3/5/25 6.310W/2 1

Wall Follower

$\frac{d}{dt} \Rightarrow$

$$d'(t) = V\,\theta(t)$$
$$\theta'(t) = \gamma\, c(t)$$
$$c(t) = K_p\left(d_d(t) - d(t)\right)$$

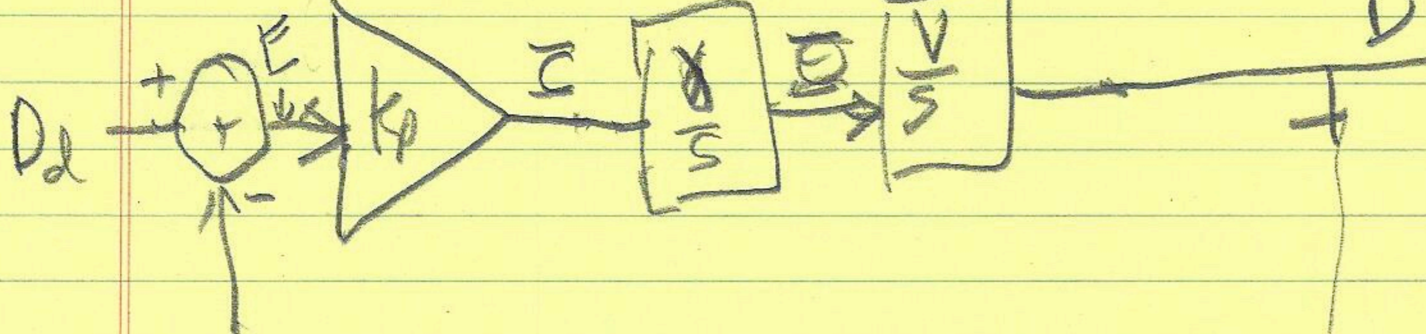
$$d[n] = d[n-1] + \Delta T\, V\, \theta[n-1]$$
$$\theta[n] = \theta[n-1] + \Delta T\, \gamma\, c[n-1]$$
$$c[n] = K_p\left(d_d[n] - d[n]\right)$$

$$\begin{bmatrix} d[n] \\ \theta[n] \end{bmatrix} = \begin{bmatrix} 1 & \Delta T V \\ -\Delta T \gamma K_p & 1 \end{bmatrix} \begin{bmatrix} d[n-1] \\ \theta[n-1] \end{bmatrix} + \begin{bmatrix} 0 \\ \Delta T \gamma K_p \end{bmatrix} d_d[n]$$

$$\lambda_1\ \lambda_2 = 1 \pm j\,\Delta T \sqrt{\gamma K_p V}$$

Assumes

$$d(t) = \underline{D}\, e^{st}$$
$$\theta(t) = \underline{\Theta}\, e^{st}$$
$$c(t) = \underline{C}\, e^{st}$$
$$s\,\underline{D}\,\cancel{e^{st}} = V\,\underline{\Theta}\,\cancel{e^{st}}$$
$$s\,\underline{\Theta}\,\cancel{e^{st}} = \gamma\,\underline{C}\,\cancel{e^{st}}$$
$$\underline{C}\,\cancel{e^{st}} = K_p\left(\underline{D}_d - \underline{D}\right)\cancel{e^{st}}$$



2R

Wall Follower

D.T.

$$d[n] = d[n-1] + \Delta T\, V\theta[n-1]$$

$$\theta[n] = \theta[n-1] + \Delta T\, \gamma\, c[n-1]$$

control

$$c[n] = K_p(-d[n])$$

$$\downarrow$$

$$\begin{bmatrix} d[n] \\ \theta[n] \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & \Delta T V \\ -\Delta T \gamma K_p & 1 \end{bmatrix}}_{eig(A)} \begin{bmatrix} d[n-1] \\ \theta[n-1] \end{bmatrix}$$

$$\lambda_1, \lambda_2 \; 1 \pm j\,\Delta T\sqrt{\gamma K_p V}$$

' means d/dt

$$d'(t) = V\theta(t)$$

$$\theta'(t) = \gamma c(t)$$

$$c(t) = K_p(-d(t))$$

Scalar

<u>guess</u> $d(t) = D e^{st}$ $\quad c(t) = C e^{st}$

<u>guess</u> $\theta(t) = \Theta e^{st}$

<u>guess</u> $c(t) = C e^{st}$

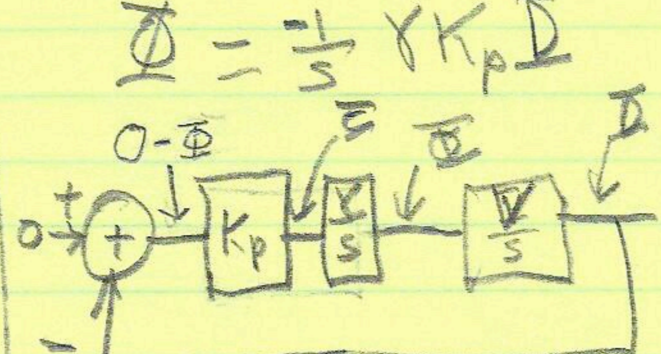
$$s D \cancel{e^{st}} = V \Theta \cancel{e^{st}}$$

