3/3/25 6.3100/2 (1)

So Far

N states

$$\underbrace{\begin{bmatrix} x[n] \end{bmatrix}}_{N} = \overbrace{\begin{bmatrix} & & \\ & A & \\ & & \end{bmatrix}}^{N} \begin{bmatrix} x[n-1] \end{bmatrix} + \begin{bmatrix} B \end{bmatrix} u[n-1] + \begin{bmatrix} B_{dist} \end{bmatrix} u_{dist}[n-1]$$

eig(A) ≡ nat freq

|eig(A)| < 1 for stability

1) A's and B's "easy" to construct
Model easy to simulate

2) We can learn things from the structure

$$(I-A)^{-1} B u[\infty] = x[\infty] \quad \text{zero disturbance} \qquad u[n] = u[\infty]$$

$$(I-A)^{-1} B_d u_d[\infty] = \underbrace{x_d[\infty] - x[\infty]}_{\text{error due to disturbance}} \qquad u_d[n] = u_d[\infty]$$

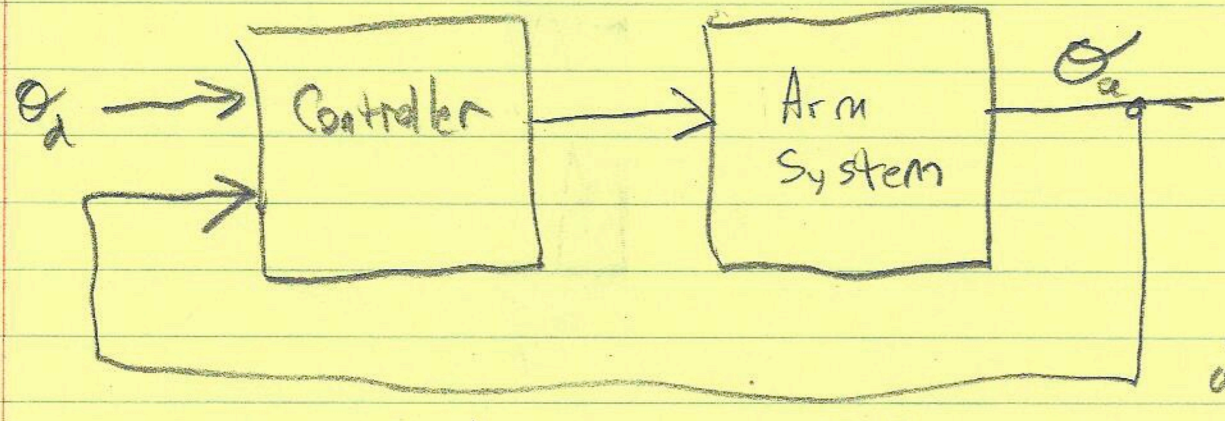
3) Examine λ vs K_p, K_d, K_i (using graphs)

