Marginal Maglev: Post-Lab

In our fourth lab (Marginal Maglev), we designed a lead controller for a magnetic levitation system. In this post lab exercise, we will implement a double lead controller in simulation. Assume that the open loop transfer function of the magnetic levitation system is given by

$$H(s) = \frac{\gamma(-\lambda_E)}{(s - \lambda_E)(s^2 - \gamma_{ay})}$$

and (for ease of grading) please use the following parameter values:

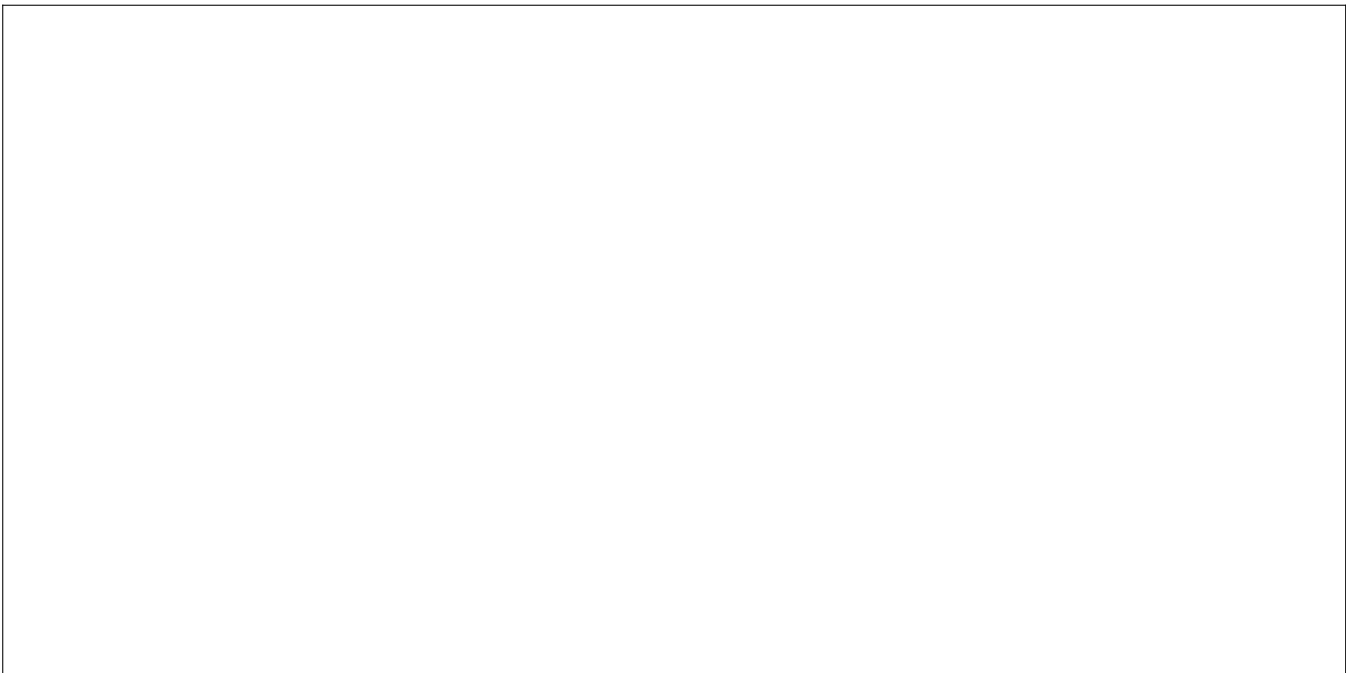$$\gamma = 1250$$
$$\gamma_{ay} = 1000$$
$$\lambda_E = -125$$

Part 1. Start by implementing a PD controller that can stabilize this system. What values of K_p and K_d result?

$K_p = $

$K_d = $

Please show a bode plot of the open loop transfer function $H(s)K(s)$. You should report the phase margin and gain margin from your simulation. In addition, please show the system step response. You may find the MATLAB functions `margin()` and `step()` useful.

