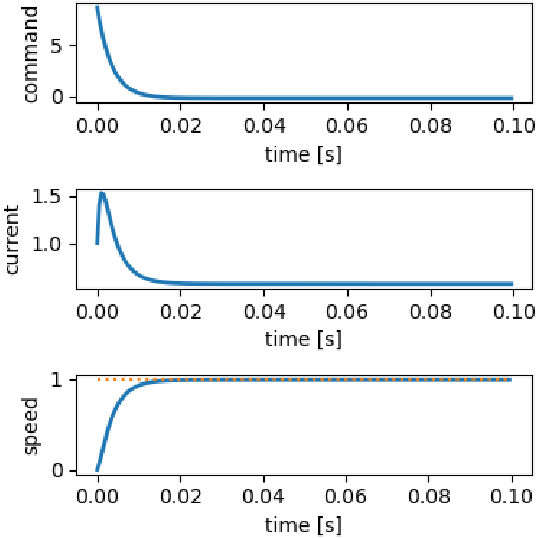
Try LQR with `Q = diag([1,1])` and `R = 1`.



How could we improve the response time for changes in speed?

Choosing the Feedback Matrix K

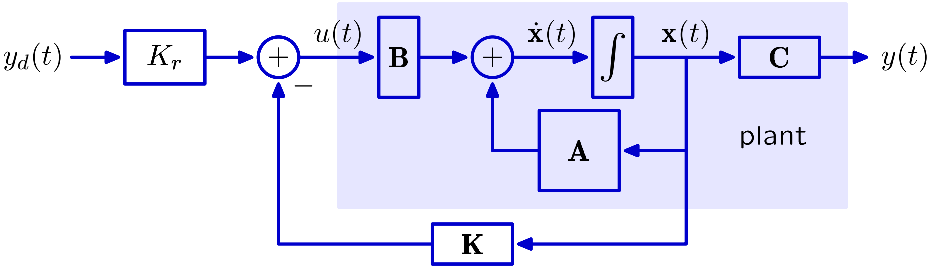
Try increasing the penalty on speed: `Q = diag([1,100])` and `R = 1`.



Definitely faster! Any drawbacks?

Effects of Sensor Noise

Feedback control can be significantly degraded by noise that is introduced by the sensors that provide information about the plant to the controller.



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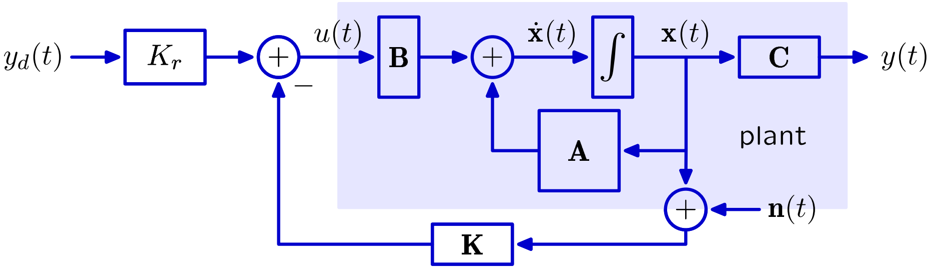
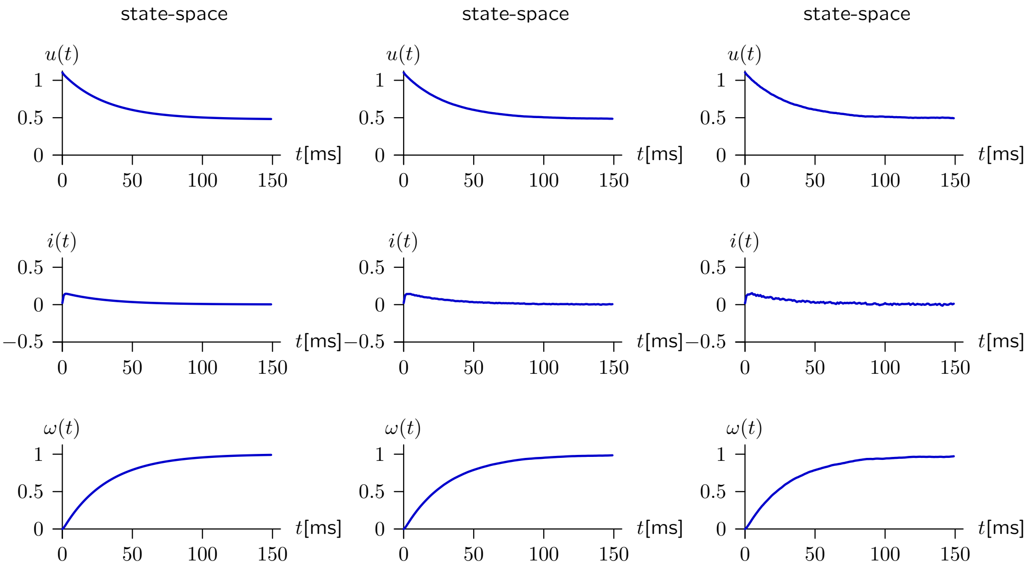


