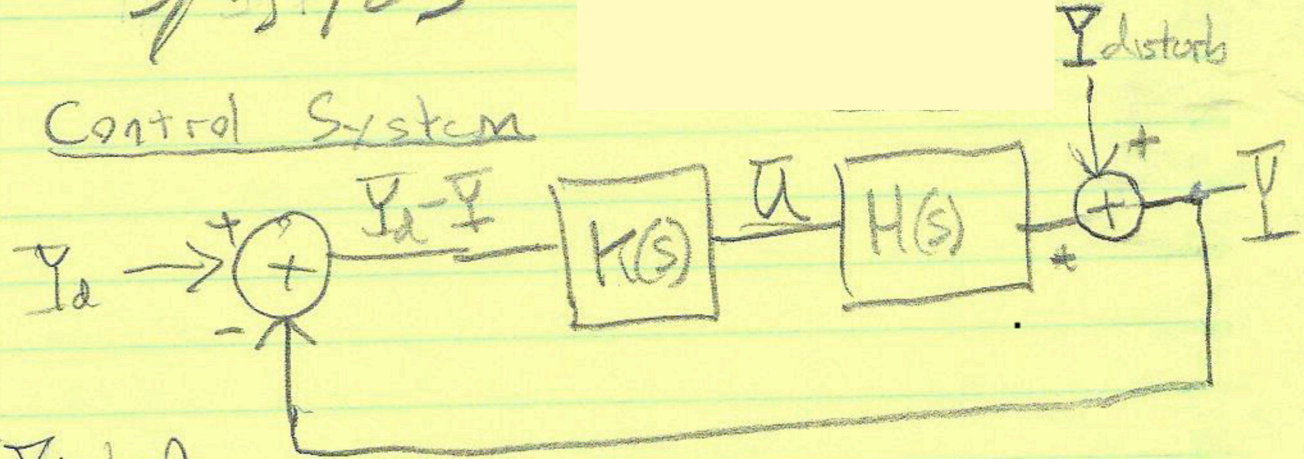
3/19,31/25

Control System



Ydist = 0

$$Y = \frac{K(s)H(s)}{1+K(s)H(s)} Y_d$$

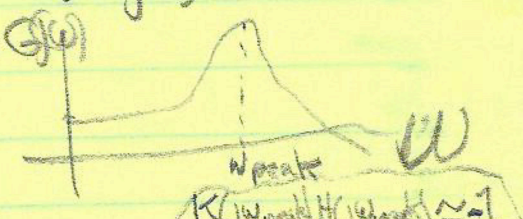
In S.S.S.

$$Y = \underbrace{\frac{K(j\omega)H(j\omega)}{1+K(j\omega)H(j\omega)}}_{G(j\omega)} Y_d$$

Good $K(j\omega)H(j\omega) >> 1 \Rightarrow G(j\omega) = 1 \quad Y = Y_d$

Bad $K(j\omega)H(j\omega) = -1 \Rightarrow G(j\omega) \to \infty$

Not as Bad $K(j\omega)H(j\omega)$ near $-1 \Rightarrow G(j\omega)$



Yd = 0

In S.S.S.

$$Y = \underbrace{\frac{1}{1+K(j\omega)H(j\omega)}}_{G_{dist}(j\omega)} Y_{dist}$$

Good $\to K(j\omega)H(j\omega) >> 1 \Rightarrow G_{dist}(j\omega) \cong 0$

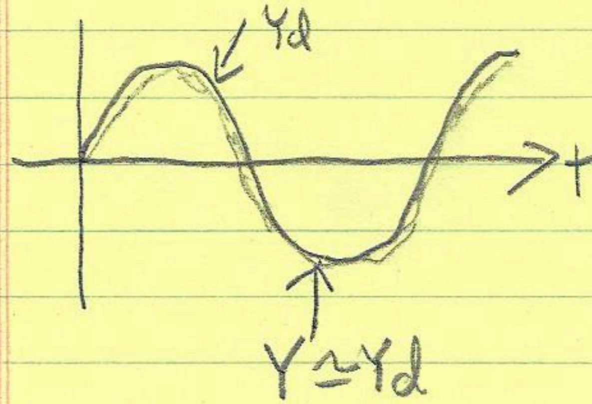
Bad $\to K(j\omega)H(j\omega) = -1 \Rightarrow G_{dist}(j\omega) \to \infty$

Not as bad $K(j\omega)H(j\omega)$ near -1 Gdist is large.