__But__ P, I & D control ideas were given how do we come up with new ideas.

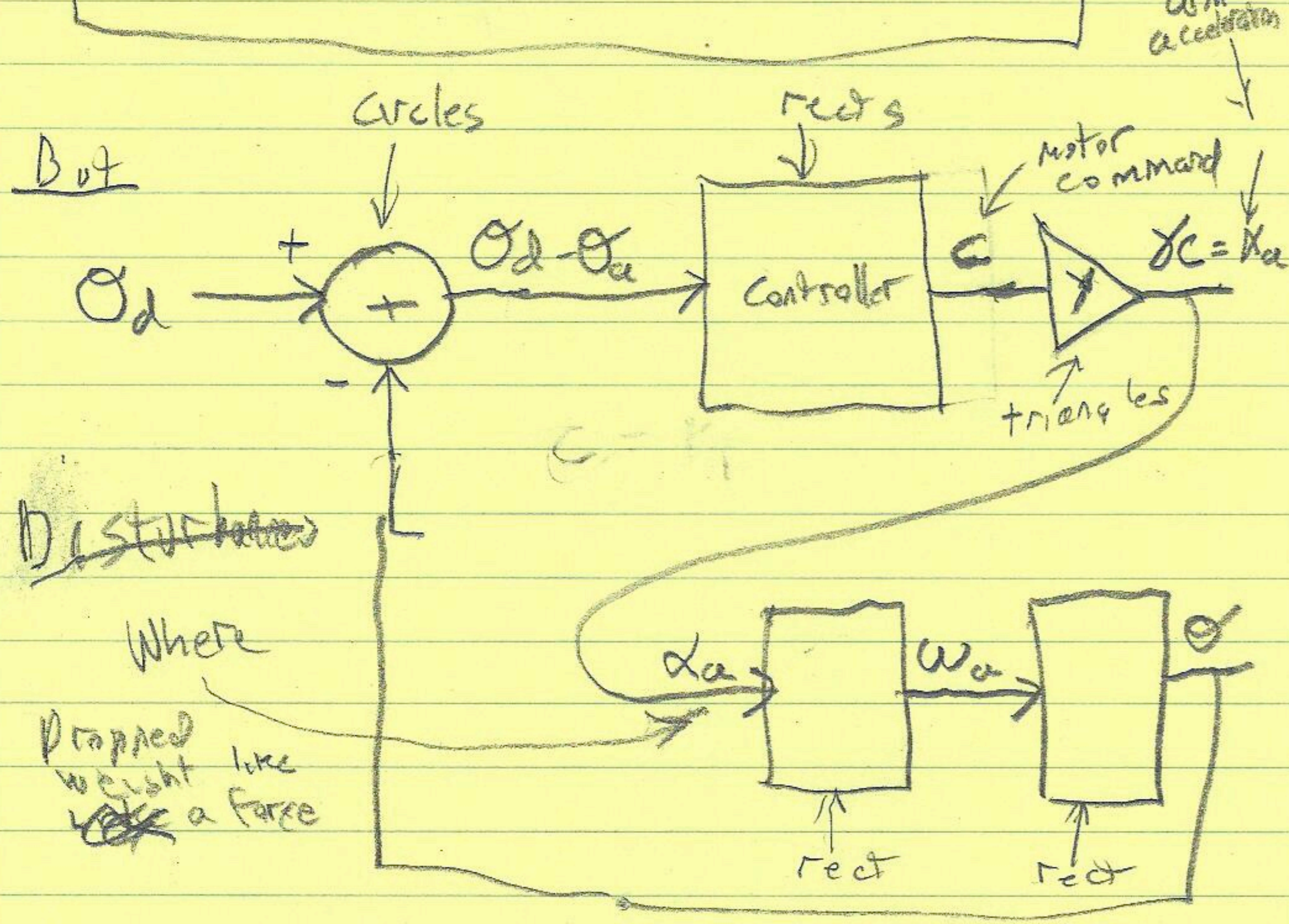
__And__ How do we examine "not s.s." dynamic disturbances,

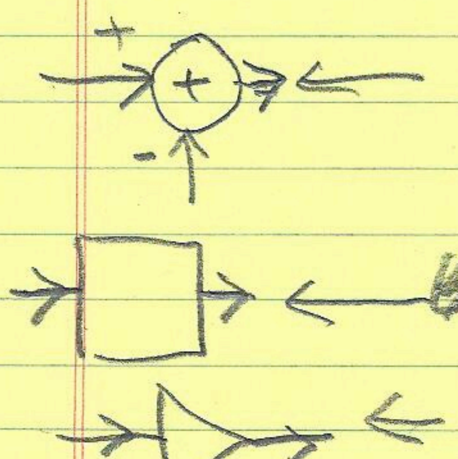
Propeller Arm - Block Diagram

2



But





Summers

I/O Box
or
Tri
(for multiplication? "Gains")

What goes in here?
cont / compose { difference equations!

