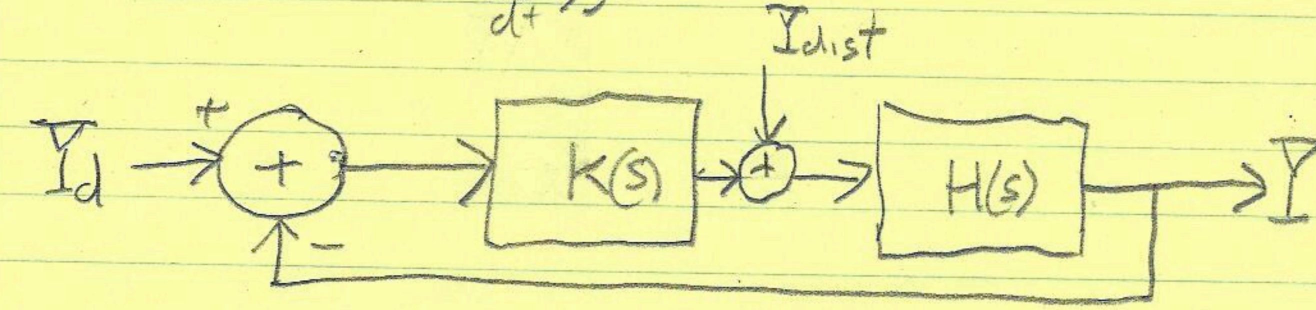
6.310 3/17/25

<u>Summary</u>

Differential Eqns $\xrightarrow{\frac{d}{dt} \to s}$ Transfer fncs $\rightarrow$ Block Diagrams



$$I = G(s) I_d + G_{dist}(s) I_{dist}$$

$$G(s) = \frac{K(s)H(s)}{1+K(s)H(s)} \quad G_{dist}(s) = \frac{H(s)}{1+K(s)H(s)}$$

<u>Simplify</u> (Assume $Y_{dist} = 0$)

$$I = G(s) I_d = \frac{b_{\hat{L}} s^{\hat{L}} + b_{\hat{L}-1} s^{\hat{L}-1} + \ldots + b_0}{a_L s^L + a_{L-1} s^{L-1} + \ldots + a_0} I_d$$

<u>Cross - Multiply</u>

$$(a_L s^L + a_{L-1} s^{L-1} + \ldots + a_0) I = (b_{\hat{L}} s^{\hat{L}} + b_{\hat{L}-1} s^{\hat{L}-1} + \ldots + b_0) I_d$$

<u>Diff Eqn</u>

$$a_L \frac{d^L}{dt^L} y(t) + \ldots + a_0 y(t) = b_{\hat{L}} \frac{d^{\hat{L}}}{dt^{\hat{L}}} y_d(t) + \ldots + b_0 y_d(t)$$

if $y_d(t) = 0 \Rightarrow y(t) = \sum_{i=1}^{L} \alpha_i e^{\lambda_i t}$ for α_i's to match initial conds

Nat. Freqs. $\Rightarrow \lambda_i$ roots of $a_L \lambda^L + a_{L-1} \lambda^{L-1} + \ldots + a_0 = 0$

if $y_d(t) = e^{j\omega t}$ $\quad y(t) = \sum_{i=1}^{L} \hat{\alpha}_i e^{\lambda_i t} + G(j\omega) e^{j\omega t}$

Summary's Summary

<u>if</u> $y_d(t) = e^{j\omega t}$

$$y(t) = \sum_{i=1} \alpha_i e^{\lambda_i t} + G(j\omega) e^{j\omega t}$$

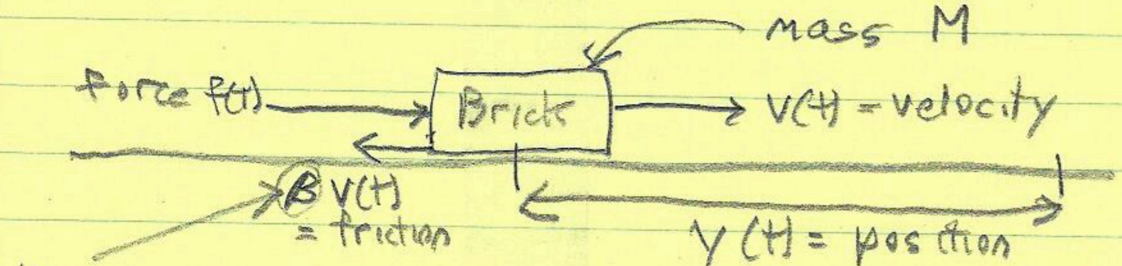
Sinusoidal steady state

if $\underset{\text{for all } i}{Re(\lambda_i) < 0}$ <u>then</u> $\sum_{i=1}^{L} \alpha_i e^{\lambda_i t} \underset{\text{as } t \to \infty}{\to} 0$

<u>How do we design a controller so</u>

$Re(\lambda_i) << 0$ (fast decay)?

