3/2/26 (1)

System in D.T.

$$\begin{array}{cc} & \\ N_{eqns} & \begin{bmatrix} X[n] \end{bmatrix} \end{array} = \begin{array}{c} N_{vars} \\ \begin{bmatrix} A(\vec{K}\text{'s}) \end{bmatrix} \end{array} \begin{bmatrix} x[n-1] \end{bmatrix} + \begin{bmatrix} B \end{bmatrix} u[n-1] + \begin{bmatrix} B_d \end{bmatrix} u_d[n-1]$$

K_p
K_d
K_i
Gains

$\text{eig}(A) = \lambda(A) \equiv$ evals $=$ nat freqs
$sp(A) < 1$ for stability $(|\lambda_i| < 1 \;\forall\, i)$

I) A & B matrices easy to construct
II) Model is easy to simulate
III) Can learn some things from structure

i.e. $\quad X[\infty] = \overbrace{(I - A)^{-1}}^{\leftarrow \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}} B\, u[\infty]$ $\quad$ if $u[\infty] = X_d[\infty]$

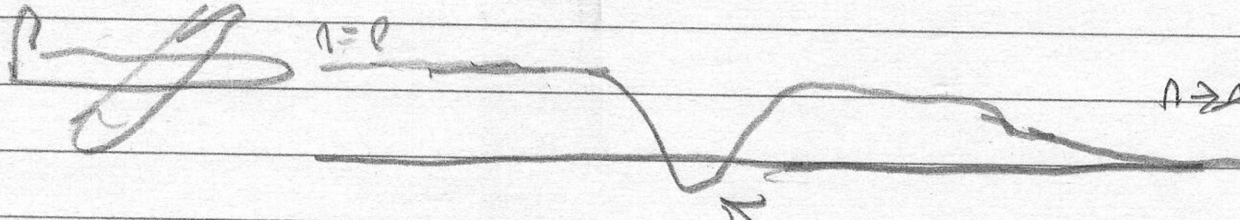
$\Updownarrow$ conflicting

$$X_d[\infty] - X[\infty] = \underbrace{(I - A)^{-1} B_d}_{\text{want to be zero}} u_d[\infty]$$

IV Easy for computer to compute $\lambda(A(\vec{K}))$ versus K's and plot graphs.