Part 2. The main issue with a PD controller is that it amplifies high frequency signals. If the physical system cannot dissipate high frequency disturbances effectively, then a PD controller may not perform well. Next, we will implement a lead controller. A lead controller can increase a system's phase margin without significantly amplifying high frequency signals. First, let's implement a lead controller in the form:

$$K_{lead}(s) = K_0 \left(\frac{s_p}{s_z} \right) \left(\frac{s - s_z}{s - s_p} \right)$$

This requires 3 parameters: K_0, s_p, and s_z. For this problem, please use

$$K_0 = 5$$
$$s_p = -900$$
$$s_z = -45$$

What are the low-frequency and high-frequency gains for this lead controller?

low-frequency gain:

high-frequency gain:

What is the maximum phase gain (and its corresponding frequency) of this lead controller?

$\phi_{max} =$

corresponding frequency:

On the same bode plot, please show the transfer function of $H(s)$, $K_{lead}(s)$, and $H(s)K_{lead}(s)$.

What is the phase margin of the open-loop system $K_{lead}H(s)$? Is this a stable system?

phase margin:

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stable?

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Please show the step response of the system with this lead controller.

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Part 3. While a lead controller can be used to stabilize the maglev system, we would like to explore ways to improve the system's phase margin. Let's try a double-lead controller:
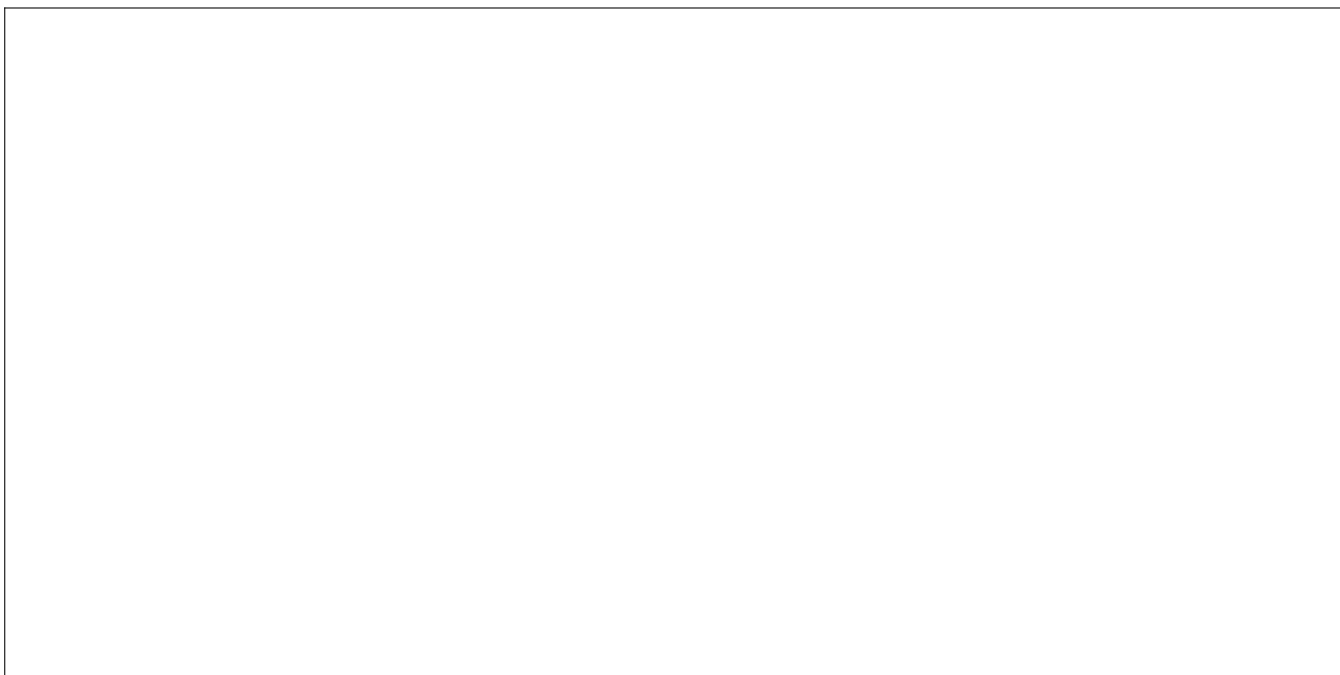
$$K_{dl} = \left(\sqrt{K_0} \left(\frac{s_p}{s_z} \right) \left(\frac{s - s_z}{s - s_p} \right) \right)^2$$

