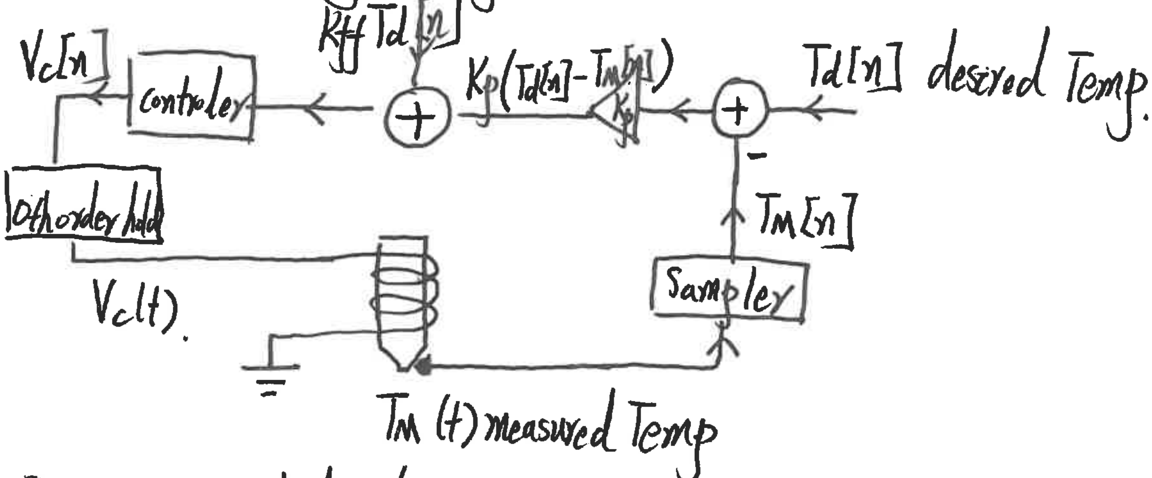
02/12/25.

* Determine system parameters.



Recap: Include loss in 1st order system:

From physics: $\dfrac{T_M[n]-T_M[n-1]}{\Delta T} \sim$ heat input + heat loss

ΔT: sampling interval, e.g. 66.6 usec.   heat input: $\gamma u[n-1]$ command   heat loss: $-\beta T_M[n-1]$ higher T, faster dissip.

$$T_M[n] = T_M[n-1] - \beta \Delta T \, T_M[n-1] + \gamma \Delta T \, u[n-1]$$

(note ~~β contains sign~~ "−" the sign in front of β.)

$$T_M[n] = (1 - \beta \Delta T) T_M[n-1] + \gamma \Delta T \, u[n-1]$$

Standard 1st order D.E. recall $\underbrace{y[n]}_{\text{output}} = \lambda y[n-1] + \gamma \underbrace{u[n-1]}_{\text{input}}$

Two strategies to design control command:

a). Feedback controller. $u[n] = K_p(T_d[n] - T_M[n])$.

b). Feedforward controller (as we mention at the beginning of semester)

Given $T_d[n]$ $\longrightarrow$ calculate $u[n]$ through pre-calibrated curve/lookup table.

$$u[n] = K_{ff}\, T_d[n].$$

In the most general form, we can include both F.B. and F.F. control.

$$u[n] = K_p\,(T_d[n] - T_M[n]) + K_{ff}\, T_d[n].$$

substitute $u[n]$ into $T_M[n]$ expression:

$$T_M[n] = (1 - \beta\Delta T - \gamma K_p \Delta T)\, T_M[n-1] + (\gamma K_p \Delta T + \gamma K_{ff} \Delta T)\, \underbrace{T_d[n-1]}_{\text{Input}} \quad ①$$

Compare with standard form of D.E. $y[n] = \lambda y[n-1] + \gamma^* u[n-1]$.

