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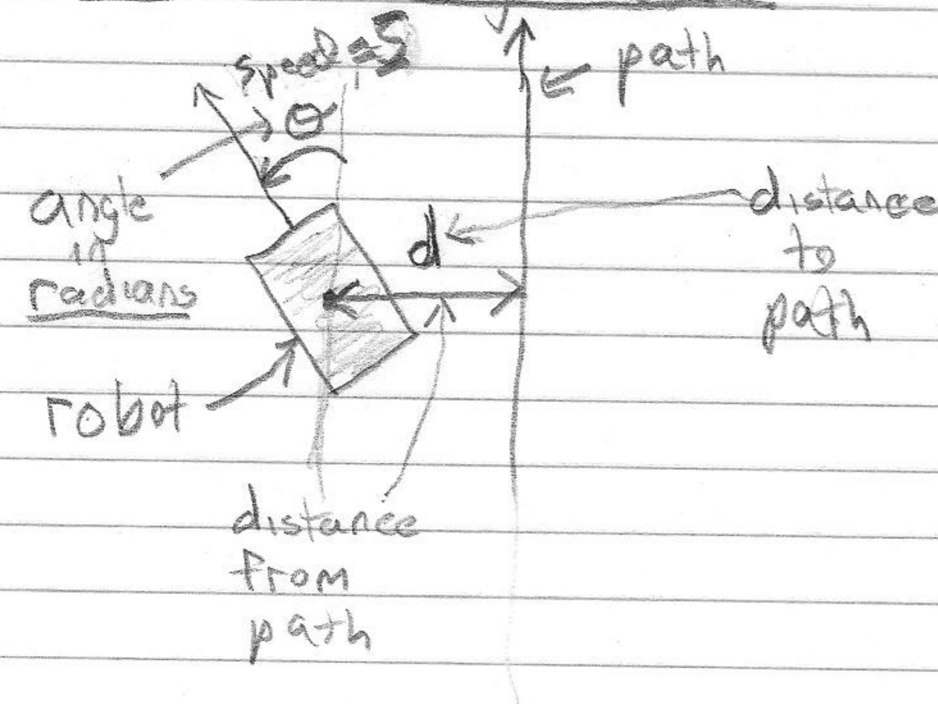
Path Following Robot



S = Robot Forward Speed

θ = robot angle

d = distance to path

ΔT = time btwn samples

ω = robot rotation speed (control!)

Equations

$\simeq \theta$ if θ is small and in radians!

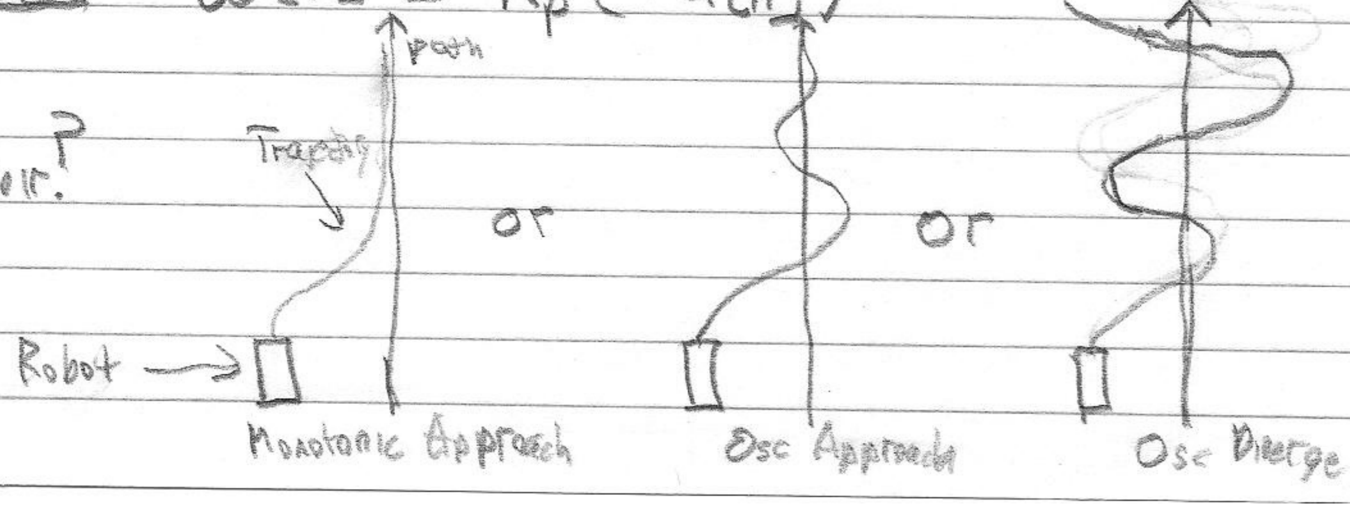
$$d[n] = d[n-1] + \Delta T \, \underline{S} \, \sin\theta$$