D.T.

$$\cancel{y}[n] = \lambda y[n-1] + \gamma u[n-1]$$

General Solution

$$y[n] = \lambda^n y[0] + \sum_{m=0}^{n-1} \lambda^{n-1-m} \gamma u[m]$$

(Proof by induction)

Implies Linearity

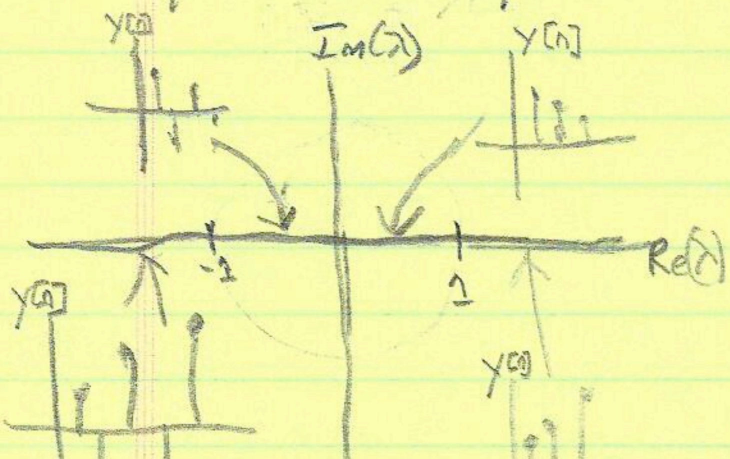
ZIR Case

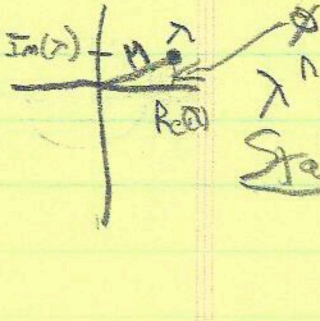
$$y[n] = \lambda y[n-1]$$

$$y[n] = Y \lambda^{n-1}$$

Scalar

$$y[n] = Y \lambda^n = \lambda y[n-1]$$





$$\lambda^n = (M_\lambda e^{j\theta_\lambda})^n = M_\lambda^n e^{jn\theta_\lambda}$$

Stability

$$|\lambda| < 1$$

C.T

$$\frac{d}{dt} y(t) = \lambda y(t) + \gamma u(t)$$

General Solution

$$y(t) = e^{\lambda t} y(0) + \int_0^t e^{\lambda(t-\tau)} u(\tau) d\tau$$

(Proof by differentiating and using integration by parts)

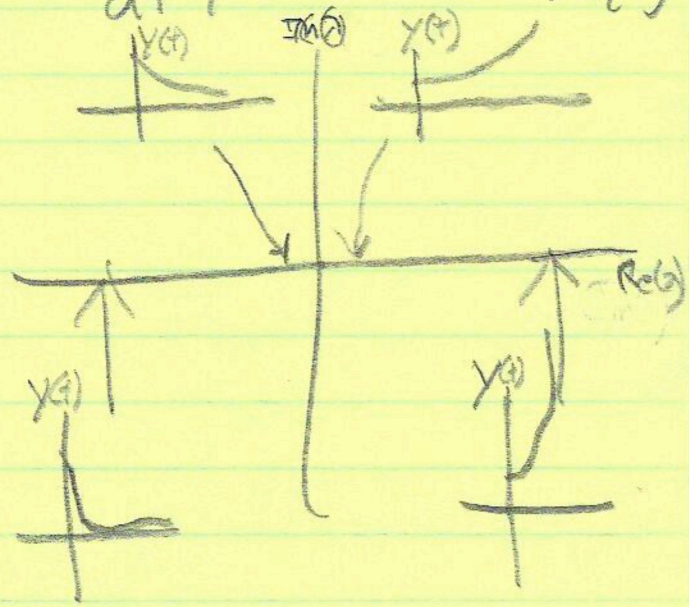
Implies Linearity

ZIR Case

$$\frac{d}{dt} y(t) = \lambda y(t)$$

$$y(t) = Y e^{\lambda t}$$

$$\frac{d}{dt} y(t) = \lambda Y e^{\lambda t} = \lambda \cancel{Y} y(t)$$



Stability

$$\mathrm{Re}(\lambda) < 0$$

$$\lambda = a + jb$$

$$e^{\lambda t} = e^{(a+jb)t} = e^{at}(\cos bt + j \sin bt)$$

Speed Control DT

perturbation in speed

$$\omega[n] = \omega[n-1] + -\Delta T\left[\beta\,\omega[n-1] - \gamma\, c[n-1]\right]$$