In the last lecture, we introduced a model for sensor noise.

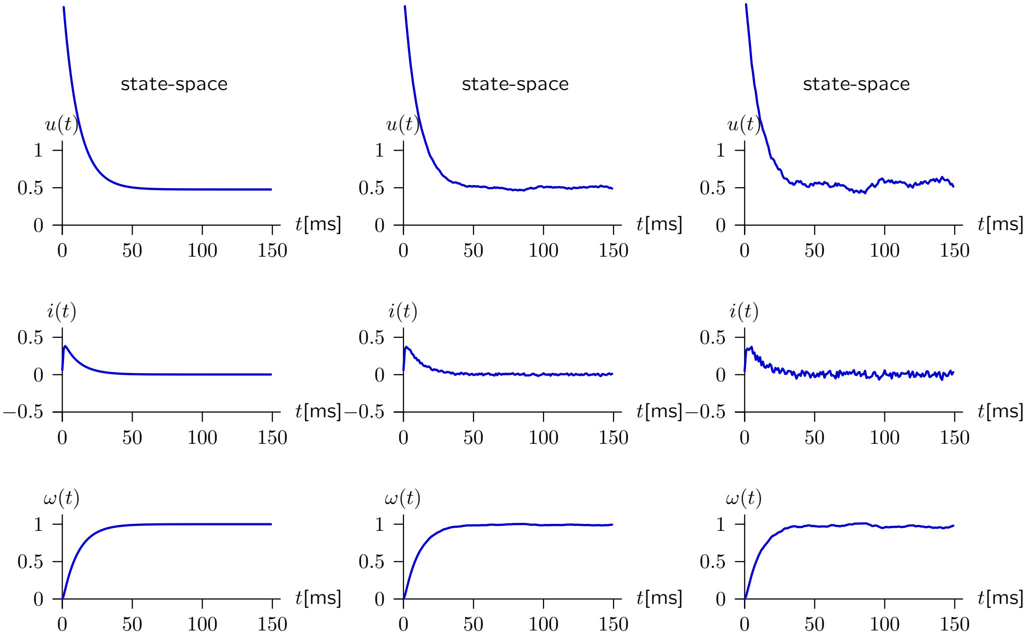
Effect of K on Noise Performance when Q=diag(1,1)

Low (left), medium (center), and high (right) values of $\mathbf{n}(t)$.