What are the low-frequency and high-frequency gains of this controller?

low-frequency gain:

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high-frequency gain:

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Please implement this double-lead controller and show the transfer functions $H(s)$, $K_{dl}(s)$, and $H(s)K_{dl}(s)$.

Has the phase margin increased? What are the pros and cons of this double lead controller?

Part 4. Now please design a double-lead controller that satisfies the following two constraints:

1. $|K_{new}(s \to 0)| = K_0 = 5$
2. $|K_{new}(s \to \infty)| = 100$

Your double-lead controller should have the form:

$$K_{new} = K_0 \frac{s_{p1}(s - s_{z1})s_{p2}(s - s_{z2})}{s_{z1}(s - s_{p1})s_{z2}(s - s_{p2})}$$

Your goal is to choose the two pairs of poles and zeros so that you can improve the phase margin without increasing the high frequency controller gain. This design process is iterative. Please show the bode plot of the new system $H(s)K_{new}(s)$.

What is the new phase margin and what are your values of s_{z1}, s_{z2}, s_{p1}, and s_{p2}?

ϕ_{new} :

s_{z1}:

s_{z2}:

s_{p1}:

s_{p2}:

Part 5. To implement the double-lead controller on a microcontroller (like the Teensy), we need discrete-time equations to approximate its behavior. Recall the form of the single-lead controller

$$e(t) \longrightarrow \boxed{K_0 \left(\frac{s_p}{s_z}\right)\left(\frac{s - s_z}{s - s_p}\right)} \longrightarrow c(t)$$

and the corresponding differential equation:

$$\dot{c}(t) - s_p c(t) = K_0 \left(\frac{s_p}{s_z}\right)(\dot{e}(t) - s_z e(t))$$

We can use the forward Euler method to develop a discrete-time approximation:

$$\frac{c[n] - c[n-1]}{\Delta T} - s_p c[n-1] = K_0 \left(\frac{s_p}{s_z}\right)\left(\frac{e[n] - e[n-1]}{\Delta T} - s_z e[n-1]\right)$$

By re-arranging the terms, we can find an update equation for $c[n]$ in terms of the previous controller output $c[n-1]$ and the current and previous error inputs $e[n]$ and $e[n-1]$:

$$c[n] = (1 + s_p \Delta T)c[n-1] + K_0 \left(\frac{s_p}{s_z}\right)\left(e[n] - (1 + s_z \Delta T)e[n-1]\right)$$

Develop a similar update equation for the output $c[n]$ of a discrete-time, double-lead controller:

$$e(t) \longrightarrow \boxed{\sqrt{K_0} \left(\frac{s_{p1}}{s_{z1}}\right)\left(\frac{s - s_{z1}}{s - s_{p1}}\right)} \xrightarrow{f(t)} \boxed{\sqrt{K_0} \left(\frac{s_{p2}}{s_{z2}}\right)\left(\frac{s - s_{z2}}{s - s_{p2}}\right)} \longrightarrow c(t)$$

Hint: Assume an intermediate signal $f(t)$ and that your controller will have access not only to the previous value of the controller output ($c[n-1]$) and the current and previous values of the error signal ($e[n]$ and $e[n-1]$) but also the previous value of the intermediate signal ($f[n-1]$).

Enter your equations in the box below.