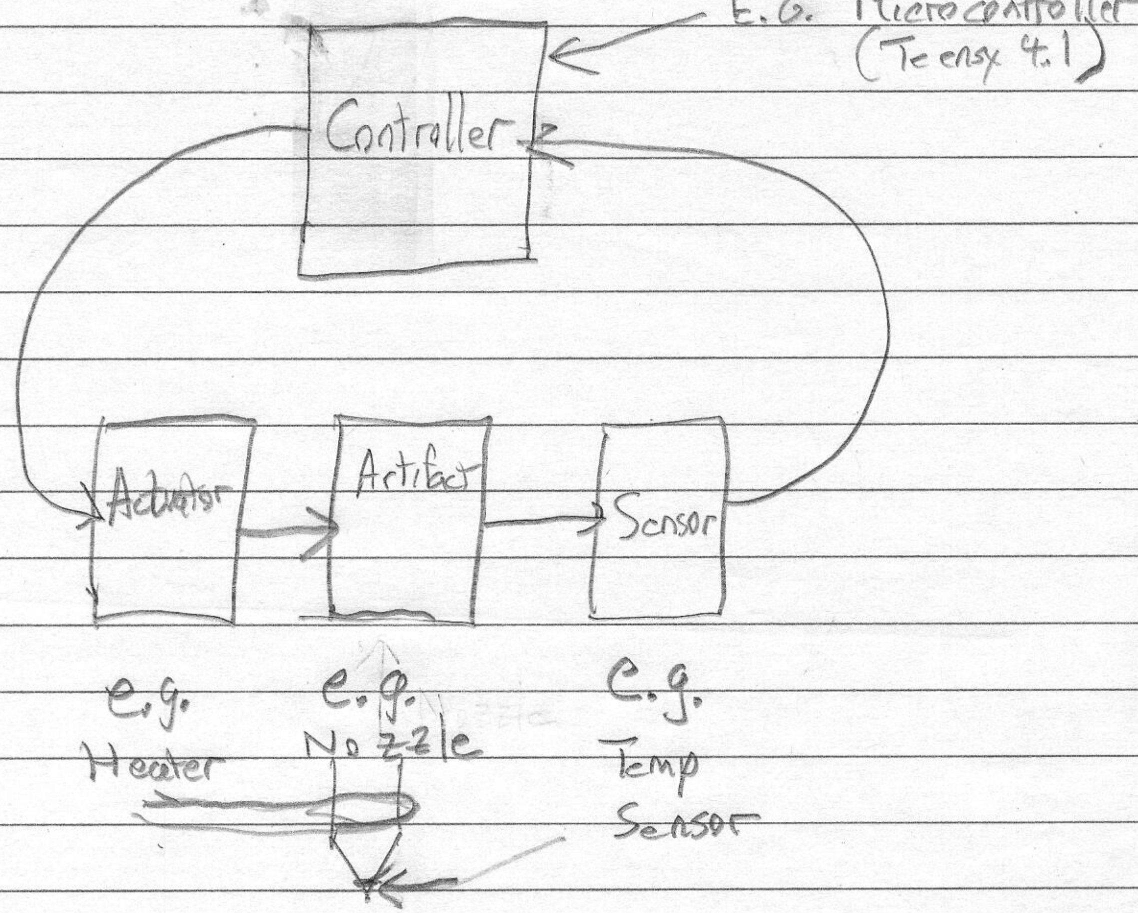
6.3100/2 2/4/26

Generic Control System



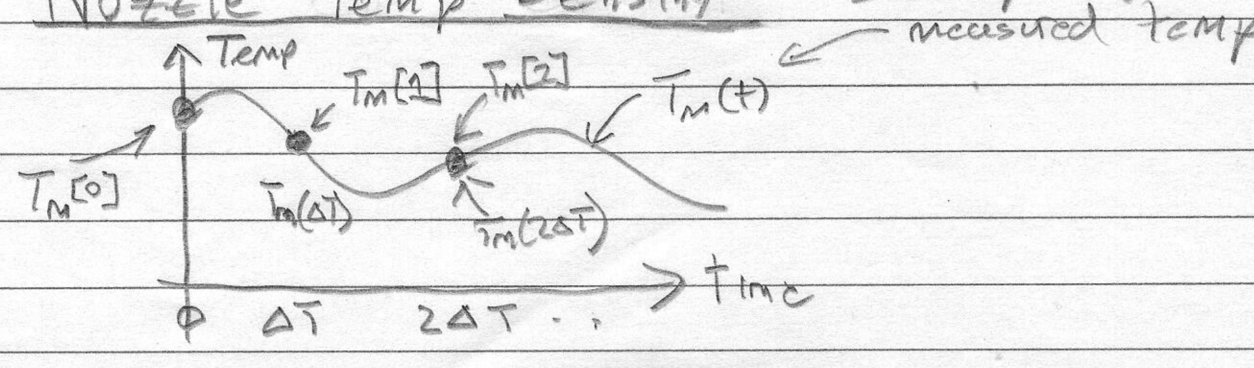
Clocked System

Every ΔT:

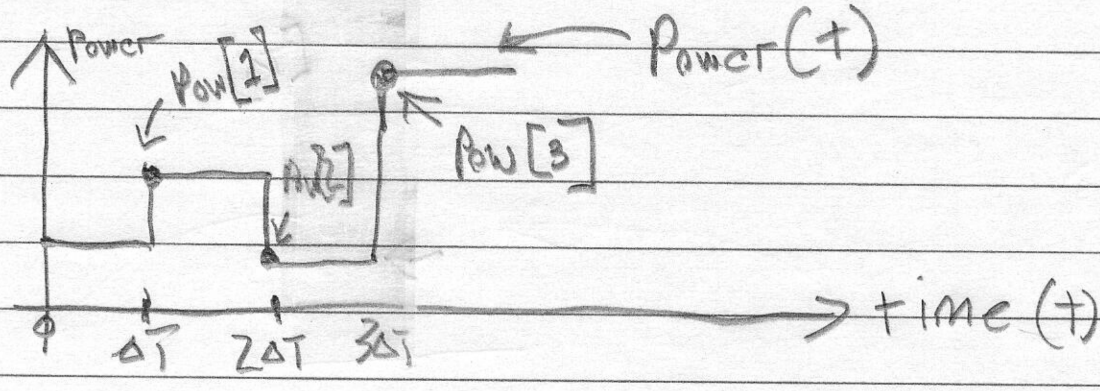
Read Senor
Compute Actuator Upate
Update the actuator

Assume
Calculation
Happens
Instantly

Nozzle Temp Sensing — Sampling



Nozzle Heater (actuator)



Control

Proportional Control (Simplest!)

$$Pow[n] = \underbrace{K_p}_{\text{Proportional Gain}} \left(T_d[n] - \underbrace{T_m[n]}_{\text{Read sensor}} \right)$$

Updated power

compute actuator update

Model

$$T_m[n] = T_m[n-1] + \Delta T \, \gamma_{th} \, P_{ow}[n-1]$$

$T_m[n]$: nozzle temp at $n\Delta T$

$T_m[n-1]$: nozzle temp at $(n-1)\Delta T$

γ_{th}: summarizes many physical parameters

or

$$\frac{dT}{dt} \simeq \frac{T_m[n] - T_m[n-1]}{\Delta T} = \gamma_{th} \, P_{ow}[n-1]$$

Combining Model and Control

$$T_m[n] = T_m[n-1] + \Delta T \gamma_{th} K_p \left(T_d[n-1] - T_m[n-1] \right)$$

$$\underbrace{T_m[n] = \left(1 - \Delta T \gamma_{th} K_p\right) T_m[n-1] + \Delta T \gamma_{th} K_p T_d[n-1]}_{\text{Linear } 1^{st} \text{ Order Difference Eqn.}}$$