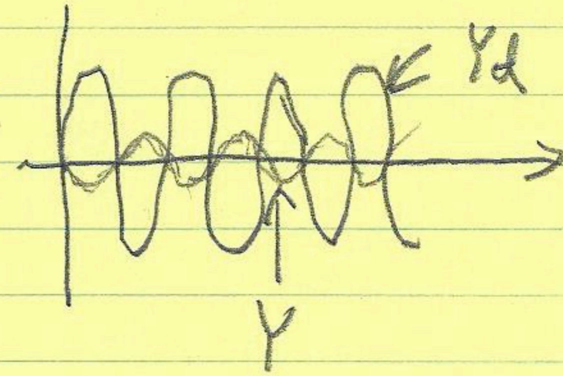
$K(j\omega)H(j\omega) = -1$ iff : $\begin{cases} \angle K(j\omega)H(j\omega) = -180 \text{ or } +180 \\ \text{When } |K(j\omega)H(j\omega)| = 1 \end{cases}$

Goals: (1) Good Tracking at low freqs
(2) Good disturbance rejection at all freqs

Need 1 $K(j\omega)H(j\omega) \gg 1$ for low frequencies



Low Frequency Tracking



At high ω
Y does not track Y_d

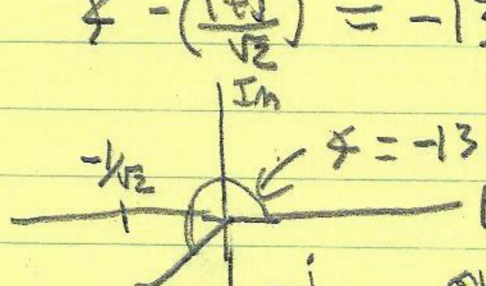
Need 2 When $|K(j\omega)H(j\omega)| = 1$

$\measuredangle K(j\omega)H(j\omega)$ must be far from $\pm 180°$

Example

Suppose $K(j\omega_{unity})H(j\omega_{unity}) = -\left(\frac{1+j}{\sqrt{2}}\right)$

$\left|\frac{1+j}{\sqrt{2}}\right| = 1 \qquad \measuredangle -\left(\frac{1+j}{\sqrt{2}}\right) = -135°$



Okay!

$$|G(j\omega)|\Big|_{\omega=\omega_{unity}} = \left|\frac{K(j\omega)H(j\omega)}{1+K(j\omega)H(j\omega)}\right|\Bigg|_{\omega=\omega_{unity}} = \left|\frac{-(1+j)/\sqrt{2}}{1-(1+j)/\sqrt{2}}\right| \simeq 1.3$$

PID Controller

$$K_p + K_d \frac{d}{dt} + K_i \int$$

$$\Downarrow$$

$$K_p + K_d s + K_i \frac{1}{s}$$

$$\Downarrow$$

$$K_p + K_d j\omega + K_i \frac{1}{j\omega}$$

Which term makes $K(j\omega)H(j\omega) >> 1$ for small ω

make K_p larger

increases
$K(j\omega)H(j\omega)$
for all ω

make K_i larger

$\frac{K_i}{j\omega} \to \infty$ as $\omega \to 0$

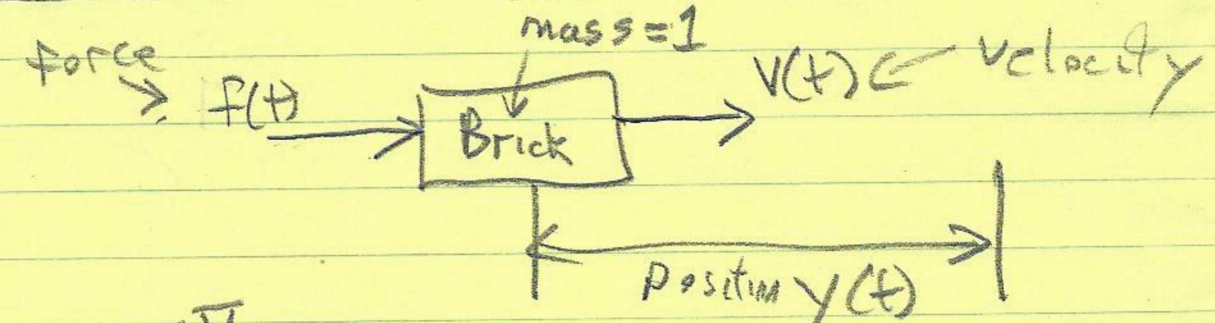
Helps more at lower frequencies

Which term helps $\angle K(j\omega_{unity})H(j\omega_{unity})$ stay away from $-180°$

