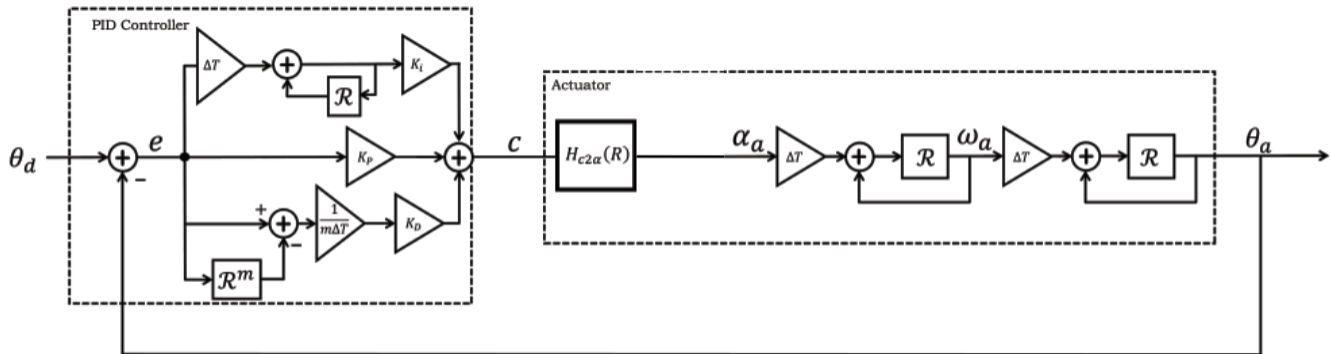


# Code of Arms: Post-Lab – Solutions

## Problems

### 1. System (Transfer) Function

In Lab 2 (Code of Arms), we studied a propellor arm system that is controlled by a PD or PID controller. The system block diagram is shown below:



Determine the system function (aka transfer function) for this block diagram as follows. Express your answers in terms of the operator  $\mathcal{R}$ .

1a. Let  $C = K(\mathcal{R})E$ . Determine an expression for  $K(\mathcal{R})$  in terms of the controller constants  $K_p$ ,  $K_d$ ,  $K_i$ , and  $m$  as well as the step size  $\Delta T$ .

$$K(\mathcal{R}) = \boxed{K_p + K_d \left( \frac{1 - \mathcal{R}^m}{m\Delta T} \right) + K_i \left( \frac{\Delta T}{1 - \mathcal{R}} \right)}$$

1b. Let  $A_a = H_{c2a}(\mathcal{R})C$ . Determine an expression for  $H_{c2a}$  for the "Non-Instant" motor model from part 7 of the lab.

$$H_{c2a}(\mathcal{R}) = \boxed{\frac{-\gamma\beta\Delta T\mathcal{R}}{1 - (1 + \beta\Delta T)\mathcal{R}}}$$

1c. Let  $\Theta_a = H_{a2\theta}(\mathcal{R})A$ . Determine  $H_{a2\theta}$ .

$$H_{a2\theta}(\mathcal{R}) = \boxed{\frac{(\Delta T\mathcal{R})^2}{(1 - \mathcal{R})^2}}$$

1d. Let  $\Theta_a = H_c(\mathcal{R})\Theta_d$ , where  $H_c(\mathcal{R})$  represents the "closed-loop" transfer function. Determine  $H_c(\mathcal{R})$  in terms of  $K(\mathcal{R})$ ,  $H_{c2a}(\mathcal{R})$ , and  $H_{a2\theta}(\mathcal{R})$ .

$$H_c(\mathcal{R}) = \boxed{\frac{K(\mathcal{R}) \times H_{c2a}(\mathcal{R}) \times H_{a2\theta}(\mathcal{R})}{1 + K(\mathcal{R}) \times H_{c2a}(\mathcal{R}) \times H_{a2\theta}(\mathcal{R})}}$$