$$\theta[n] = \theta[n-1] + \Delta T \, \omega[n-1]$$

we get to control

Spose $\omega[n] = K_p(-d[n])$

Robot Behavior?



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Matrix Form

$$\underbrace{\begin{bmatrix} d[n] \\ \theta[n] \end{bmatrix}}_{x[n]} = \underbrace{\begin{bmatrix} 1 & S\Delta T \\ 0 & 1 \end{bmatrix}}_{A} \begin{bmatrix} d[n-1] \\ \theta[n-1] \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \Delta T \end{bmatrix}}_{B} u[n-1]$$

$$\vec{x}[n] = A\, x[n-1] + B\, u[n-1]$$

<u>Scalar</u> $y[n] = \lambda y[n-1] + \gamma u[n-1] \Rightarrow y[n] = \lambda^n y[0] + \sum_{m=0}^{n-1} \lambda^{n-1-m} \gamma u[m]$

scalars commute

<u>Vector</u> $x[n] = A^n x[0] + \sum_{m=0}^{n-1} A^{n-1-m} B u[m]$

matrices don't commute

<u>Now include</u> $\omega[n] = K_p(-d[n])$

$$\begin{bmatrix} d[n] \\ \theta[n] \end{bmatrix} = \begin{bmatrix} 1 & S\Delta T \\ -\Delta T K_p & 1 \end{bmatrix} \begin{bmatrix} d[n-1] \\ \theta[n-1] \end{bmatrix}$$

indep of n!

$$\begin{bmatrix} d[n] \\ \theta[n] \end{bmatrix} = \begin{bmatrix} V \end{bmatrix} \begin{bmatrix} \lambda_1^n & 0 \\ 0 & \lambda_2^n \end{bmatrix} \begin{bmatrix} V \end{bmatrix}^{-1} \begin{bmatrix} d[0] \\ \theta[0] \end{bmatrix}$$

Linear Algebra

Assum A is Diagonalizable

$A \in \mathbb{R}^{N\times N}$ is diagonalizable iff $\exists$

$\{\vec{v}_1, \ldots \vec{v}_N\}$ lin indep vectors s.t. λ

$$A\vec{v}_i = \lambda_i \vec{v}_i \qquad \text{or} \qquad A\,V = V\overbrace{\begin{bmatrix}\lambda_1 & & \\ & \ddots & \\ & & \lambda_N\end{bmatrix}}^{\lambda}$$

eigen vector — eigenvalue

N×N matrix of evectors — and V^{-1} exists

Suff Conds A is symmetric ($A = A^T$)
A has n unique evals } Not Nec

If A is Diagonalizable

$$A^k = A\cdot A\cdot A \cdots A = (V\lambda V^{-1})(V\lambda V^{-1})\cdots(V\lambda V^{-1})$$

$$= V\lambda^k V^{-1} = V\begin{bmatrix}\lambda_1^k & & \\ & \ddots & \\ & & \lambda_N^k\end{bmatrix}V^{-1}$$

$$d[n] = C_1 \lambda_1^n + C_2 \lambda_2^n$$ C_1, C_2 depend on $d[0], \theta[0]$

<u>Why</u>

$$\begin{bmatrix} d[n] \\ \theta[n] \end{bmatrix} = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix} \overbrace{\begin{bmatrix} \lambda_1 & \\ & \lambda_2 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}}^{\begin{bmatrix} P_1\lambda_1 & 0 \\ 0 & P_2\lambda_2 \end{bmatrix}} = \begin{bmatrix} V_{11}P_1\lambda_1^n + V_{12}P_2\lambda_2^n \\ V_{21}P_1\lambda_1^n + V_{22}P_2\lambda_2^n \end{bmatrix}$$

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} V \end{bmatrix}^{-1} \begin{bmatrix} d[0] \\ \theta[0] \end{bmatrix}$$