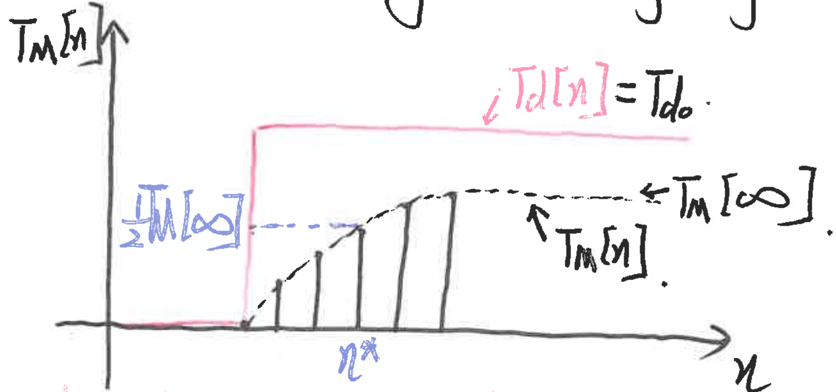
$$\lambda = 1 - \beta\Delta T - \gamma K_p \Delta T \quad \text{natural frequency.}$$

From previous discussion, need $|\lambda| < 1$ to be stable.

Real heater system is stable when no feedback is included ($K_p = 0$), why?

Determine parameters (β and γ) from measurement.

Last week in Lab, you saw step response like below (when K_p is small).



$\longleftarrow$ Fig. 1

How to extract β (dissipation rate) and γ (actuator efficiency)?

Approach 1: Solve Eq. 1, get general form of $T_M[n]$, and map it to Fig. 1.

— A lot of computation and curve fitting work!

Approach 2: Use certain key features in Fig. 1 to directly determine β and γ.

Key features A: steady state value $T_M[\infty]$.

Key feature B: How fast does the curve reach steady state.

Half time n^* $\quad T_M[n^*] = \frac{1}{2} T_M[\infty]$.

Two equations $\longleftrightarrow$ two unknowns: β and γ.

We can determine β and γ by either testing F.F. control or F.B. control (or having both of them on simultaneously).

F.F. control is easier, to be used as an example.

$K_p = 0$, $K_{ff} \neq 0$, —— step response in the absence of F.B. loop.

you'll do this in Lab 1B.

Eq. 1 $\rightarrow$ $$T_M[n] = \underbrace{(1 \overset{-}{\cancel{+}} \beta \Delta T)}_{\lambda} T_M[n-1] + \gamma K_{ff} \Delta T \, T_d[n-1] \quad ②$$

A. steady state: recall the trick: $T_M[\cancel{\infty} n] = T_M[n-1] = T_M[\infty]$.

$$\boxed{T_M[\infty] = \frac{\gamma \Delta T K_{ff}}{1-\lambda} T_{d_0} = + \frac{\gamma}{\beta} K_{ff} T_{d_0}} \quad ③$$

B. Half time to reach steady state.

Z.S.R. of Eq ②: $T_M[n] = \sum_{m=0}^{n-1} \lambda^{n-1-m} \gamma K_{ff} \Delta T\, T_{d_0}$

$$= \frac{1-\lambda^n}{1-\lambda} \gamma K_{ff} \Delta T\, T_{d_0} = (1-\lambda^n)\, T_M[\infty].$$

$$\underbrace{\frac{\gamma K_{ff} \Delta T\, T_{d_0}}{1-\lambda}}_{T_M[\infty]}$$

Half time n^*: $T_M[n^*] = \frac{1}{2} T_M[\infty]$.

$$1-\lambda^{n^*} = \frac{1}{2} \implies \lambda = e^{\frac{1}{n^*}\log\frac{1}{2}}$$