Note

$$\angle K(j\omega)H(j\omega) = \angle K(j\omega) + \angle H(j\omega)$$

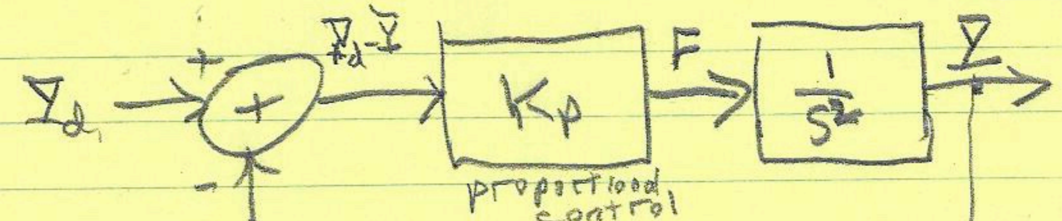
Frictionless Position Control



$$s\mathbf{Y} = \mathbf{V} \qquad\qquad s\mathbf{V} = F$$

$$\frac{d}{dt}\, y(t) = v(t) \qquad \frac{d}{dt}\, V(t) = f(t) \quad \text{mass} = 1$$

$$= \underbrace{K_p\,(y_d(t) - y(t))}_{\text{proportional control}}$$

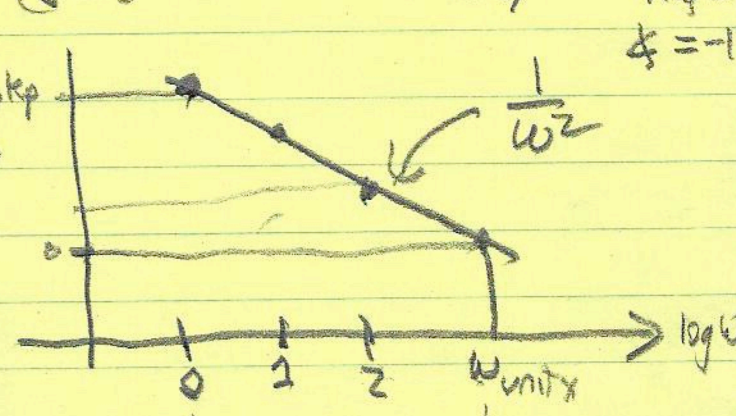


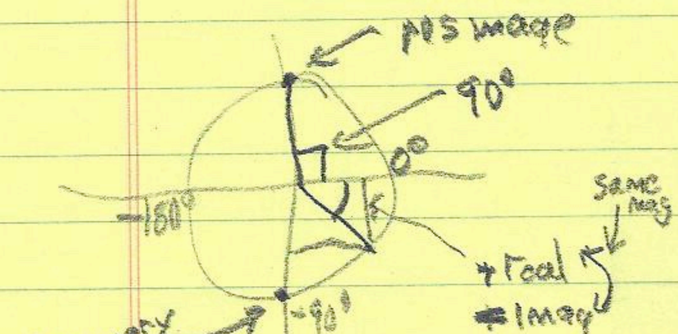
$$\sqrt{K_p} = \omega_{unity}$$

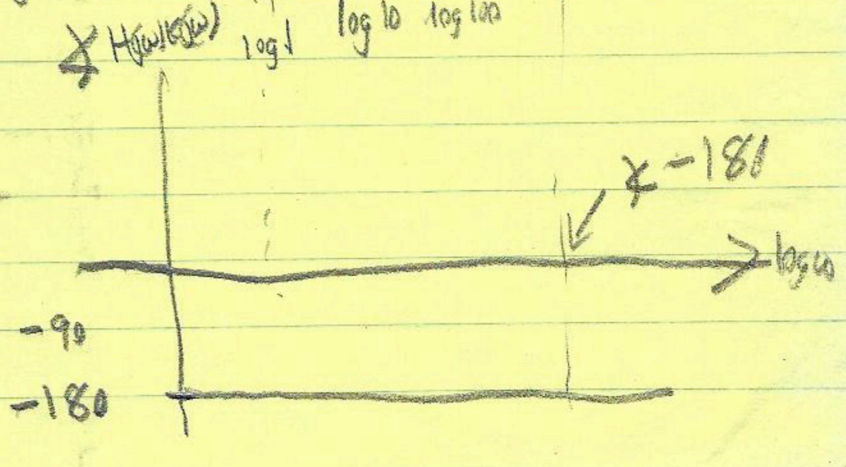
$$K(j\omega)\,H(j\omega) = \frac{K_p}{(j\omega)^2} = -\frac{K_p}{\omega_{unity}^2} = -1$$