Sliding Brick Example



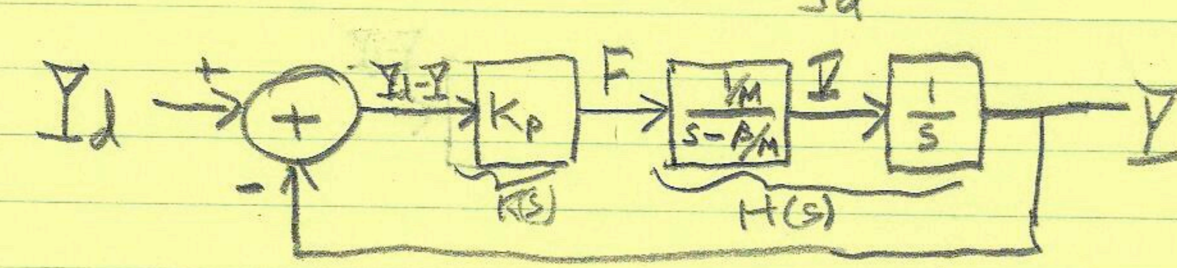
$\beta < 0$

Diff. Eqns

$$\underbrace{\frac{d}{dt} V(t)}_{s\,V} = \frac{1}{M}\Big(\underbrace{f(t)}_{F} + \underbrace{\beta V(t)}_{\beta V}\Big)$$

$$\underbrace{\frac{d}{dt} Y(t)}_{s\,Y} = \underbrace{V(t)}_{V}$$

$$\underbrace{f(t)}_{F} = K_p\big(\underbrace{Y_d(t)}_{Y_d} - \underbrace{Y(t)}_{Y}\big)$$



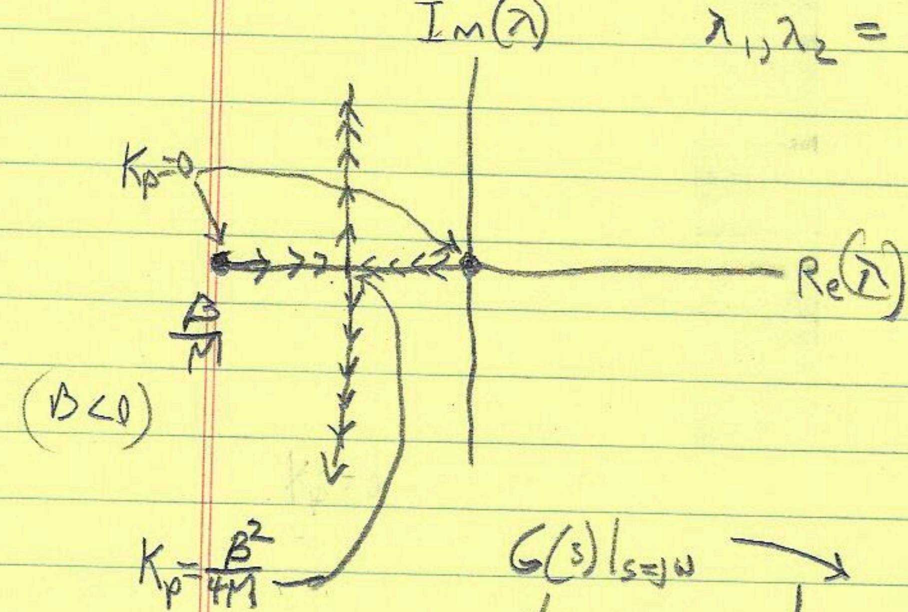
$$H(s) = \frac{1/M}{(s - \beta/M)\, s}$$

$$Y = G(s) Y_d = \frac{K(s)H(s)}{1 + K(s)H(s)} Y_d = \frac{\frac{K_p/M}{(s-\beta/M)s}}{1 + \frac{K_p/M}{(s-\beta/M)s}} Y_d$$

$$Y = \frac{K_p/M}{s^2 - \frac{\beta}{M}s + K_p/M} Y_d \qquad \lambda_1, \lambda_2 = \frac{\beta}{2M} \pm \sqrt{\frac{\beta^2}{4M^2} - \frac{K_p}{M}}$$