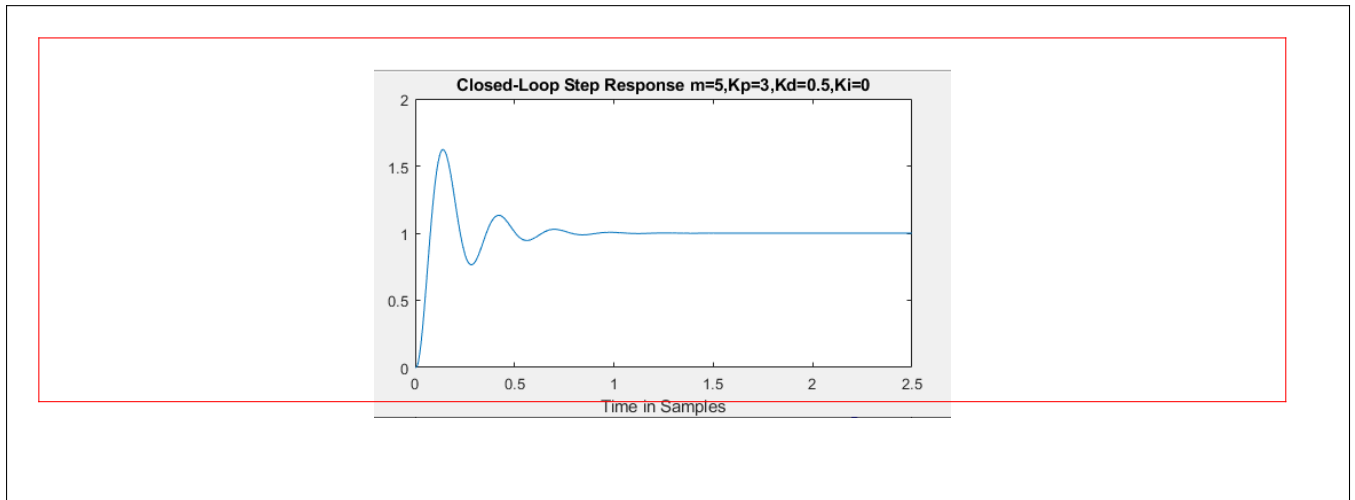
## 2. Step Responses

Compare the step responses of the PD and PID systems as follows. Set the input  $\theta_d[n]$  to a unit step signal

$$\theta_d[n] = \begin{cases} 1 & \text{if } n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Then use your favorite computer language to compute the step response under the following conditions.

2a. PD controller. Set  $K_i = 0$  and use the control parameters that you used in lab. Paste a plot of the step response in the box below.

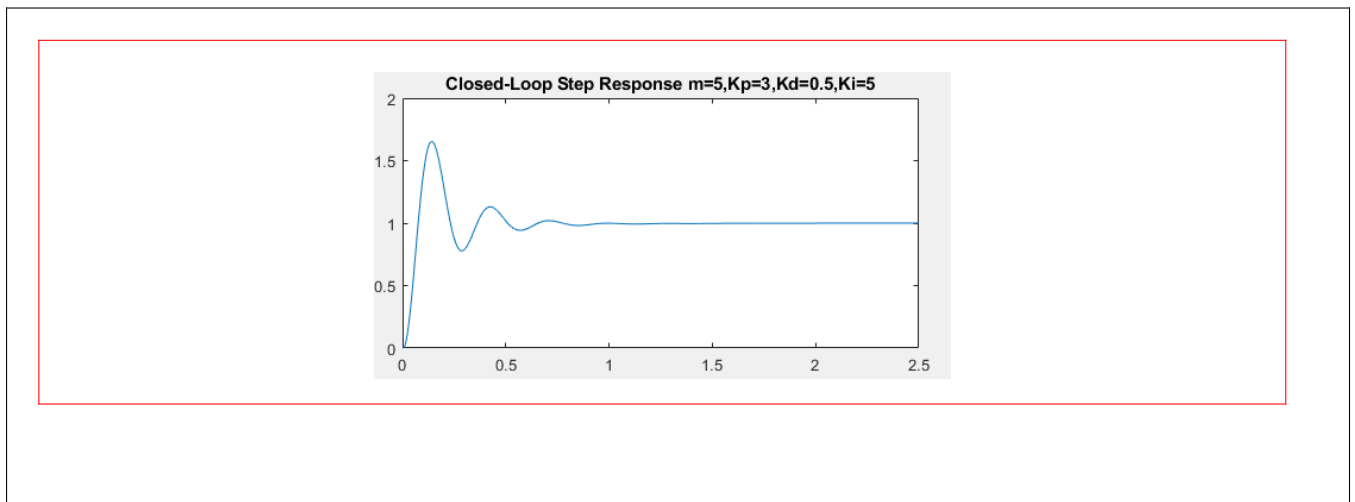


What is the steady-state error of this system?

steady-state error =

0

2b. PID controller. Now set  $K_i$  to the value you used in lab and calculate the response with a PID controller.