General 1^{st}-Order Linear Diff Eqn

"Input" (Usually known): $u[0], u[1], \ldots$

"Output" (to be computed): $y[0], y[1]$

initial condition

LFODE

$$\forall n \qquad y[n] = \lambda\, y[n-1] + \gamma\, u[n-1]$$

$n=1 \qquad y[1] = \lambda y[0] + \gamma u[0]$

$n=2 \qquad y[2] = \lambda y[1] + \gamma u[1] = \lambda(\lambda y[0] + \gamma u[0]) + \gamma u[1]$

$$= \lambda^2 y[0] + \lambda \gamma u[0] + \gamma u[1]$$

$\vdots$

For arb n $\qquad y[n] = \lambda^n y[0] + \gamma \sum_{m=0}^{n} \lambda^{(n-m-1)} u[m]$

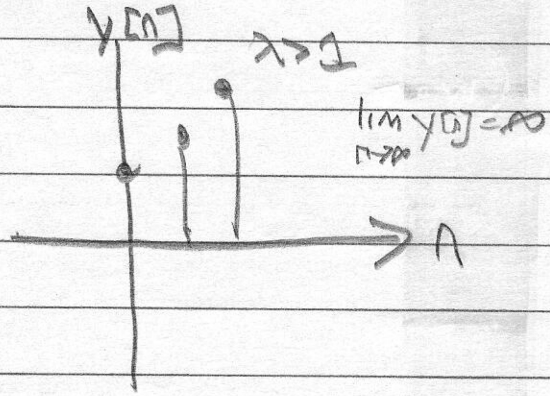
If $u[n]=0\ \forall n$ (ZIR): $\qquad y[n] = \lambda^n y[0]$

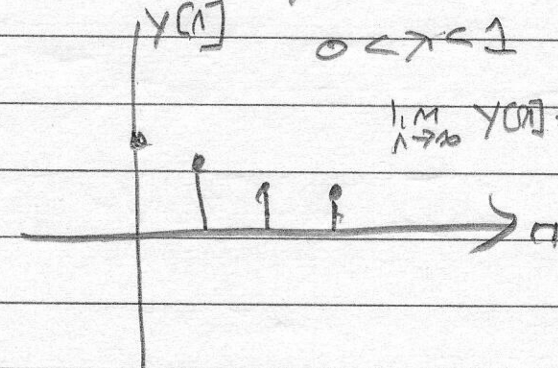
zero input response

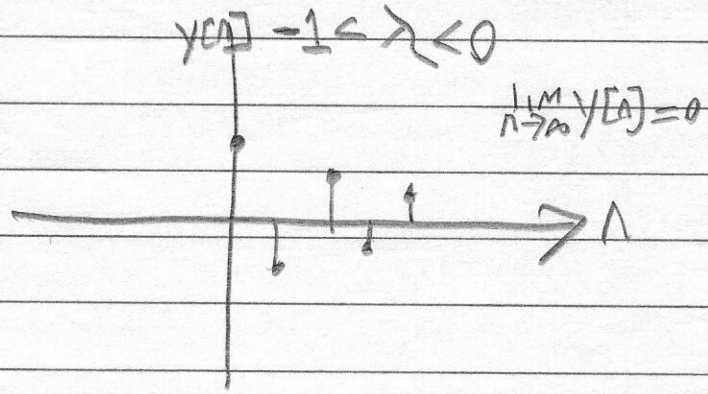
If $y[0]=0$ (ZSR): $\qquad y[n] = \gamma \sum_{m=0}^{n-1} \lambda^{(n-m-1)} u[m]$