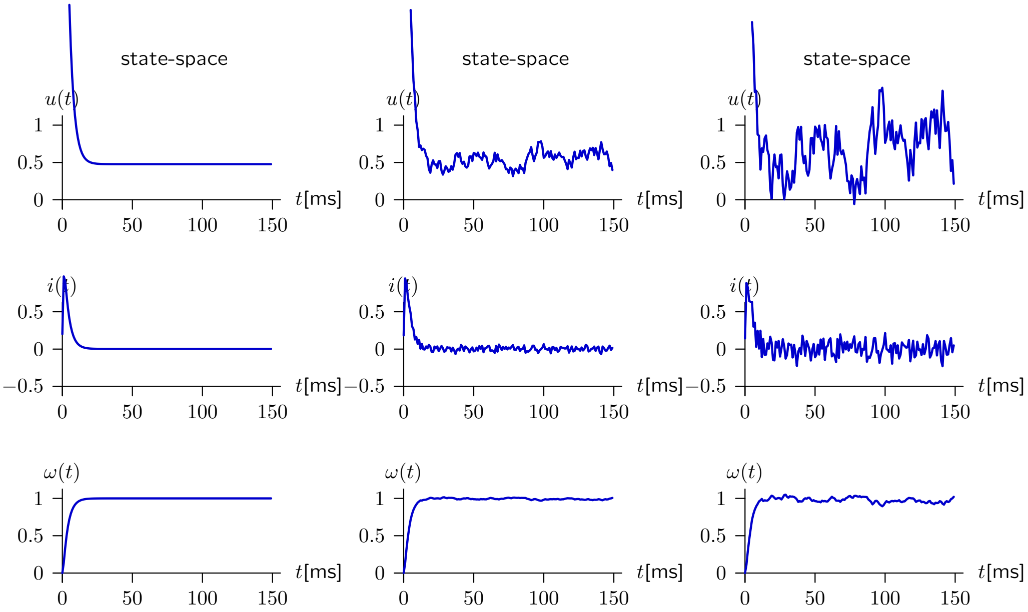
Effect of K on Noise Performance when Q=diag(1,10)

Low (left), medium (center), and high (right) values of $\mathbf{n}(t)$.



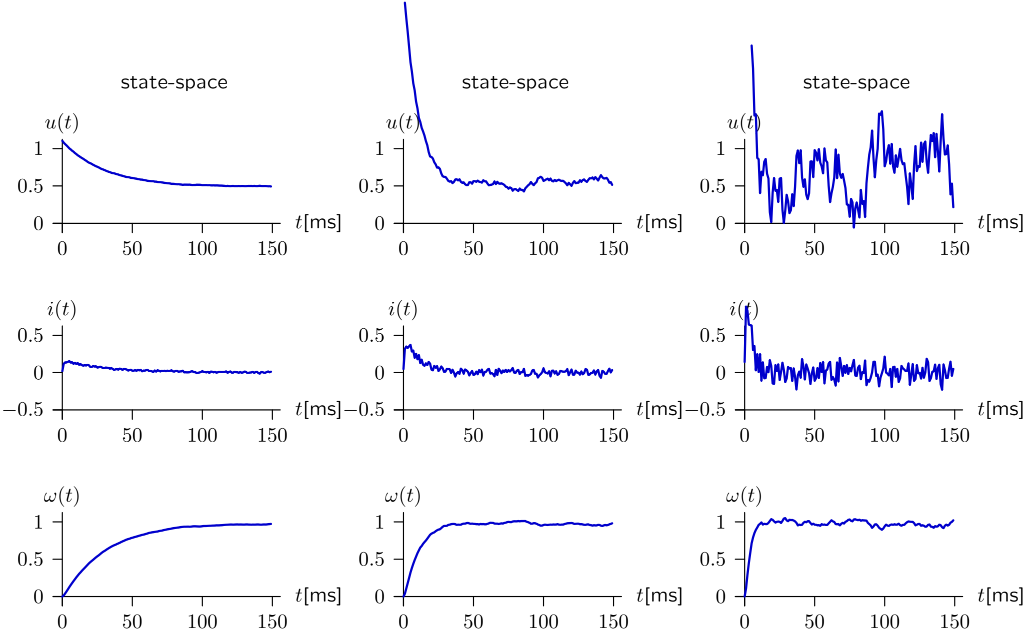
Effect of K on Noise Performance when Q=diag(1,100)

Low (left), medium (center), and high (right) values of $\mathbf{n}(t)$.