Nat freqs

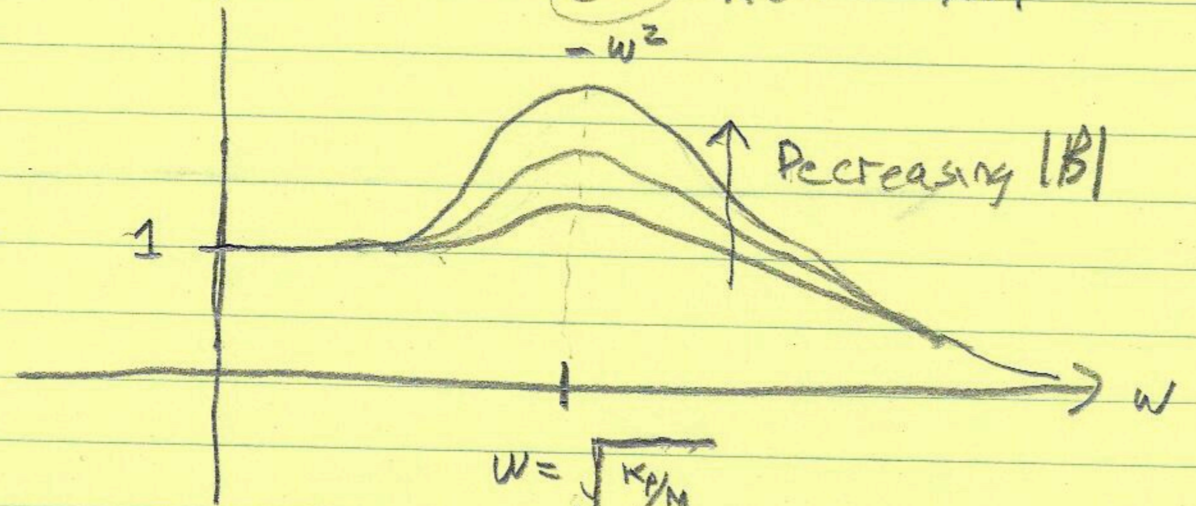
$$\lambda_1, \lambda_2 = \frac{B}{2M} \pm \sqrt{\frac{B^2}{4M^2} - \frac{K_p}{M}}$$



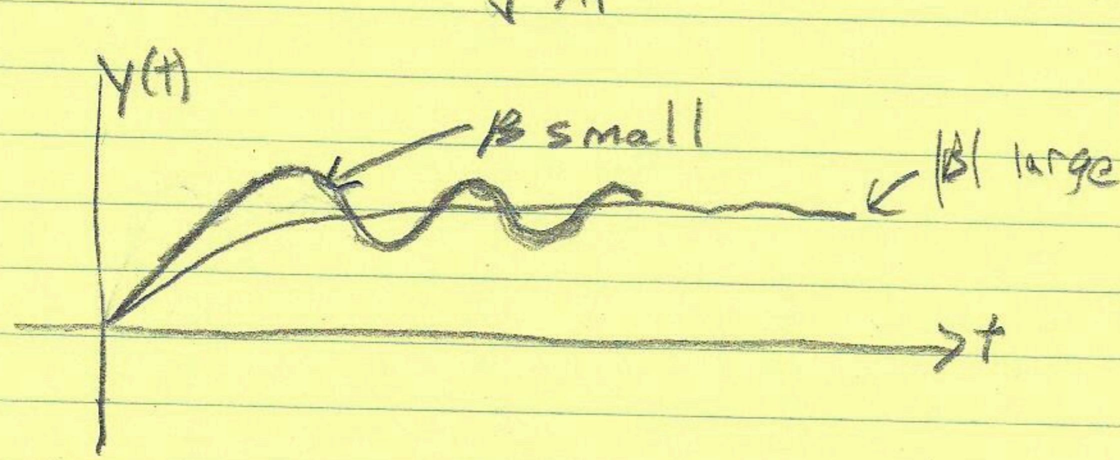
$G(s)|_{s=j\omega}$

$$|G(j\omega)| = \left| \frac{K_p/M}{\underbrace{(j\omega)^2}_{-\omega^2} - \frac{B}{M}(j\omega) + K_p/M} \right|$$