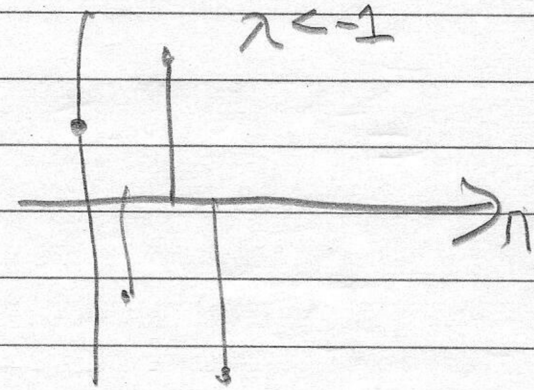
state

ZIR $\quad y[n] = \lambda y[n-1] = \lambda^n y[0]$









Steady-state response

if $u[n] = u_0 \;\forall n$ (Constant input)

&

$|\lambda| < 1$

then

$$\lim_{n\to\infty} y[n] = \lim_{n\to\infty} \underbrace{\lambda^n y[0]}_{\to 0} + \lim_{n\to\infty} \underbrace{\gamma \sum_{m=0}^{n-1} \lambda^{(n-m)-1} u_0}_{\text{sum}}$$

$$y[\infty] = \frac{\gamma}{1-\lambda} u_0$$

Easier if $y[\infty]$ exists then

$$y[n] = \lambda y[n-1] + \gamma u_0$$

$$y[\infty] = \lambda y[\infty] + \gamma u_0$$

$$y[\infty] = \frac{\gamma}{1-\lambda} u_0$$

For Nozzle Example

$$\lambda = (1 - \Delta T \gamma_{th} K_p) \qquad \gamma = \Delta T K_p \gamma_{th}$$

So $|\lambda| < 1$ iff $0 < \Delta T \gamma_{th} K_p < 2$

And if $0 < \Delta T \gamma_{th} K_p < 1$ $0 < \lambda < 1$

(No oscillations)

If $T_m[\infty]$ exists $(|1 - \Delta T \gamma K_p| < 1,\ T_d[n] = T_{d_0})$

$$T_m[\infty] = \frac{\Delta T \gamma_{th} K_p}{1 - (1 - \Delta T \gamma_{th} K_p)} T_{d_0}$$

$$= 1 \cdot T_{d_0} = T_{d_0}$$

So Using any $0 < K_p < \frac{2}{\Delta T \gamma_{th}}$

Steady-state temp matches desired temp!

&

$$K_p (T[\infty] - T_{d_0}) = 0 = pow[\infty]$$

Is that right?

Include Heat Loss

$$T_m[n] = T_m[n-1] + \Delta T\,\gamma_{th}\,\overbrace{P_{ew}[n-1]}^{K_p(T_d[n-1]-T_m[n-1])}$$

$$- \Delta T\,\underbrace{\beta\,T_m[n-1]}_{\text{Heat loss}}$$

$$T_m[n] = \underbrace{\left(1 - \Delta T\left(\gamma_{th} K_p + \beta\right)\right)}_{\lambda} T_m[n-1]$$

$$+ \Delta T\,\gamma_{th} K_p\,T_d[n-1]$$

If $|\lambda| < 1$ & $T_d[n] = T_{d0}$

$$T_m[\infty] = \left(1 - \Delta T\left(\gamma_{th} K_p + \beta\right)\right) T_m[\infty]$$

$$+ \Delta T\,\gamma_{th} K_p\,T_{d0}$$

$$T_m[\infty] = \frac{\cancel{\Delta T}\,\gamma_{th} K_p}{\cancel{\Delta T}\left(\gamma_{th} K_p + \beta\right)}\,T_{d0}$$

$$= \frac{1}{1 + \frac{\beta}{\gamma_{th} K_p}}\,T_{d0}$$

$$\simeq 1 \text{ if } K_p \gamma_{th} \gg \beta$$

Constraints

↑ Bigger K_p → Better match

But

$\Delta T \gamma_{th} K_p > 2$

Unstable

If ΔT is small K_p → larger