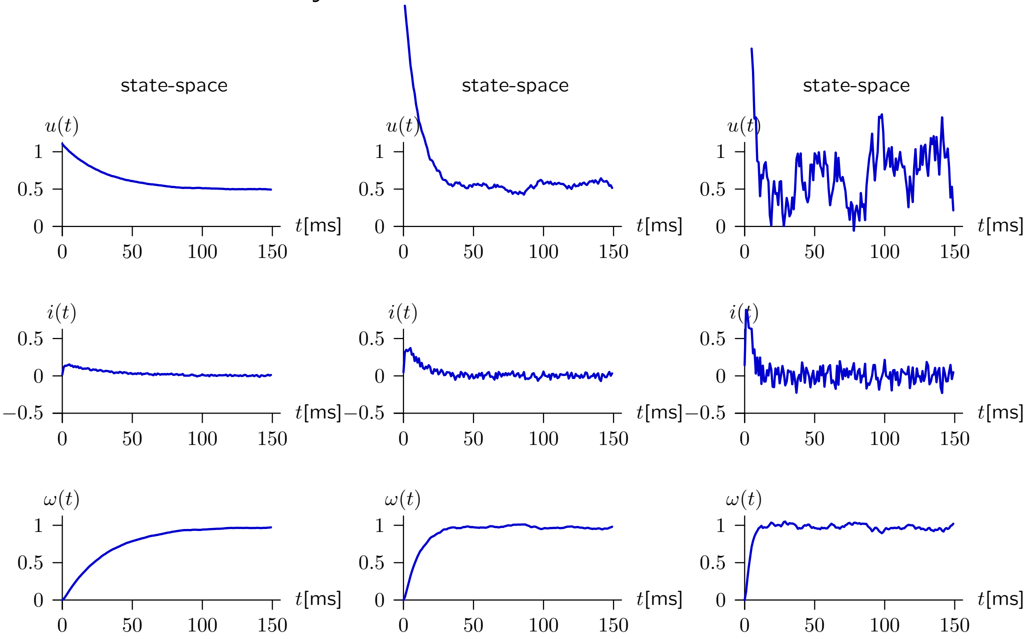
Effects of Q on Noise Performance

Results for Q=diag(1,1) (left), Q=diag(1,10), and Q=diag(1,100) (right) with **same amount of sensor noise**.



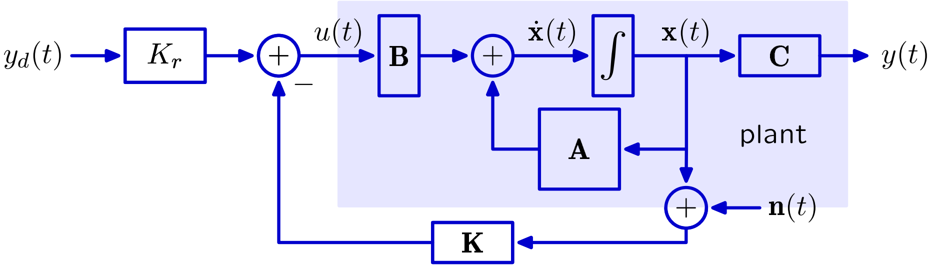
Effects of Q on Noise Performance

Increasing the velocity penalty increases **speed** of response but also increases **noise** sensitivity.



