6.3100 3/30/26 ①

Controller Design So Far

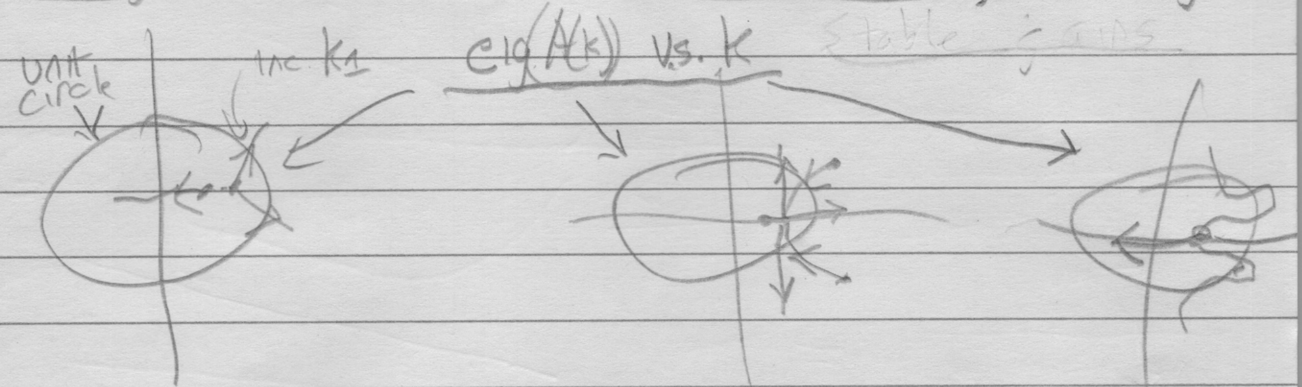
$\vec{K} = \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix}$ $k_1 \leftarrow K_d$, $k_2 \leftarrow K_i$, $k_3 \leftarrow K_p$

In D.T.

$$\vec{x}[n] = \underbrace{A(\vec{K})}_{\text{Depends on controller}} x[n-1] + B(\vec{K}) x_d[n-1] + B_d(K) x_{dist}[n-1]$$

Design with 1-Parameter loci plots, find stable gains

Won't Scale to More Gains



K_1 varies, K_2, K_3 fixed K_3, K_1 fixed, K_2 varies K_3 varies, K_1, K_2 fixed

Ad-Hoc approach for disturbance rej.: e.g. increase K_p

Steady-State accuracy improvement: use K_i (Add state)

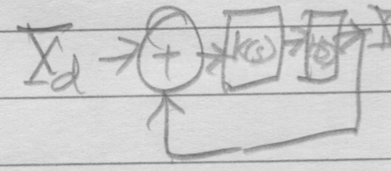
$$\underbrace{(I + A(\vec{K}))^{-1} B(K)}_{\text{relates } X_d[\infty] \text{ to } X[\infty]} = \begin{bmatrix} 1 \\ \epsilon \\ \epsilon \end{bmatrix}$$

Want $X_{d_1}[\infty] = X_1[\infty]$

In C.T.

Block diagrams & Transfer functions

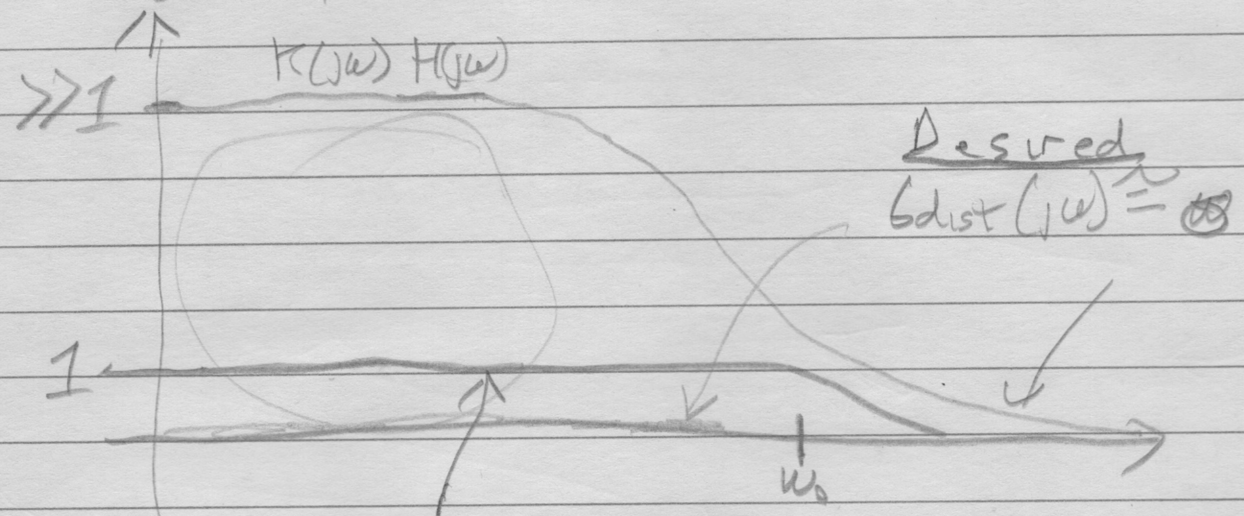
$$G(s) = \frac{K(s)\,H(s)}{1 + K(s)H(s)}$$