But

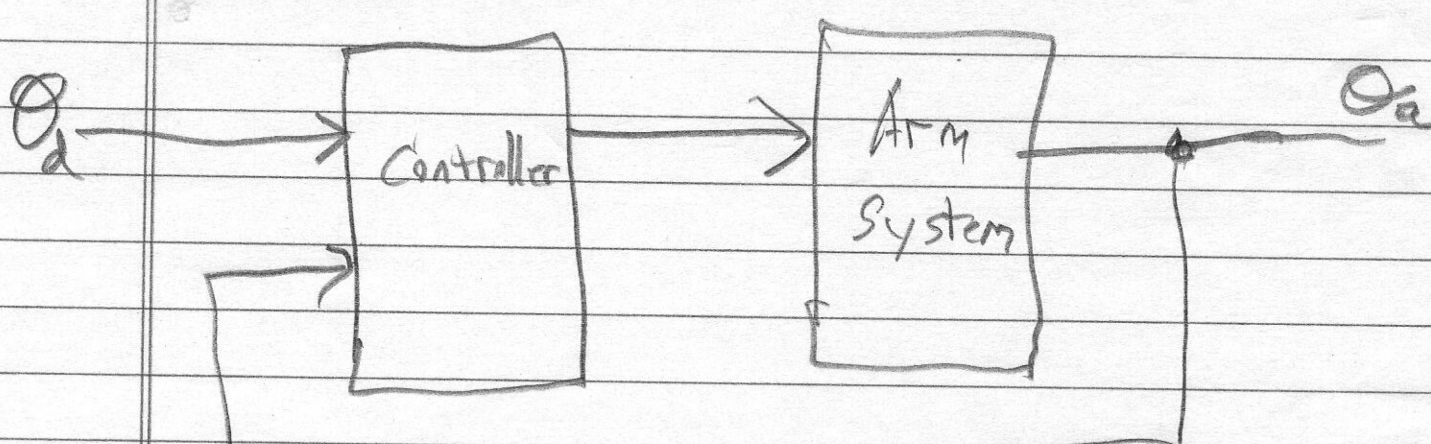
- What about between $n=0$ & $n=\infty$



- P.I.D. was given, how do we invent new approaches

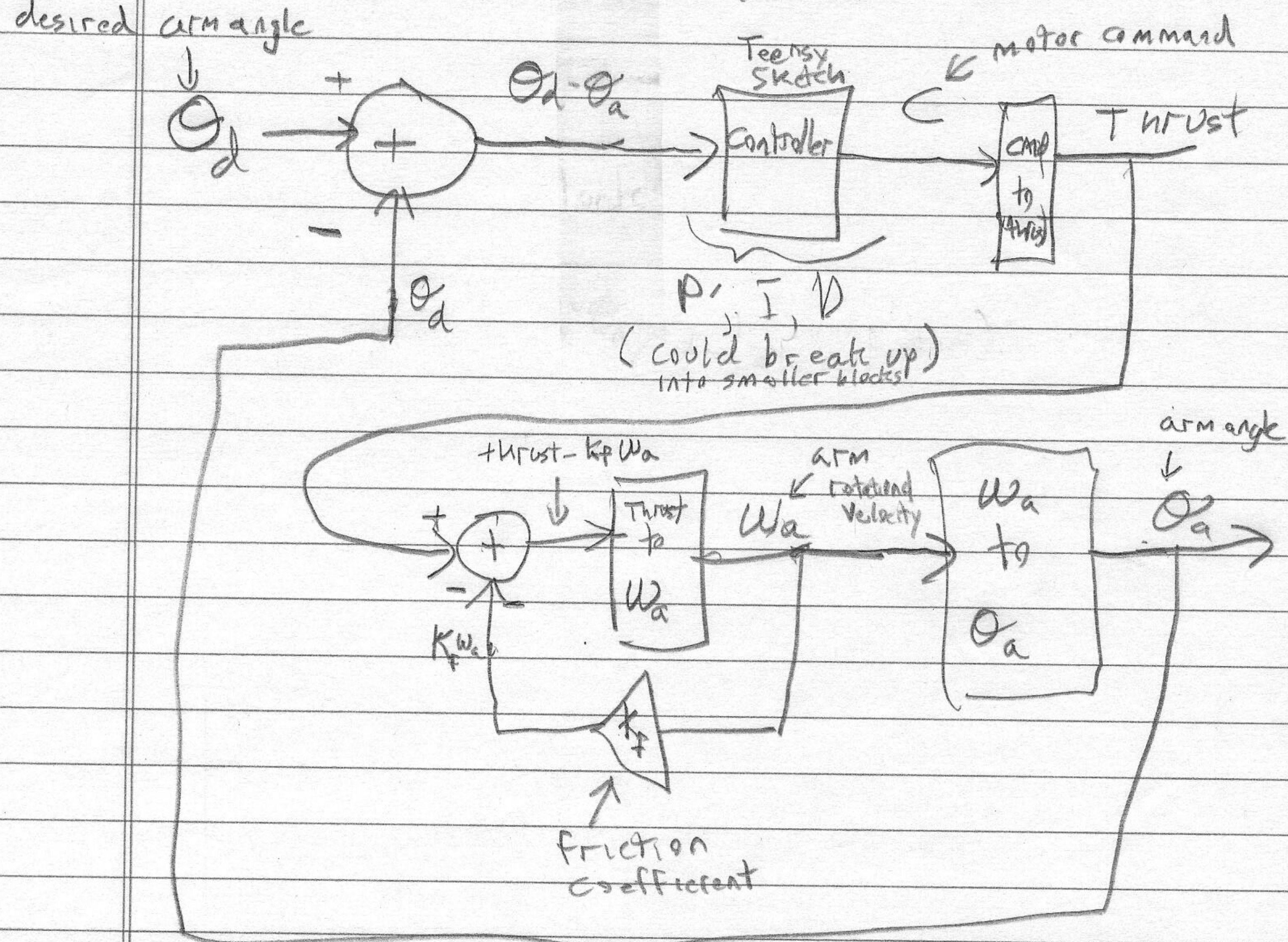
- Matrix approach is all-at-once, how do we build up detail in the model, in disturbance, in control
  PD → PID
  Instant torque → Torque buildup