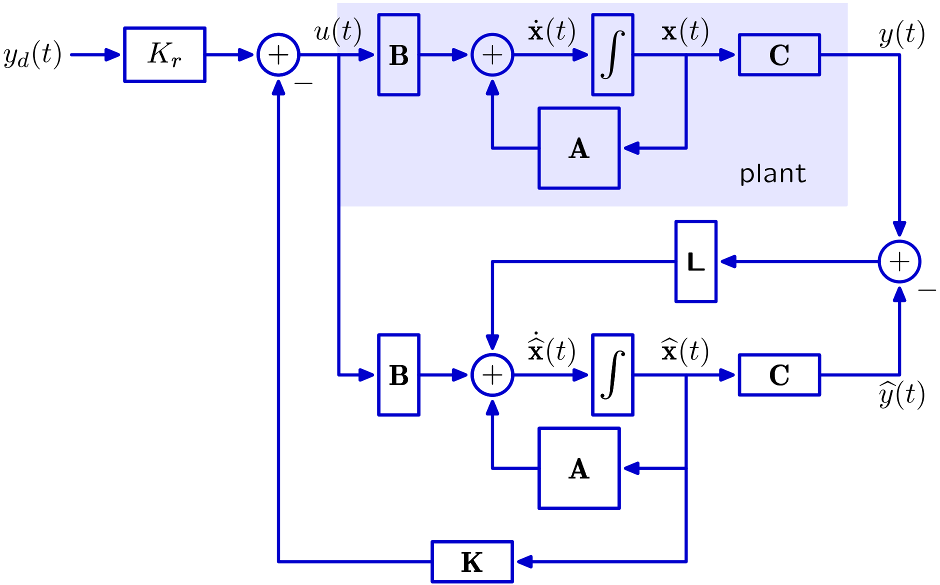
Check Yourself

Why does increasing the penalty on speed errors increase noise sensitivity?



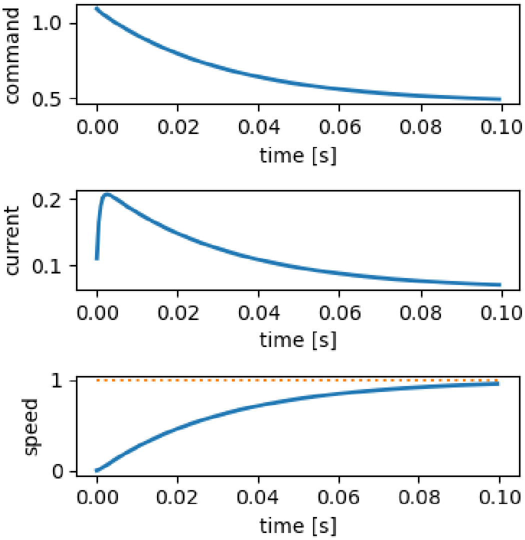
Design Tradeoffs with an Observer

Are there similar trends with an observer? Start by analyzing without noise.



Choosing the Matrix L

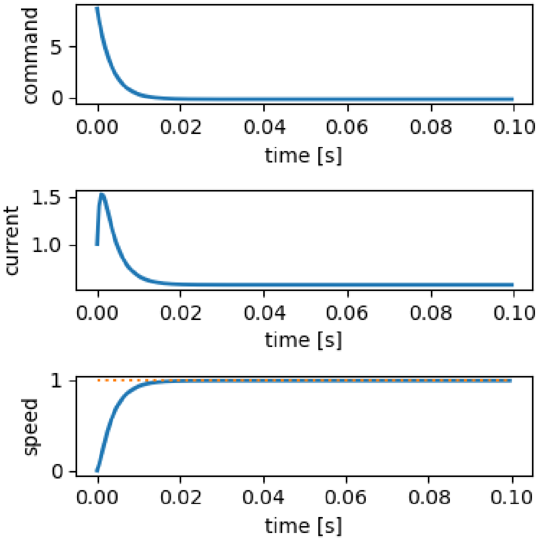
Try LQR with Q=diag(1,1) and R=1 for both $\mathbf{K}$ and $\mathbf{L}$.



Similar to result with state-space model.

Choosing the Matrix L

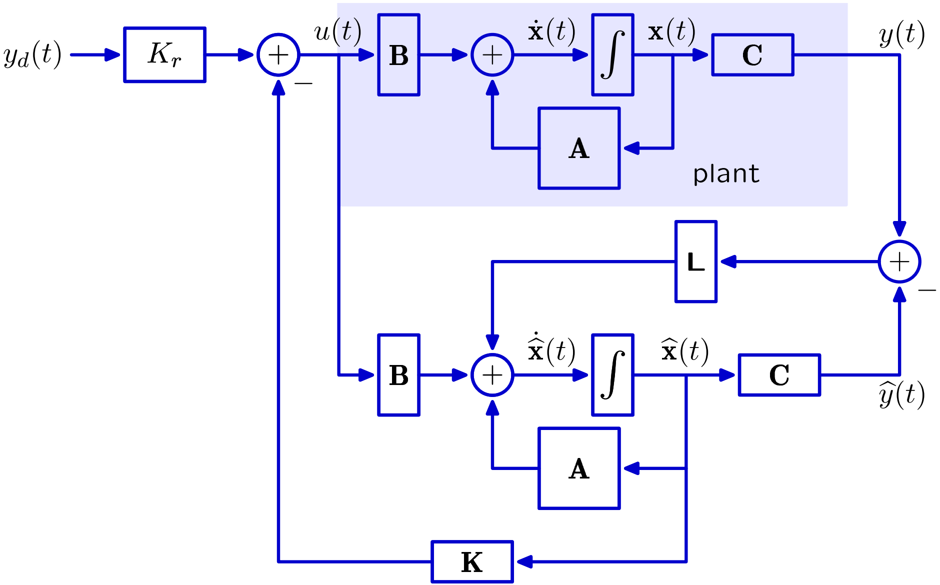
Try Q=diag(1,100) for both $\mathbf{K}$ and $\mathbf{L}$.