$$G_{dist}(s) = \frac{1,\ H(s),\ K(s)}{1 + K(s)H(s)}$$

Design For Stability: $Re(\text{roots of } (1+K(s)H(s))) < 0$

Frequency-response & Sino Steady State



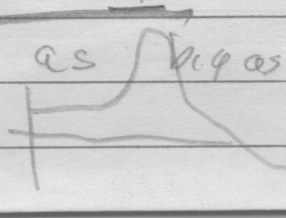
Desired: $G(j\omega) = 1$ for $\omega < \omega_0$

Designed $K(j\omega)$, so

- $K(j\omega)H(j\omega) >> 1 \quad \omega < \omega_0$
  - Good disturb reject
  - Good Tracking

Maybe Stable?

- Avoid $K(j\omega)H(j\omega) = -1$
  - Phase Margin: as big as possible
  - Avoids $G(j\omega)$, $G_{dist}(j\omega)$

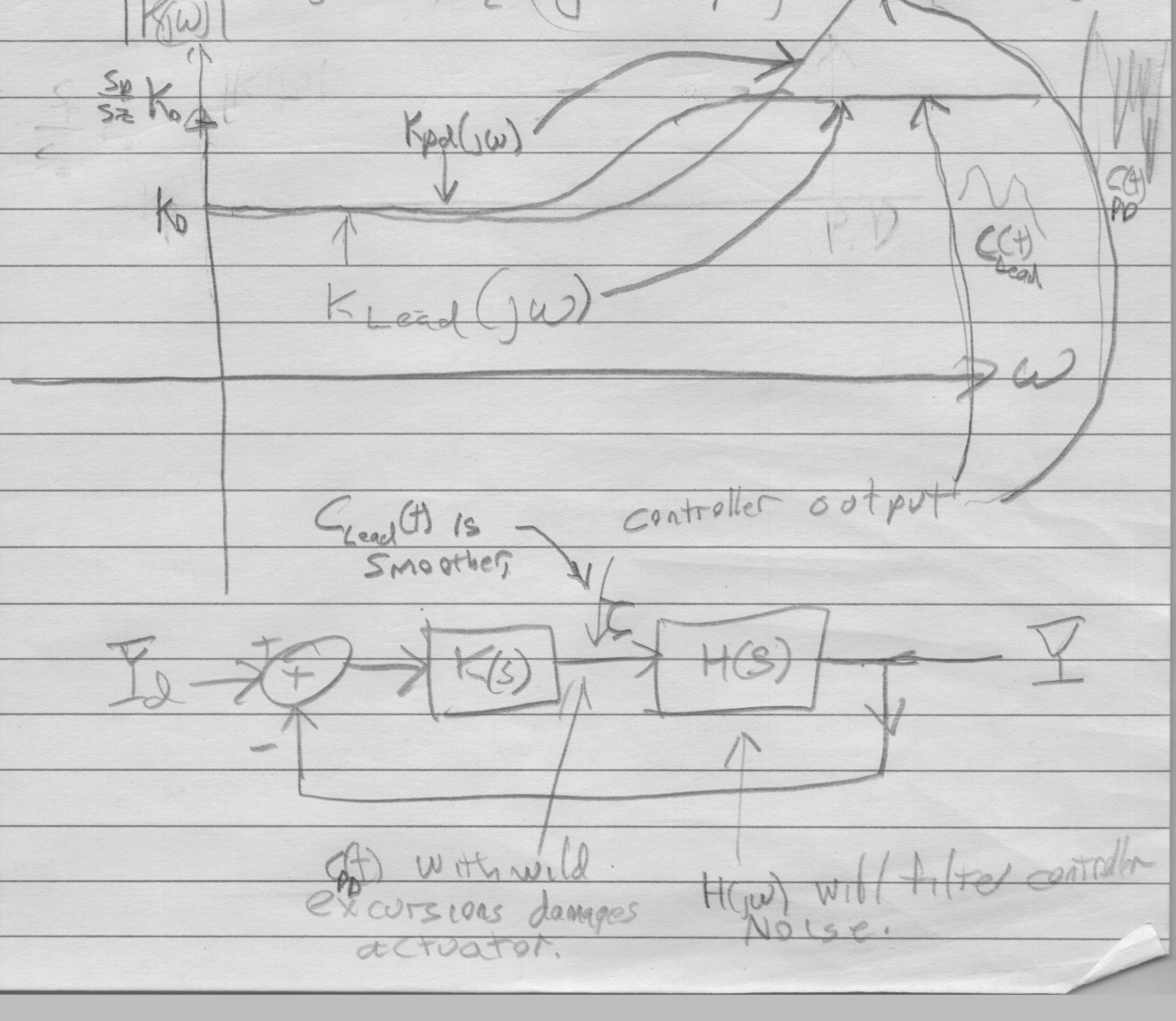


Pole location, Phase Margin, etc are proxies for our real performance metrics, and we have almost no way of including costs.

Not so obvious what metrics should be.

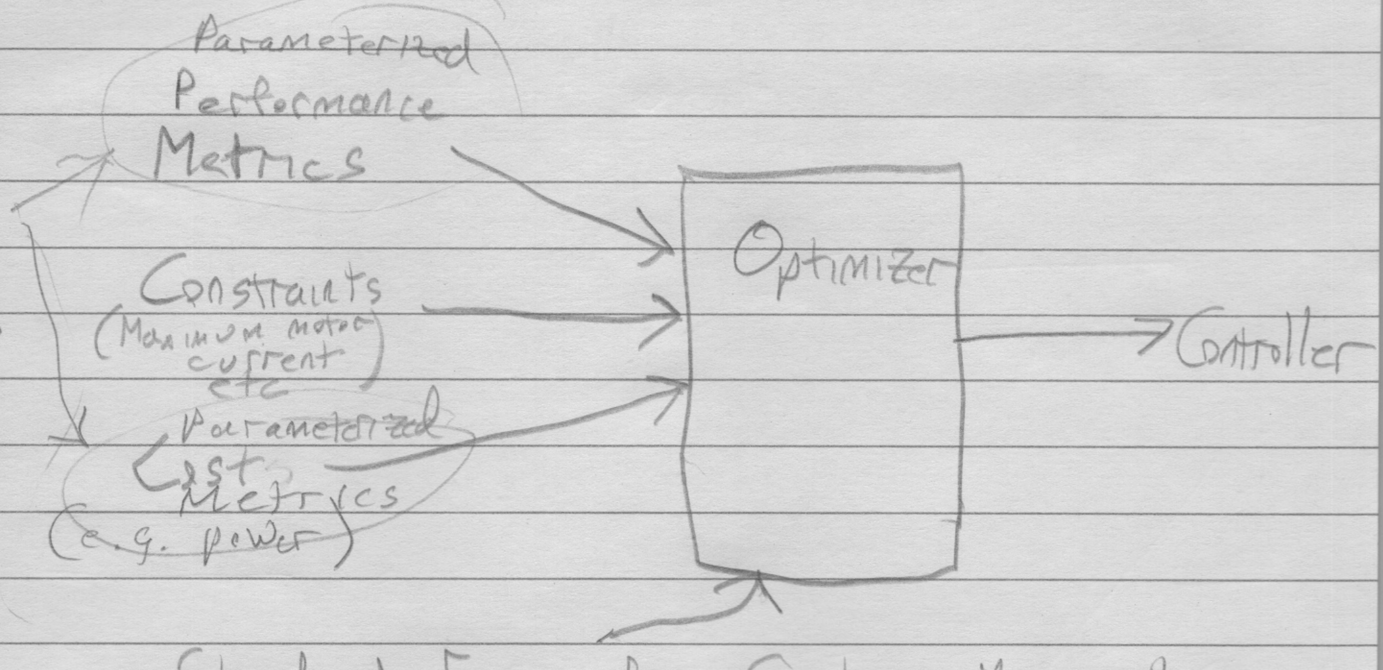
Example Replacing PD with Lead addresses cost