$$s \Theta \cancel{e^{st}} = \gamma C \cancel{e^{st}}$$

$$C \cancel{e^{st}} = -K_p D \cancel{e^{st}}$$

$$D = \frac{V}{s}\Theta$$

$$\Theta = \frac{-1}{s}\gamma K_p D$$



$$D = \frac{V}{s}\Theta = \frac{V}{s}\frac{\gamma}{s} C$$

$$= -\frac{V}{s}\frac{\gamma}{s} K_p D$$

$$\left(1 + \frac{V\gamma K_p}{s^2}\right) D = 0$$

$$s^2 + V\gamma K_p = 0$$

$$s_{1,2} = \pm\sqrt{-V\gamma K_p}$$

3 R

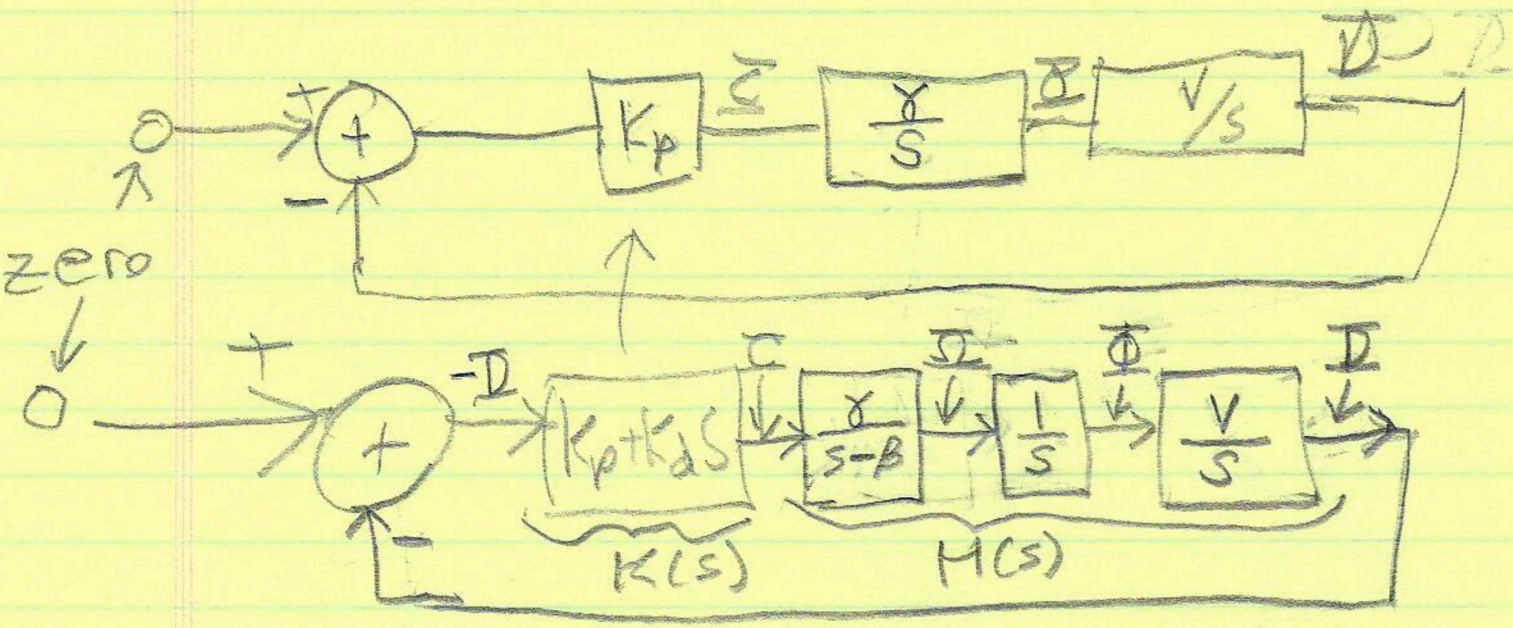
More Complete Model PD Control

$d'(t) = V\,\theta(t) \Rightarrow s\,D = V\,\Theta$

$\theta'(t) = \omega(t)$ $\qquad s\,\Theta = \Omega +$

$\omega'(t) = \gamma\,c(t) + \beta\,\omega(t)$ $\qquad s\,\Omega = \gamma\,C + \beta\,\Omega$

$c(t) = K_p(-d(t)) + K_d(-d'(t))$



$$K(s)H(s) = \frac{(K_p + K s)\gamma}{(s-\beta)s^2}$$