$$c[n] = K_p\left(\underbrace{\omega_d[n]}_{=0} - \omega[n]\right)$$

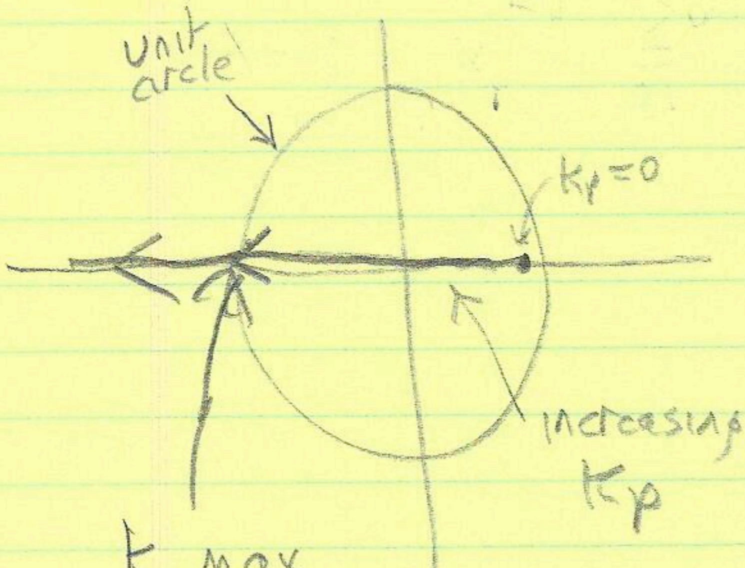
or

$$\frac{\omega[n] - \omega[n-1]}{\Delta T} = -\left[\beta\,\omega[n-1] + \gamma K_p\,\omega[n-1]\right]$$

$$\omega[n] = \left(1 - \Delta T\left(\beta + \gamma K_p\right)\right)\omega[n-1]$$

$$\omega[n] = \lambda^n\,\omega[0]$$

$$\lambda = 1 - \Delta T\left(\beta + \gamma K_p\right)$$



K_p max

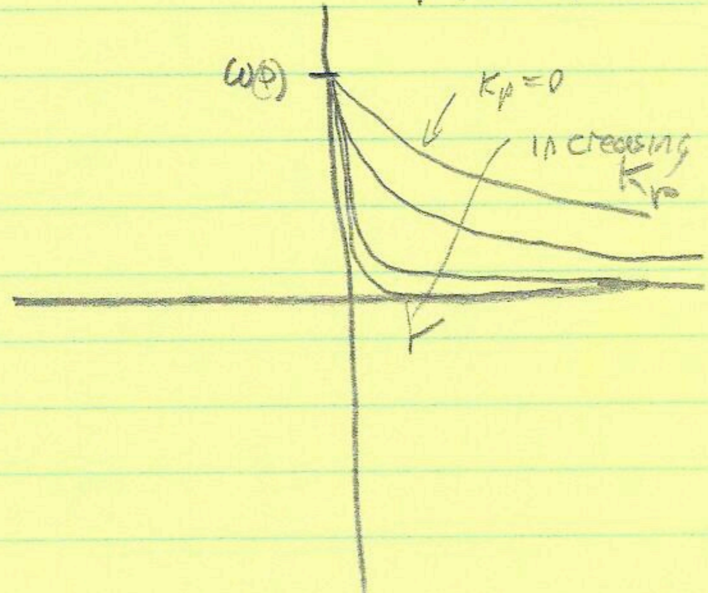
$$-\Delta T\left(\beta + \gamma K_p\right) = -2$$

$$K_{p_{max}} = \left(\frac{2}{\Delta T} - \beta\right)\Big/\gamma$$

$$\frac{d}{dt}\omega(t) = \left(-\beta - \gamma K_p\right)\omega(t)$$

$$\omega(t) = e^{(-\beta - \gamma K_p)t}\,\omega(0)$$

pos pos pos



Why?

$$\frac{d}{dt}\omega(t) \text{ is } \lim_{\Delta t \to 0} = \frac{\omega[t+\Delta t] - \omega[t]}{\Delta t}$$