Example



No info Here

Instead - Block Diagram for Arm



What goes in the Blocks?

E.g. cmd → thrust $\omega_a \to \theta_a$ thrust → ω

Inside Blocks Difference Eqns in each block do not compose easily!

1) How do we compose blocks?

2) How do we characterize the blocks!

zero friction

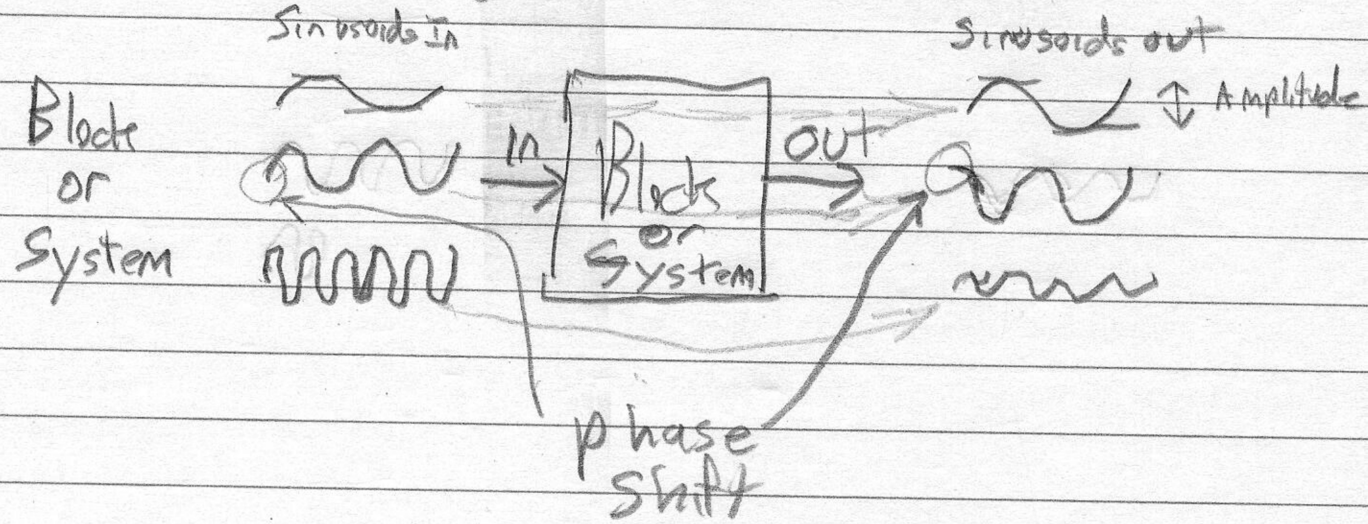
e.g. $\begin{cases} \omega[n] = \omega[n-1] + \Delta T \,\text{thrust}[n-1] \\ \theta[n] = \theta[n-1] + \Delta T \,\omega[n-1] \end{cases}$

Not Obvious!