Freq Response



Step Response



Alternative View

$$G(s) = \frac{K(s)H(s)}{1+K(s)H(s)}\Bigg|_{s=j\omega}$$

$$\approx \frac{K(j\omega)H(j\omega)}{1+K(j\omega)H(j\omega)}$$

if $K(j\omega)H(j\omega) >> 1 \Rightarrow G(j\omega) \doteq 1$

Good

$$y(t) = \underbrace{G(j\omega)}_{\approx 1} \underbrace{e^{j\omega t}}_{y_d(t)} = \underbrace{e^{j\omega t}}_{y(t)}$$

$\Rightarrow y(t) \simeq y_d(t)$ in sinusoidal steady state

Good

if $K(j\omega)H(j\omega) \simeq -1 \Rightarrow |G(j\omega)| \to \infty$

Very Bad

$$y(t) = G(j\omega)e^{j\omega t} >>> e^{j\omega t}$$

$$y(t) \neq y_d(t)$$

huge

Pick $K(s)$ so that $K(j\omega)H(j\omega)$ far from -1

$K(j\omega)H(j\omega) = -1$ iff

a) $|K(j\omega)H(j\omega)| = 1$

b) $\measuredangle K(j\omega)H(j\omega) = -180° = -\pi$ radians

If $H(s)$ models a physical system

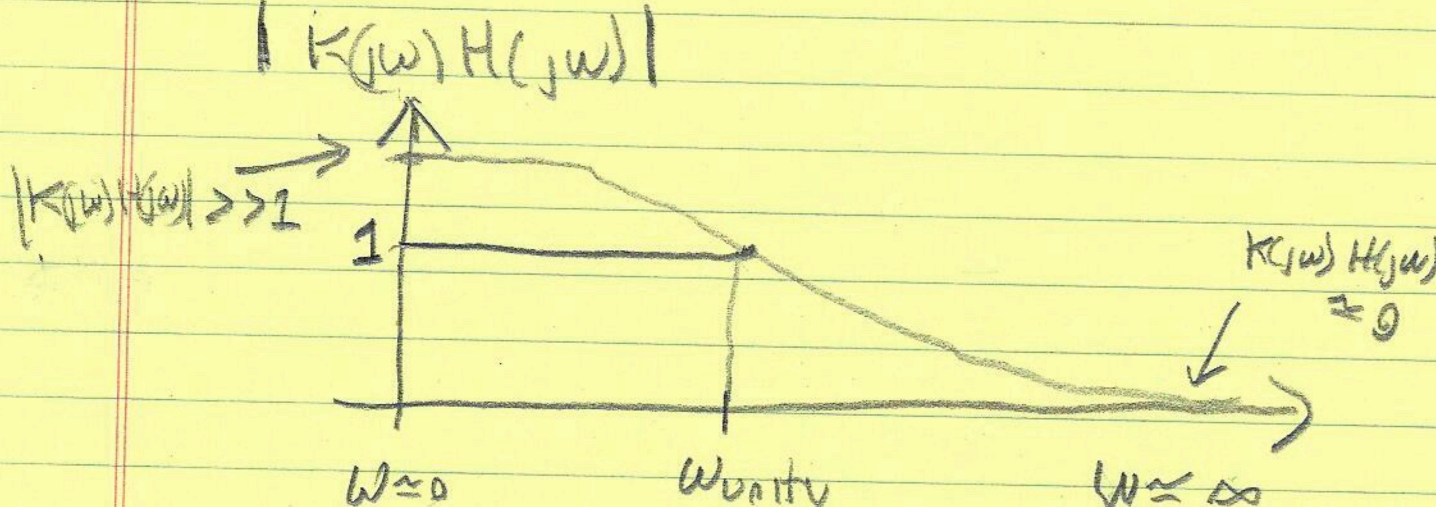
$$\lim_{\omega \to \infty} |H(j\omega)| \to 0$$ (Physical systems don't respond instantly fast)

Usually

$$\lim_{\omega \to \infty} |K(j\omega) \; H(j\omega)| \to 0$$

$K(j\omega)$: Might increase with ω $(K_p + K_d j\omega)$

$H(j\omega)$: goes to zero with ω fast enough



So Pick $K(j\omega)$ so that at $\omega = \omega_{unity}$

a) $|K(j\omega_{unity}) \; H(j\omega_{unity})| = 1$

b) $\measuredangle K(j\omega) \; H(j\omega)$ is far from -180°