What is the steady-state error of this system?

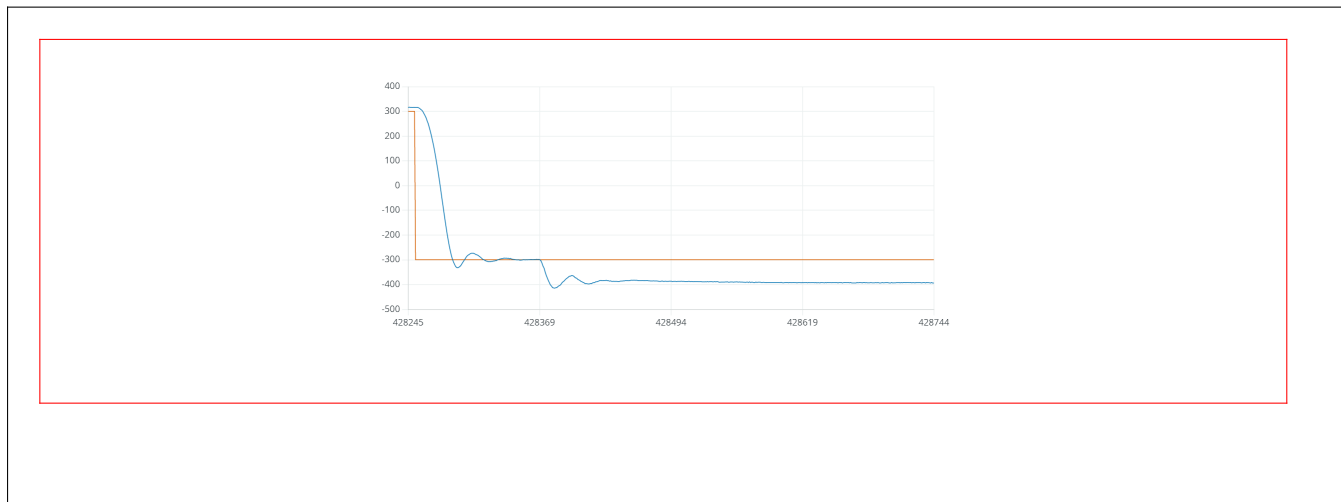
steady-state error =

0

### 3. Disturbance Measurements

From a modeling point of view, adding the integral term doesn't help much for this system. However, the integral term helps significantly when dealing with disturbances. You have collected data when a weight is dropped onto the propellor arm. Let's compare the disturbance rejection properties for the PD and PID controllers.

3a. Show the experimental data of your PD controller when a weight is added.



3b. Show the experimental data of your PID controller when a weight is added.



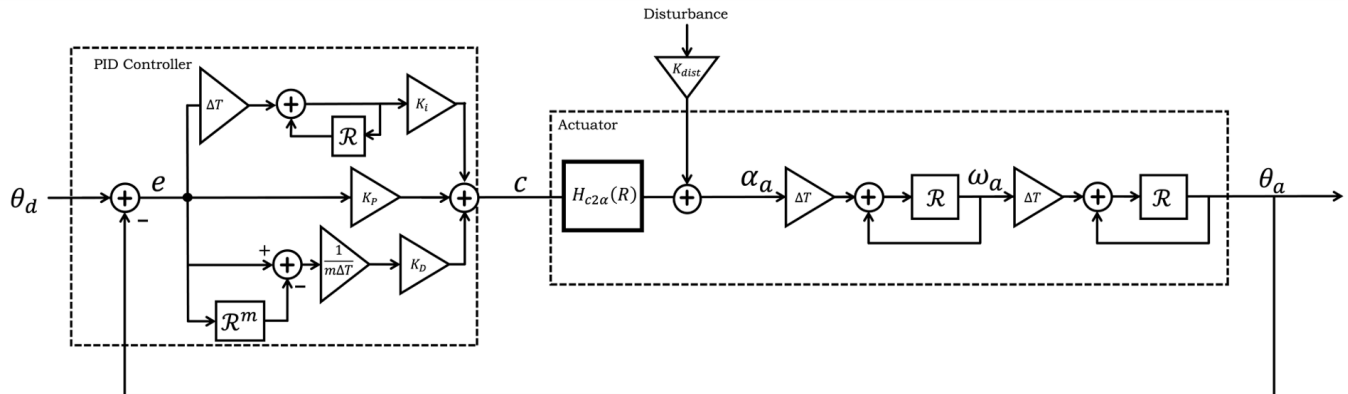
3c. Which controller provides better disturbance rejection?