Again, similar to state-space result: higher gain $\rightarrow$ faster response.

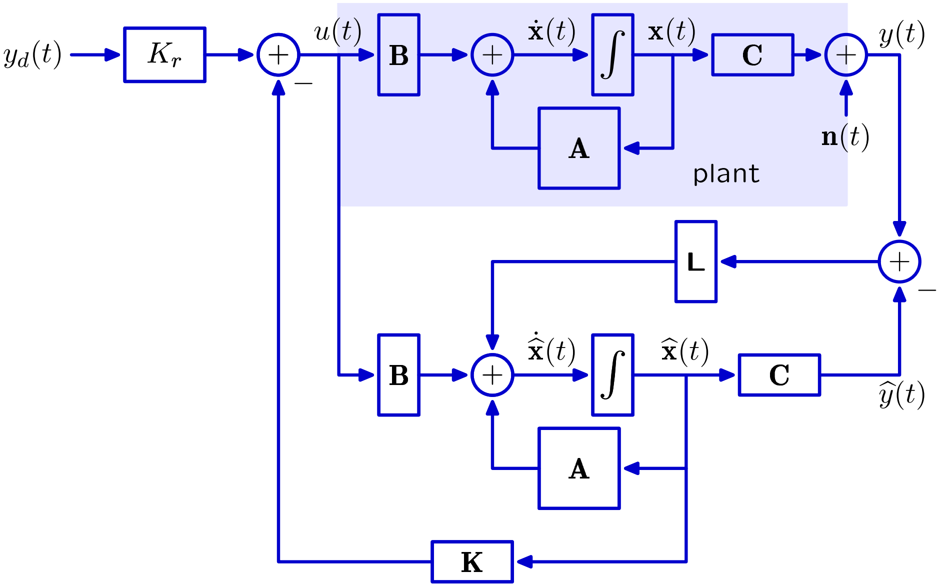
Effects of Sensor Noise

Model effects of noise on the observer system.



Effects of Sensor Noise

Sensor noise at the ouput is an important source of noise.



Choosing K and L

Find the eigenvalues associated with $\mathbf{K}$ and $\mathbf{L}$ for different Q and R.