$$K(j\omega) = K_p \frac{s_p}{s_z}\left(\frac{j\omega + s_z}{j\omega + s_p}\right) \text{ vs. } K_p + K_d j\omega$$



What we would Like

What are Good Metrics?



Standard Form for System Description

Standard Form for CT S.I.S.O.

$$\frac{d}{dt}\vec{X}(t) = A\vec{X}(t) + B\,u(t) \qquad \text{System}$$

$$y(t) = [c_1 \ \dots \ c_N] \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix}$$

(A is $N \times N$, B is $N \times 1$)

Feed Back Control

$$u(t) = K_r\, y_d(t) - [K_1 \ K_2 \ K_3 \ K_N] \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix}$$

$$= K_r\, y_d(t) - \vec{K}\vec{X}$$

$$\left.\begin{aligned} \dot{\vec{X}} &= (A - BK)\vec{X} + B K_r y_d \\ y &= CX + D y_d \end{aligned}\right\} A, B, C, D \text{ Defines a System \& controller}$$

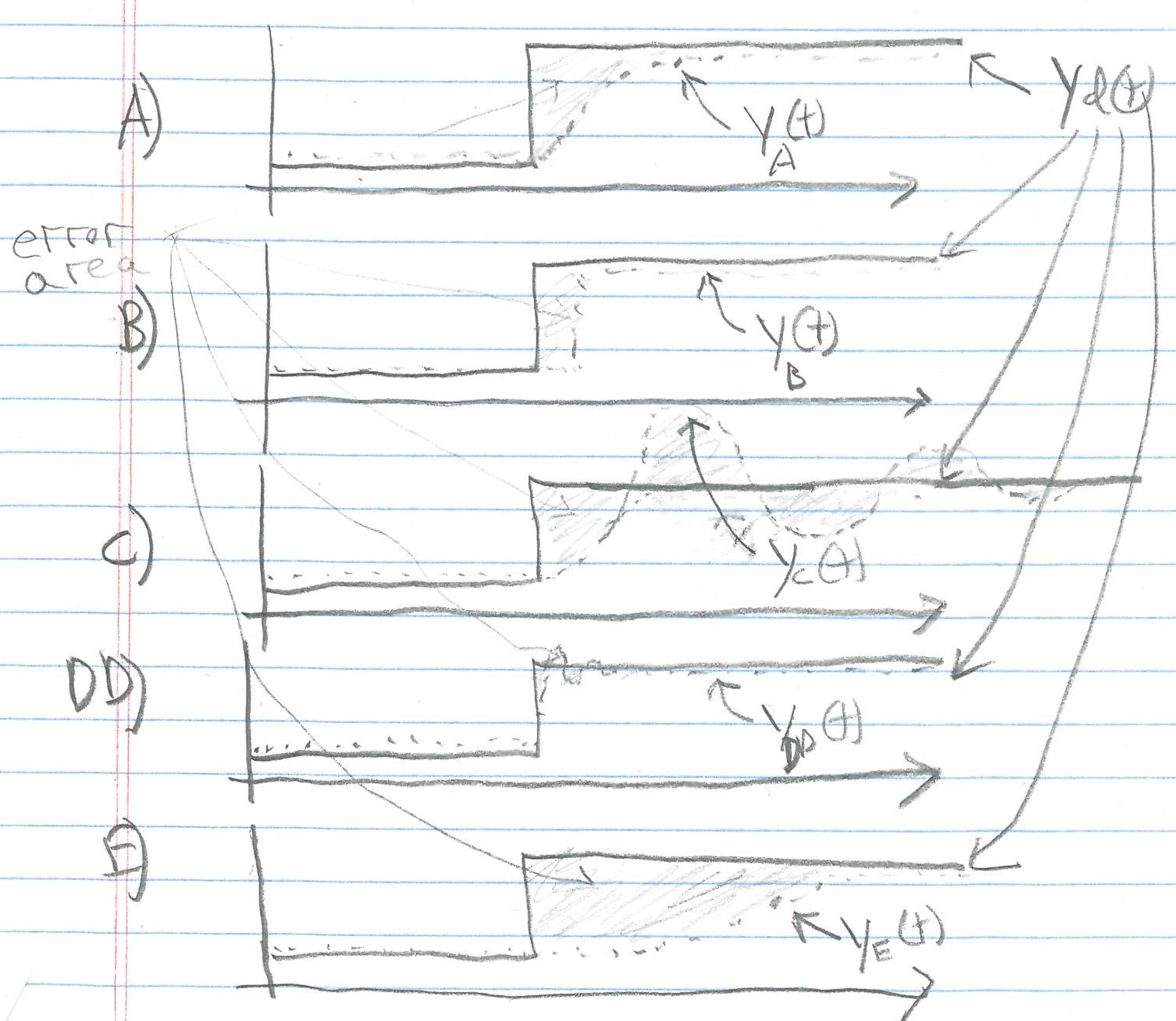
Depends

Not every possible controller
"Integral?" (unless we add states)

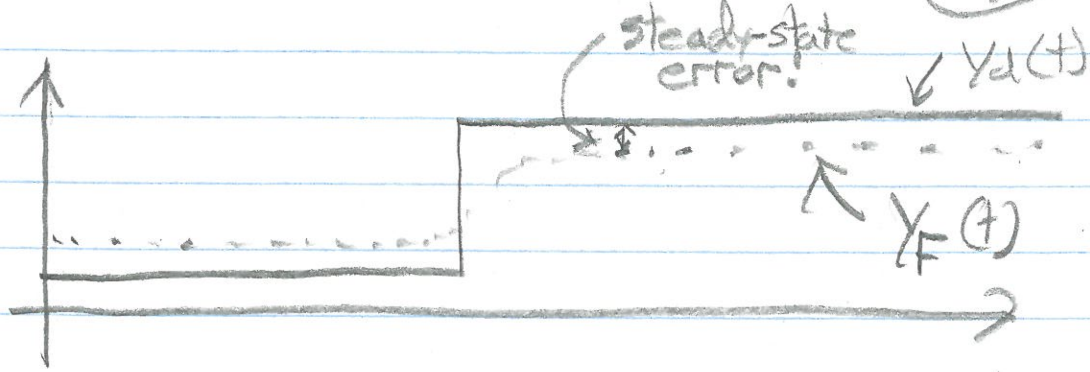
Phase Margin, Pole location, etc are <u>indirect</u> measures of tracking error

Direct Tracking Metric?

Example Closed-Loop Step Response



F)



Candidate Metrics Controller Should Minimize

1) $$\max_{\tau \in [0,\infty)} \left| y_d(\tau) - y(\tau) \right|$$

<u>Bad Choice</u>: Same value for Case A, B, C, D, E, and smaller for Case F

2) $$\int_0^\infty \left(y_d(\tau) - y(\tau) \right) d\tau$$ Integral of error

<u>Bad Choice</u>: Value is small for case "C", because positive and negative areas cancel

<u>But</u>: Is infinite in case F, because of steady-state error!

3) $$\int_0^\infty (y_d(\tau) - y(\tau))^2 \, d\tau$$

Good Choice:

Small for A, B, DD
Large for C, E,
Infinite for F!

Differentiable: (so better than $\int |y_d(\tau) - y(\tau)| d\tau$)

But: Can still be small when there is overshoot (Case DD)

If we like the integral of the squared error metric

$$\int_0^\infty \left(y_d(\tilde{\tau}) - y(\tilde{\tau})\right)^2 d\tilde{\tau} = \text{integral of squared error}$$

Can we design a controller, $K(s)$, that minimizes the integral of the squared error?

Not by hand, but using computational tools, yes.