<u>Find C_1 & C_2 ?</u> (Calc Evec, Evec^{-1}, multiply or)

Know $d[0] = C_1 \lambda_1^0 + C_2 \lambda_2^0 = C_1 + C_2$

<u>Calc</u> $d[1] = d[0] + \Delta T\, \theta[0] = C_1\lambda_1 + C_2\lambda_2$

<u>Two equations in Two unknowns</u>

$\boxed{\text{Solve for } C_1, C_2}$

Find Lambdas

~~Determinent Formula~~

$$\det\left(\lambda I - A\right) = 0 \Rightarrow \lambda \text{ is an eval}$$

2x2 case

$$\det\left(\begin{bmatrix} \lambda-1 & S\Delta T \\ -\Delta T K_p & \lambda-1 \end{bmatrix}\right) = 0$$

$$(\lambda-1)^2 + (\Delta T)^2 S K_p = \lambda^2 - 2\lambda + (1+\Delta T^2 S K_p) = 0$$

Speed > 0

$$\lambda_1, \lambda_2 = 1 \pm \sqrt{1 - (1+\Delta T^2 S K_p)}$$

$K_p > 0$

$$= 1 \pm \sqrt{-(\Delta T)^2 S K_p}$$

$$= 1 \pm j\, \Delta T \sqrt{S K_p}$$

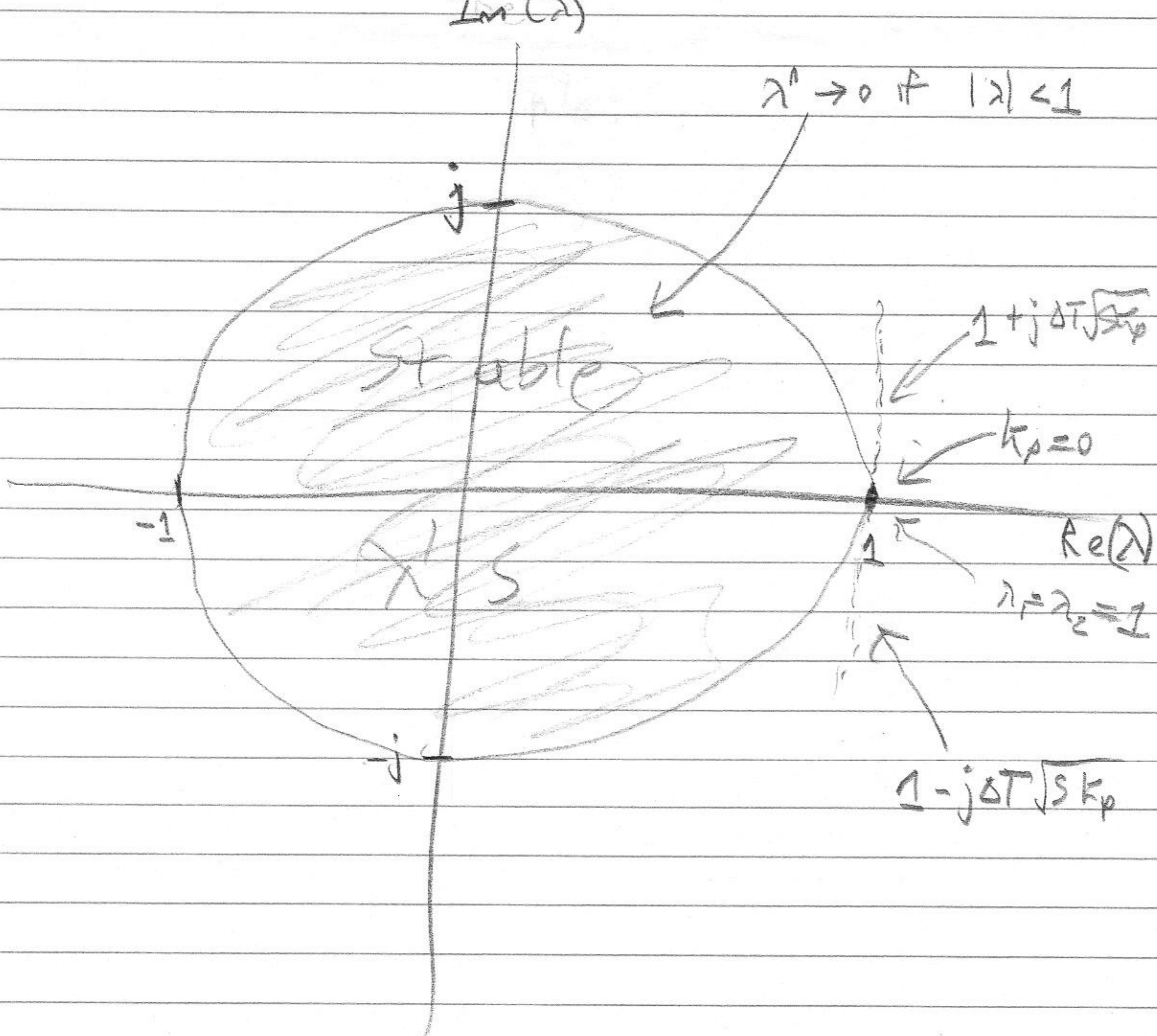
$\sqrt{-1}$

Wait