$\theta[n] = 2\theta[n-1] - \theta[n-2] + (\Delta T)^2 \,\text{thrust}[n-1]$

We Will Characterize Blocks & System with

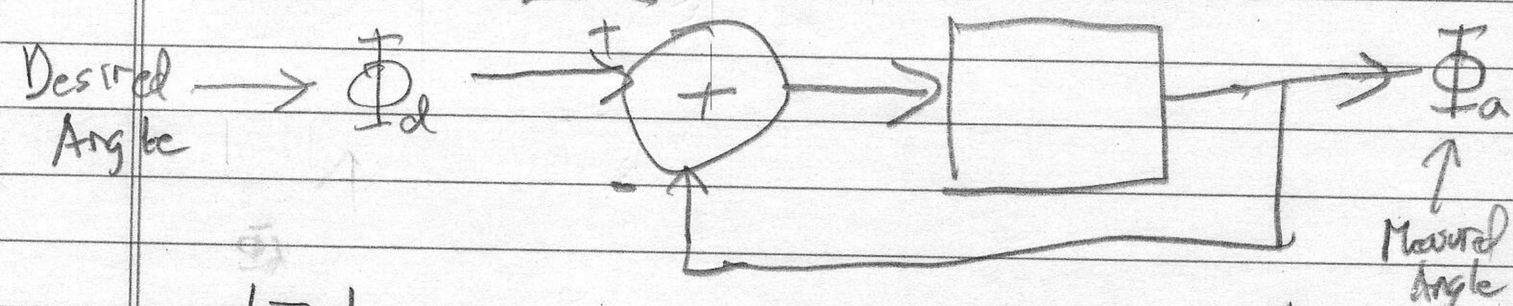
Frequency Response



For Linear Time Invariant systems: Input freq = Output freq

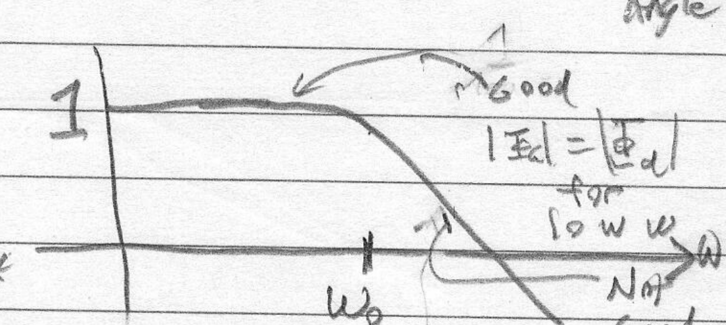
What does Freq Response Tell us

Control System



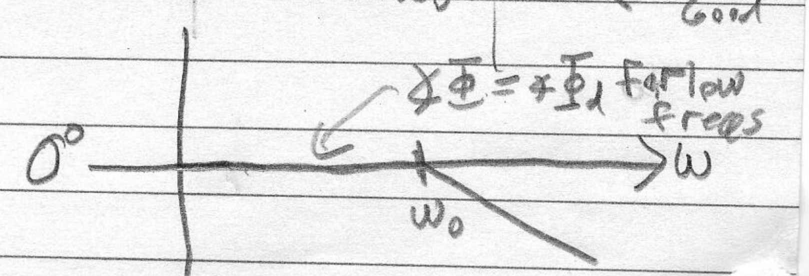
$\dfrac{\text{Output Amplitude}}{\text{Input amplitude}} = \dfrac{|\Phi_a|}{|\Phi_d|}$ versus frequency ($\omega = 2\pi f$)

radians/sec, cycles/sec



$\angle\Phi_a - \angle\Phi_d$

phase shift



Important Analysis Techniques

1) Complex Exponentials

$$e^{st} = e^{(\sigma + j\omega)t} = e^{\sigma t}\underbrace{(\cos\omega t + j\sin\omega t)}_{e^{j\omega t}}$$