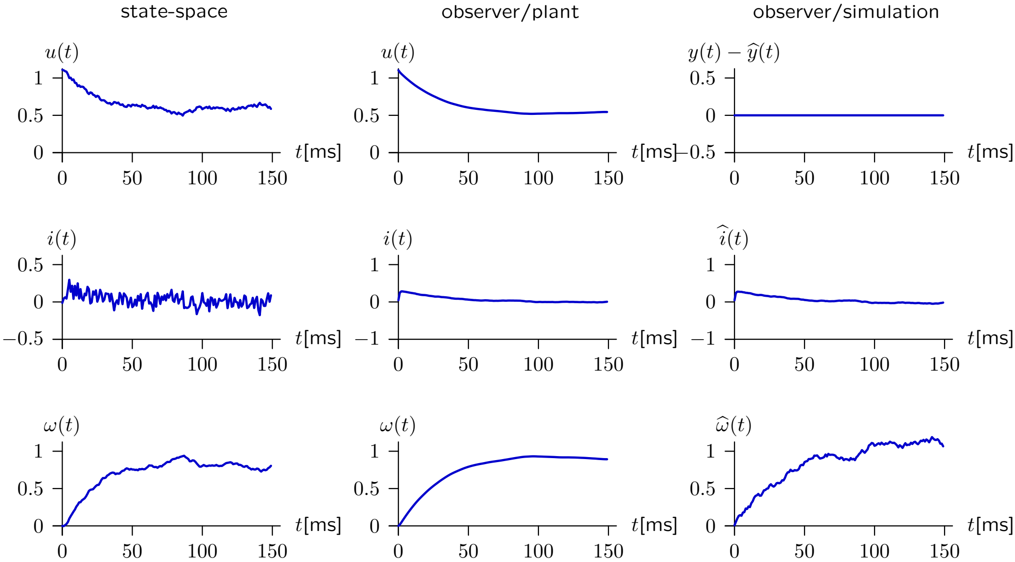
$\mathbf{Q}$	$\mathbf{R}$	$\mathbf{K}$	eig($\mathbf{A}-\mathbf{BK}$)	$\mathbf{L}$	eig($\mathbf{A^T}-\mathbf{C^T L^T}$)
[1,1]	1	[0.1597,0.6305]	-1401,-31.6	[-0.0024;0.0377]	-1387,-13.8
[1,10]	1	[0.4458,2.7199]	-1398,-91	[-0.0238;0.3662]	-1387,-14
[1,100]	1	[1.3104,9.5311]	-1370,-292	[-0.2146;3.3063]	-1387,-17
[1 1000]	1	[3.5963,31.1406]	-1060,-1060	[-1.3466;20.9981]	-1387,-35

Which of these values of Q will result in best performance?

Choosing of K and L

Choose $\mathbf{K}$ based on `Q=diag([1,1])` and `R=1`.

Choose $\mathbf{L}$ based on `Q=diag([1,1000])` and `R=1`.

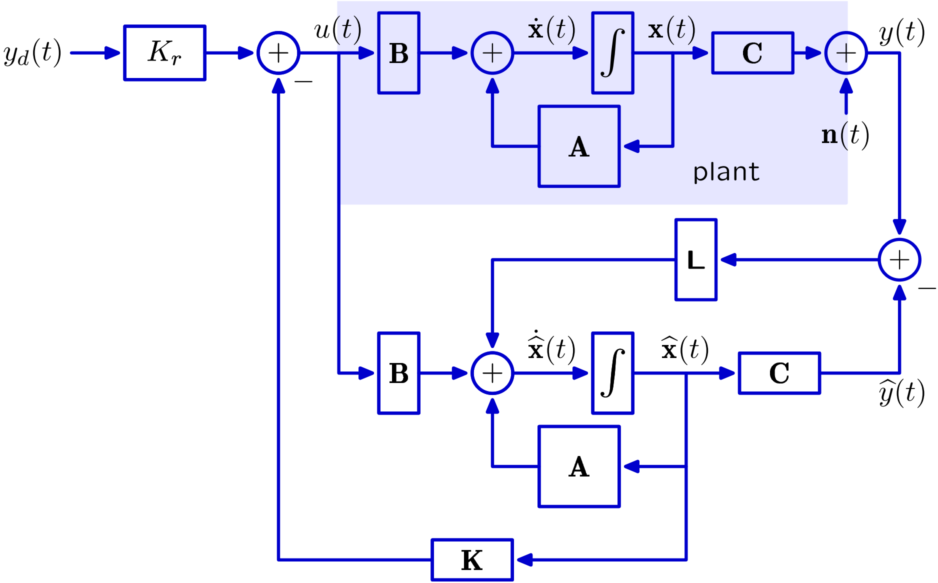


Is noise performance for the observer-based controller better or worse than that for the state-space controller?

Effects of Sensor Noise

Why is speed of state-space similar to that of observer?

Why is there less noise with observer (states are not even measured)?



Effects of K and L

As with the state-space controller, higher gains ($\mathbf{K}$) can increase speed.

Extra care is needed when designing an observer. Make sure that the observer stabilizes to the plant **before** the observer is used to provide feedback to the plant.