$$d[n] = C_1 \lambda_1^n + C_2 \lambda_2^n$$

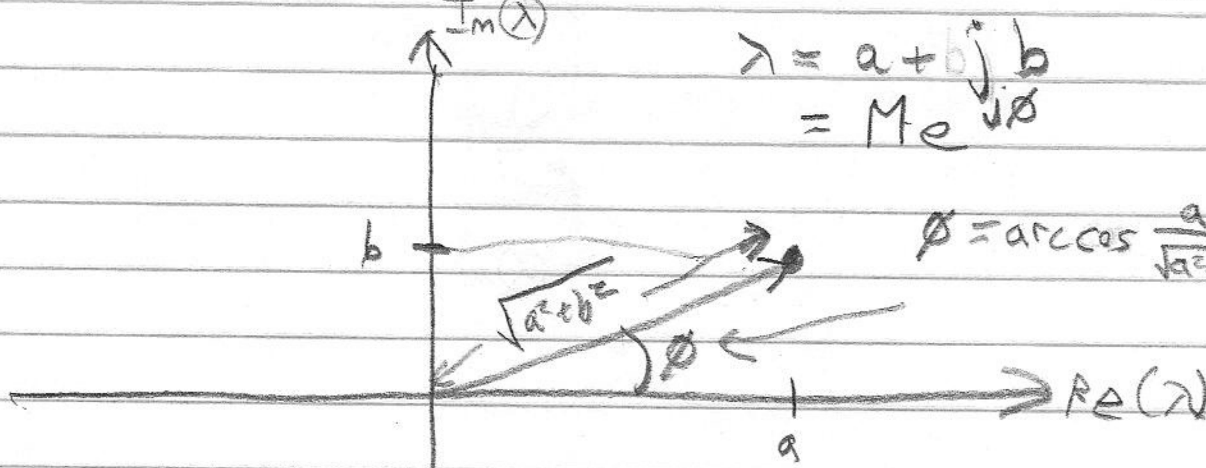
$$= C_1 \left(1 + j\,\Delta T \sqrt{S K_p}\right)^n + C_2 \left(1 - j\,\Delta T \sqrt{S K_p}\right)^n$$

The Complex Plane



<u>So</u> Robot does <u>Not</u> settle on path!

Polar Form (Brrr) (6)



$$\lambda = a + jb$$
$$= Me^{j\phi}$$

$$\phi = \arccos \frac{a}{\sqrt{a^2+b^2}}$$

$$Me^{j\phi} \equiv M(\cos\phi + j\sin\phi)$$
$$Me^{-j\phi} = M(\cos\phi - j\sin\phi)$$

$$(Me^{j\phi})^n = M^n e^{jn\phi}$$
$$= M(\cos n\phi + j\sin n\phi)$$

<u>One Period of Osallation</u>

$$n_{period}\,\phi = 2\pi \qquad n_p = \frac{2\pi}{\phi}$$

<u>if</u> $d[n] = C_1 M^n e^{jn\phi} + C_2 M^n e^{-jn\phi}$

$$C_1 = M_c e^{j\phi_c} \Leftrightarrow C_2 = M_c e^{-j\phi_c}$$

$$d[n] = M_c M^n e^{j(n\phi+\phi_c)} + M_c M^n e^{j(n\phi-\phi_c)}$$

$$= M_c M^n \cos(n\phi + \phi_c)$$