$$s = \underset{Re(s)}{\sigma} + \underset{Im(s)}{j\omega}$$

complex conjugate

$$\frac{1}{2}\left(\underbrace{(e^{st})}_{e^{\sigma t}e^{j\omega t}} + \underbrace{(e^{st})^{*}}_{e^{\sigma t}e^{-j\omega t}}\right) = \frac{1}{2}e^{\sigma t}(\cos\omega t + j\sin\omega t) + \frac{1}{2}e^{\sigma t}(\cos\omega t - j\sin\omega t) = e^{\sigma t}\cos\omega t$$

notice imaginary term

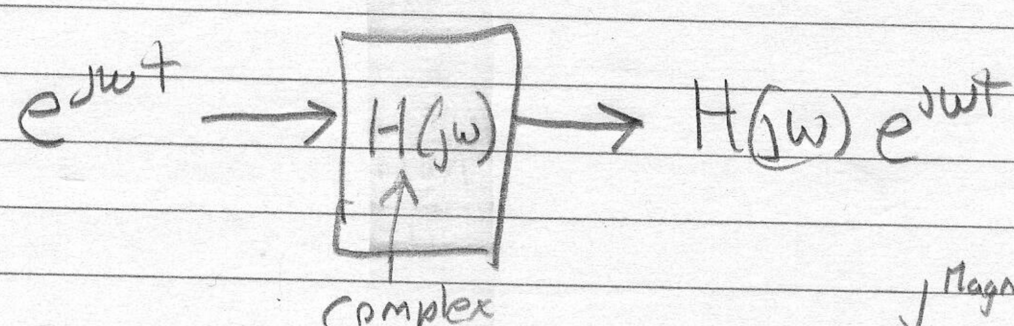
2) Derivatives of e^{st}

$$\frac{d}{dt}e^{st} = s e^{st} \Rightarrow \frac{d}{dt}e^{j\omega t} = j\omega e^{j\omega t}$$

Compare to a <u>difference</u> of e^{st}

$$\frac{d}{dt}e^{st} \simeq \underbrace{\frac{e^{s(n\Delta T)} - e^{s((n-1)\Delta T)}}{\Delta T}}_{\text{Discrete time approximation}} = \underline{\text{Messy}}!$$

3) Description of Frequency Response



<u>Use Polar Form:</u> $H(j\omega) = \overset{\text{Magnitude}}{M(j\omega)}\, e^{j\overset{\text{Phase}}{\phi(j\omega)}}$

$$H(j\omega)e^{j\omega t} = \underbrace{M(j\omega)}_{\substack{\text{Magnitude} \\ \text{(Amplitude)} \\ \text{changes}}}\ \underbrace{e^{j\phi(j\omega)}\ e^{j\omega t}}$$

$$= e^{j(\omega t + \phi(j\omega))}$$

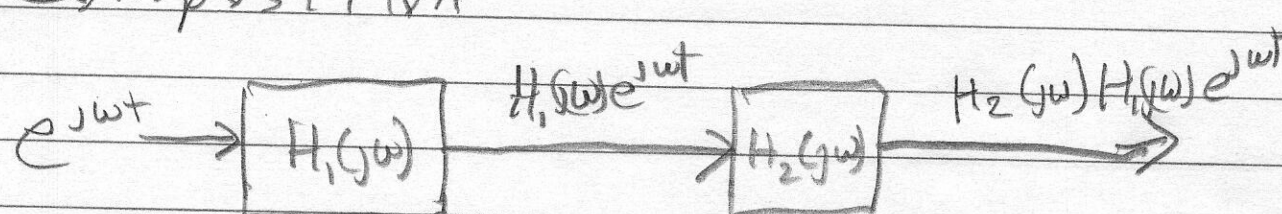
$$= \cos(\omega t + \phi(j\omega)) - j\sin(\omega t + \phi(j\omega))$$

<u>Frequency Response Representation</u>

Complex Number with frequency dependent Mag & Phase

$$H(j\omega) = M(j\omega)e^{j\phi(j\omega)}$$

4) Composition



<u>Easy!</u>