PD or PID:

It should be clear that the PID controller has better disturbance rejection.

#### 4. Disturbance Model

We can model the effect of the disturbance by adding a new input pathway as shown below.



Determine the disturbance transfer function  $H_{dist}$  as follows

$$\Theta_a = H_{dist}(\mathcal{R})M_{dist}$$

where  $M_{dist}$  represents the disturbance input. When you write the disturbance transfer function, you can set the other input  $\Theta_d$  to 0.

4a. Derive the disturbance transfer function.

$$H_{a2\theta}(\mathcal{R})(-H_{c2a}(\mathcal{R})K(\mathcal{R})\Theta_a + K_{dist}M_{dist}) = \Theta_a$$

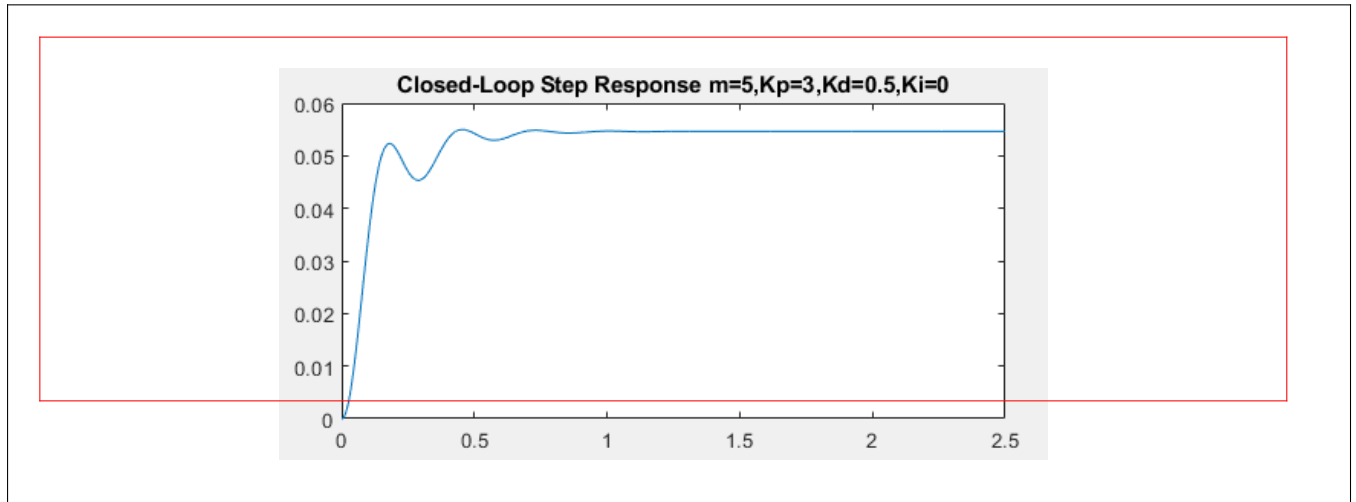
$$\Theta_a(-H_{a2\theta}(\mathcal{R})H_{c2a}(\mathcal{R})K(\mathcal{R}) - 1) = -K_{dist}M_{dist}H_{a2\theta}(\mathcal{R})$$

$$\Theta_a = \left( \frac{K_{dist}H_{a2\theta}(\mathcal{R})}{H_{a2\theta}(\mathcal{R})H_{c2a}(\mathcal{R})K(\mathcal{R}) + 1} \right) M_{dist}$$

$$H_{dist}(\mathcal{R}) = \frac{K_{dist}H_{a2\theta}(\mathcal{R})}{H_{a2\theta}(\mathcal{R})H_{c2a}(\mathcal{R})K(\mathcal{R}) + 1}$$

Implement this disturbance function in your favorite programming language. Simulate the system disturbance rejection with a PD controller and PID controller. In the simulations, you can model dropping a weight as a step function. Use  $K_{dist} = 10$ .

4b. Disturbance rejection with a PD controller.



4c. Disturbance rejection with a PID controller.