Mag = 1
$\measuredangle = -180$

$20 \log | K(j\omega)\, H(j\omega) | \rightarrow$







__If__ H(S) models a physical

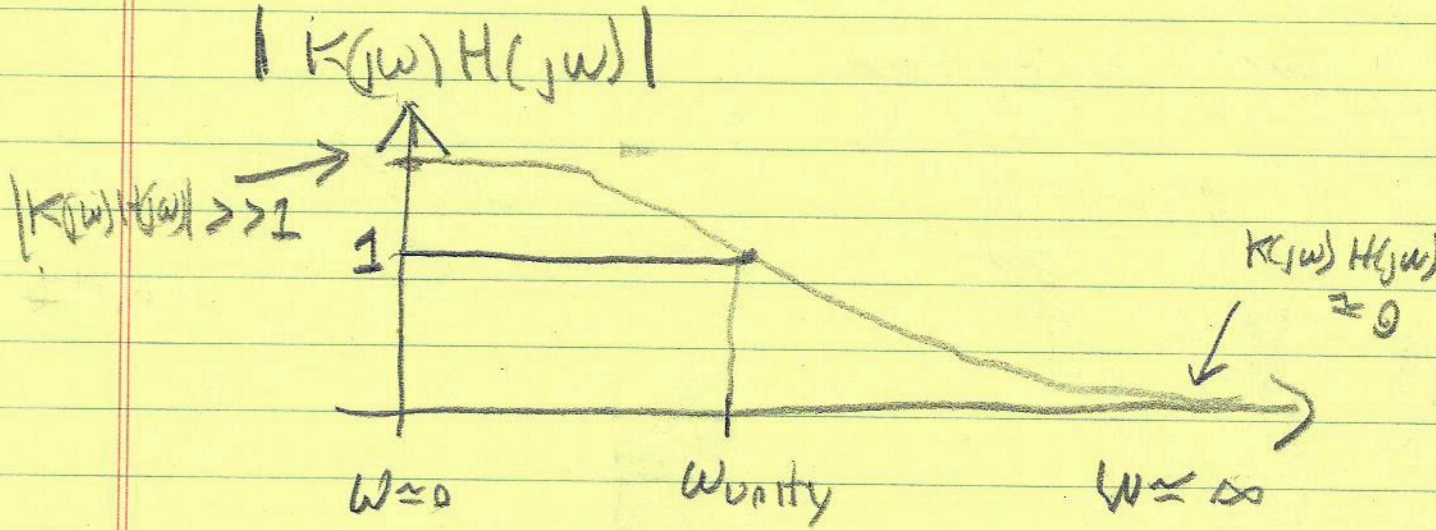
$\lim_{\omega \to \infty} |H(j\omega)| \to 0$ (Physical sy
respond i

__Usually__

$\lim_{\omega \to \infty} |K(j\omega)\, H(j\omega)| \to 0$

K(jω): Might increase with ω $(K_p + K_d j\omega)$

H(jω): goes to zero with ω fast enough



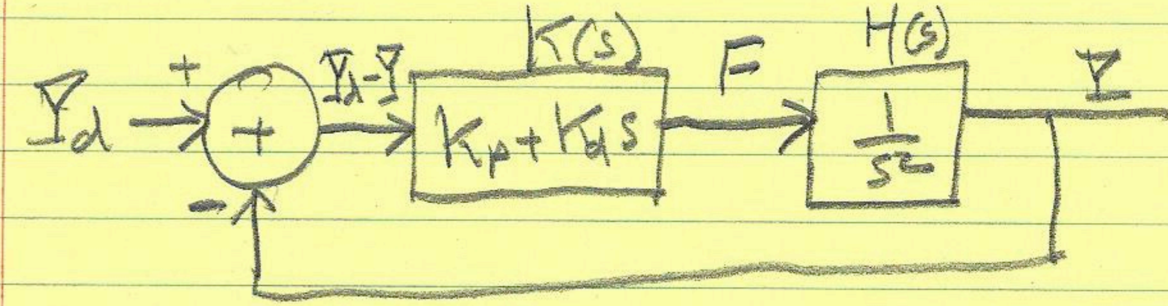
__So__ Pick $K(j\omega)$ so that at $\omega = \omega_{unity}$

a) $|K(j\omega_{unity})\, H(j\omega_{unity})| = 1$

b) $\angle K(j\omega)\, H(j\omega)$ is far from $-180°$

Use a PD controller

6



$$K(j\omega)H(j\omega) = \frac{K_p + K_d j\omega}{(j\omega)^2} = -\frac{K_p + j\omega K_d}{\omega^2}$$

$$= -\frac{K_p}{\omega^2} - j\frac{K_d}{\omega}$$

$$\measuredangle K(j\omega)H(j\omega) \xrightarrow[\omega \to 0]{} \measuredangle -\frac{K_p}{\omega^2} = -180^\circ$$

$$\measuredangle K(j\omega)H(j\omega) \xrightarrow[\omega \to \infty]{} \measuredangle -j\frac{K_d}{\omega} = -90^\circ$$