or $1-\beta\Delta T = e^{\frac{1}{n^*}\log\frac{1}{2}}$

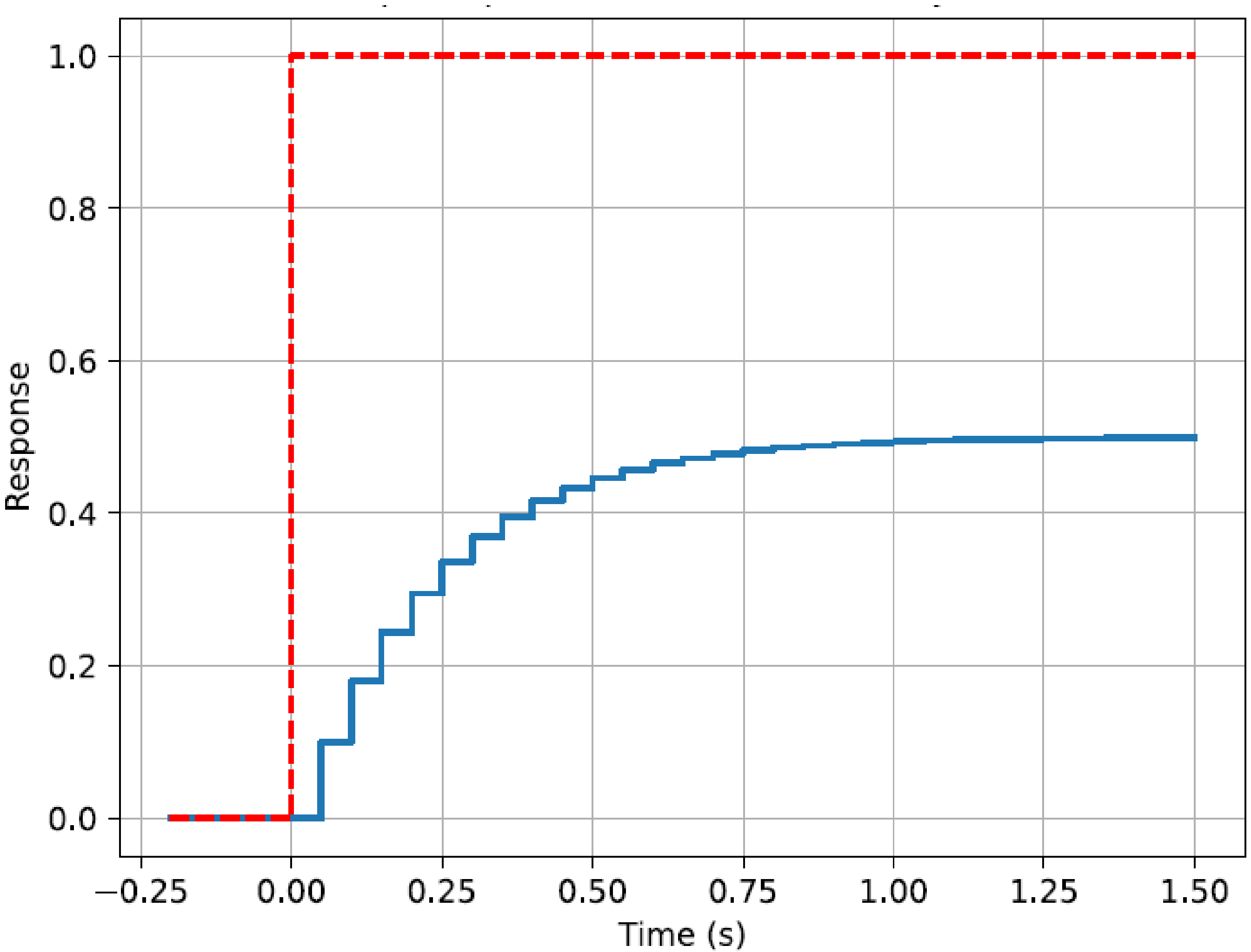
$$\boxed{\beta = -\frac{e^{\frac{1}{n^*}\log\frac{1}{2}} - 1}{\Delta T}} \quad ④.$$

Substitute into Eq. ③: $\boxed{\gamma = +\frac{\beta}{K_{ff}} \frac{T_M[\infty]}{T_{d_0}}}$ ⑤.

what you need for Lab 1B.

Exercise: See handout, determine β and γ. ($\Delta T = 66.6\,\mu\text{sec}$, $K_{ff} = 1$).

Can you estimate β and γ using measured step response from F.B. control? Try it yourself.


1.0
0.8
0.6
0.4
0.2
0.0
Response
−0.25
0.00
0.25
0.50
0.75
1.00
1.25
1.50
Time (s)